Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

1-Algebraic-functions/1.2-Trinomial-products/1.2.2-Quartic/43-1.2.2.6-P-x-d-x-^m-a+b-x^2+c-x^4-^p

Nasser M. Abbasi

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [145]. This is test number [43].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (145)	0.00 (0)
Mathematica	100.00 (145)	0.00 (0)
Maple	98.62 (143)	1.38 (2)
Mupad	98.62 (143)	1.38 (2)
Giac	98.62 (143)	1.38 (2)
Fricas	86.21 (125)	13.79 (20)
Sympy	55.17 (80)	44.83 (65)
Maxima	50.34 (73)	49.66 (72)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

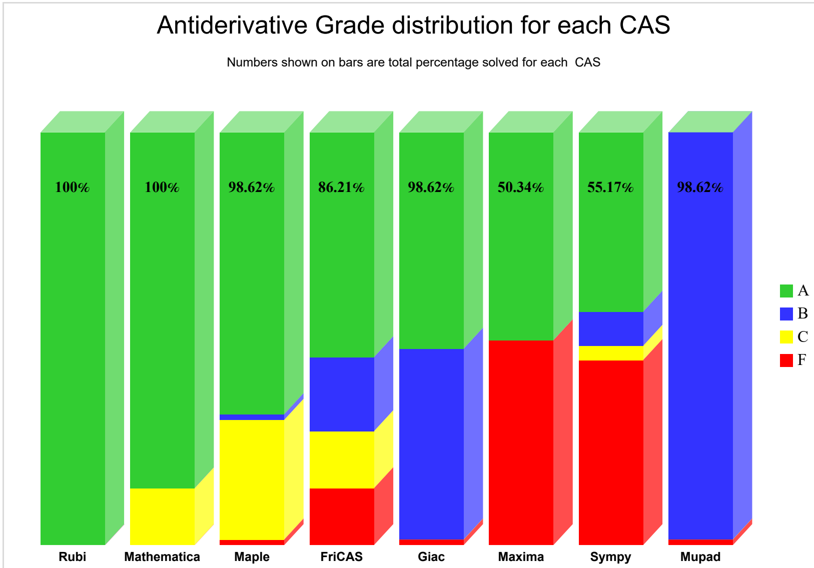
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

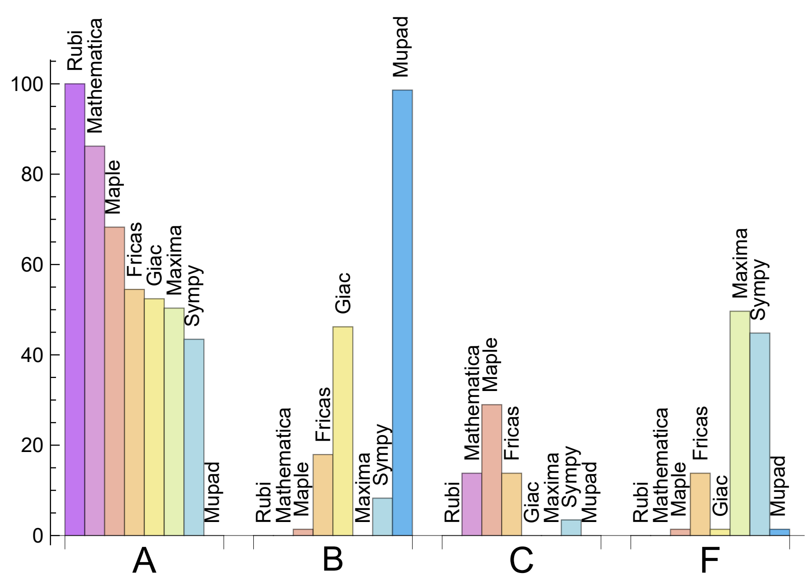
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	86.207	0.000	13.793	0.000
Maple	68.276	1.379	28.966	1.379
Fricas	54.483	17.931	13.793	13.793
Giac	52.414	46.207	0.000	1.379
Maxima	50.345	0.000	0.000	49.655
Sympy	43.448	8.276	3.448	44.828
Mupad	0.000	98.621	0.000	1.379

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	2	100.00	0.00	0.00
Mupad	2	0.00	100.00	0.00
Giac	2	100.00	0.00	0.00
Fricas	20	10.00	90.00	0.00
Sympy	65	1.54	98.46	0.00
Maxima	72	77.78	0.00	22.22

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maple	0.15
Maxima	0.24
Mathematica	0.31
Rubi	0.74
Giac	0.74
Sympy	3.73
Mupad	6.56
Fricas	10.67

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	105.48	0.95	65.00	0.89
Mathematica	202.84	0.97	139.00	1.00
Maple	202.96	0.94	101.00	0.83
Rubi	208.86	1.03	186.00	1.01
Sympy	1048.58	3.99	71.00	0.98
Giac	1770.90	5.37	228.00	1.20
Mupad	4593.26	13.26	176.00	0.96
Fricas	31130.94	122.35	143.00	1.27

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

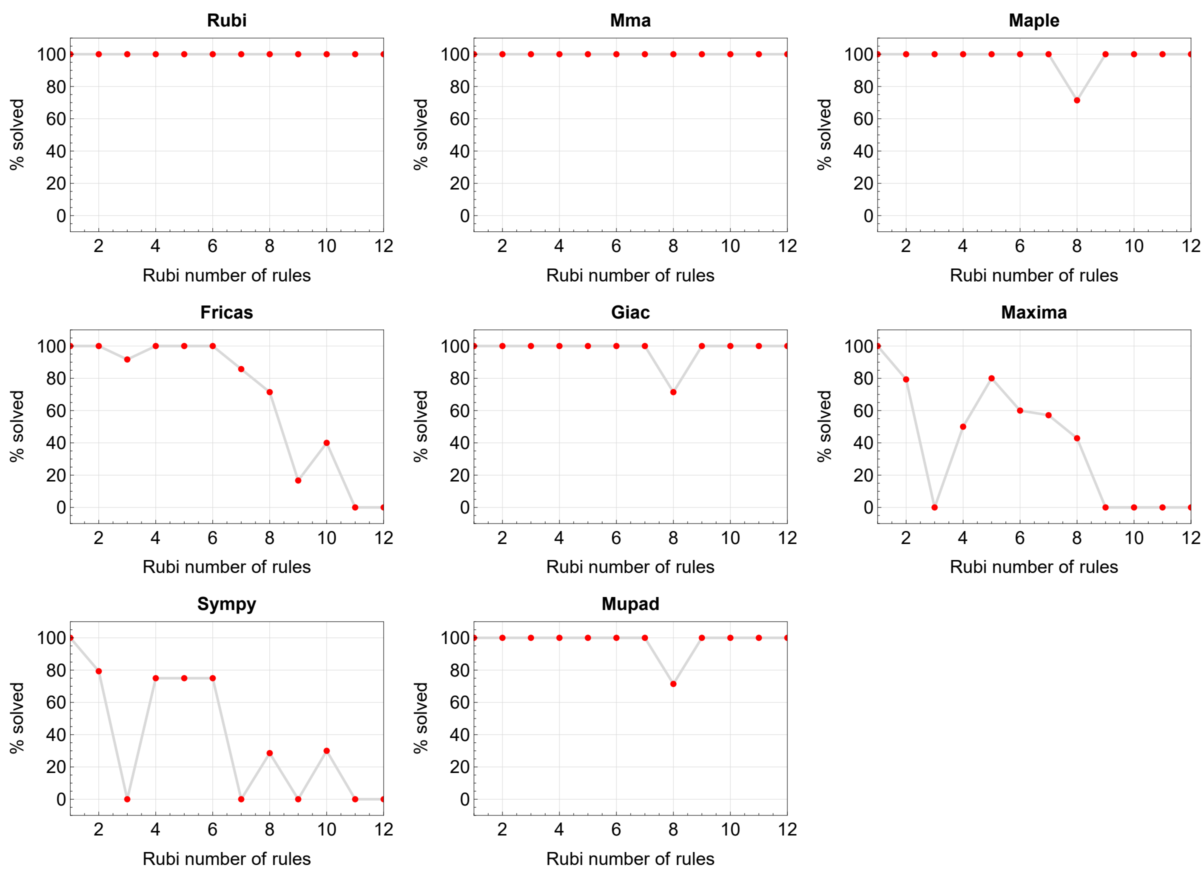


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

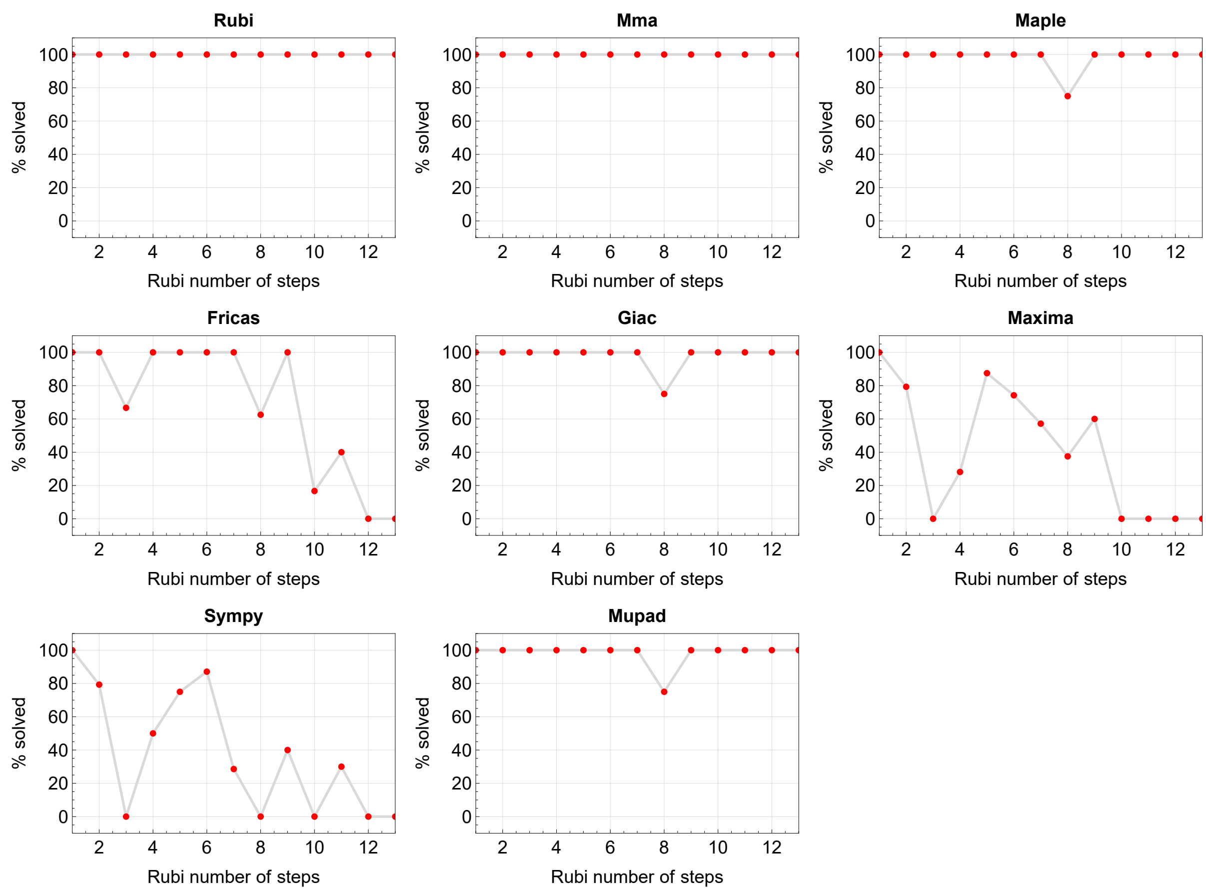


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

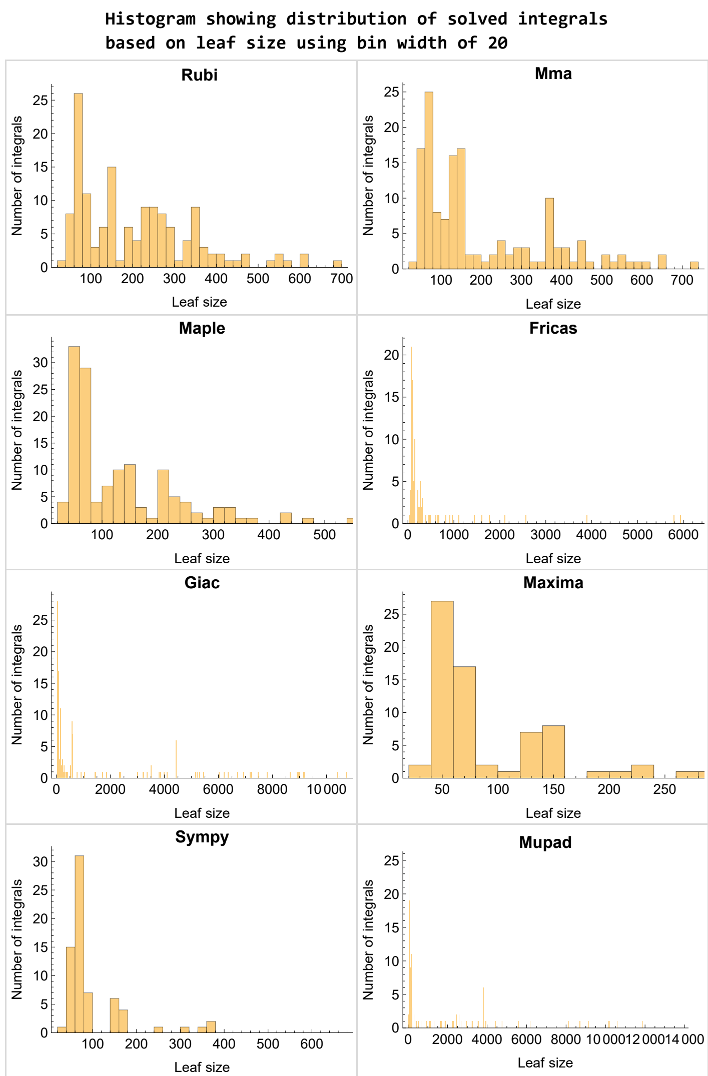


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

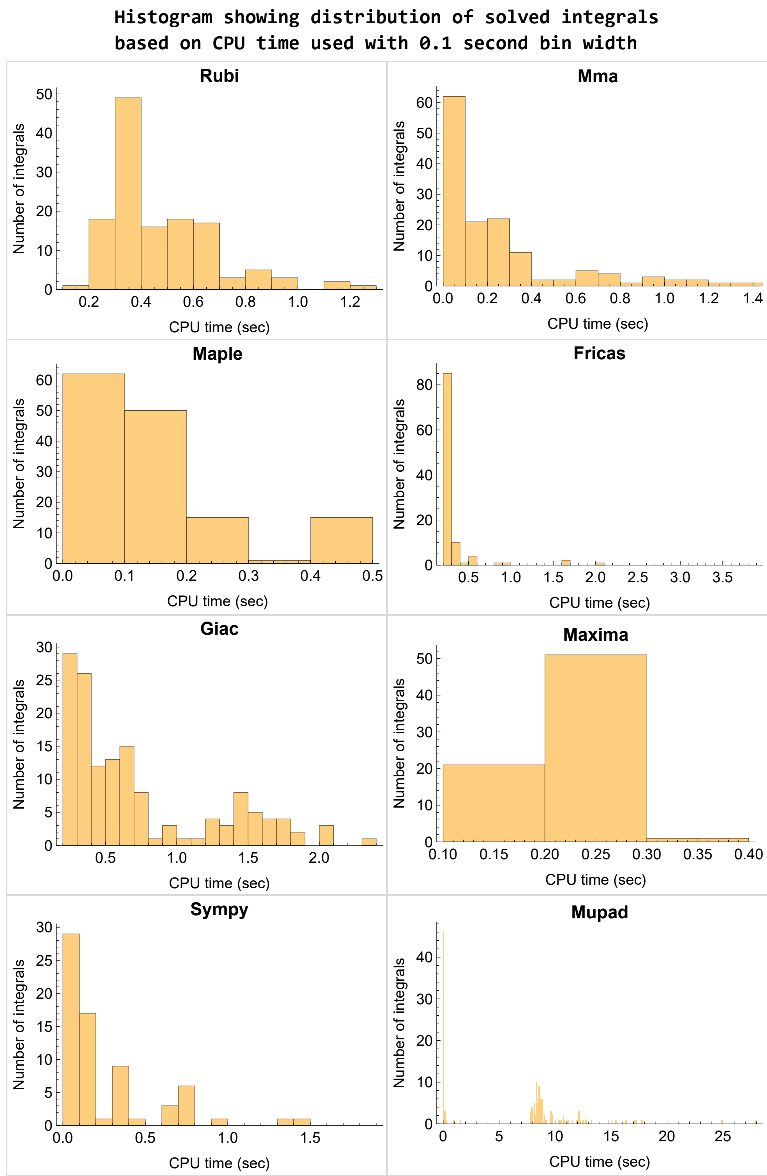


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

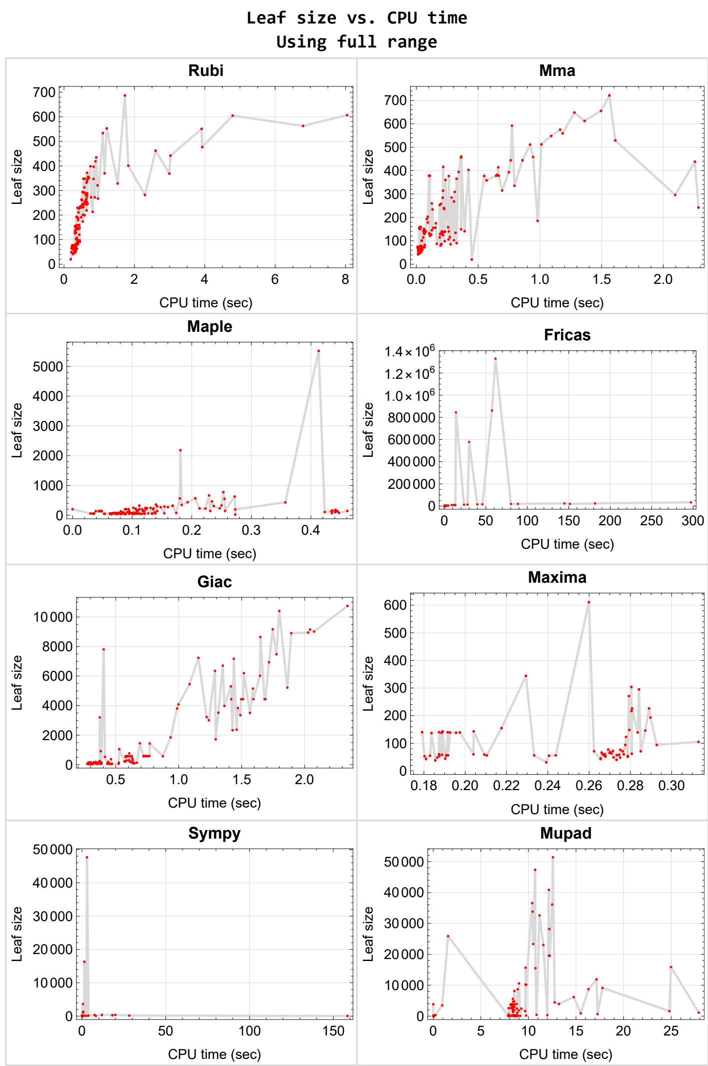


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{135, 136, 137, 138\}$

Mathematica $\{40\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

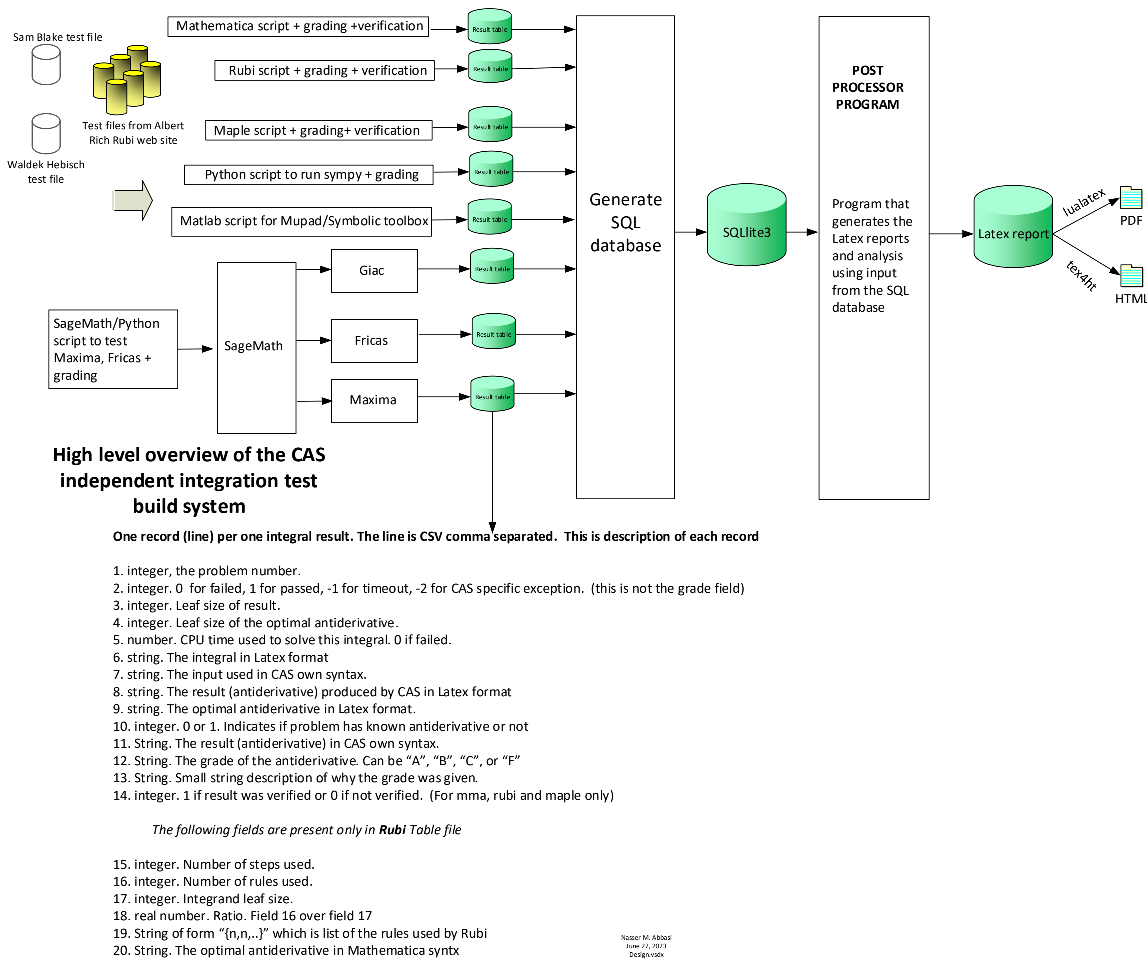
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145 }

B grade { }

C grade { 40, 41, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 26, 27, 28, 34, 35, 36, 39, 47, 48, 49, 50, 51, 52, 53, 54, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 125, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145 }

B grade { 37, 38 }

C grade { 21, 22, 23, 24, 25, 29, 30, 31, 32, 33, 42, 43, 44, 45, 46, 55, 56, 57, 68, 69, 70, 71, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 126, 127, 128, 135 }

F normal fail { 40, 41 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 47, 48, 49, 50, 51, 52, 53, 54, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 125, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145 }

B grade { 37, 38, 39, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 127, 128, 129, 130 }

C grade { 22, 23, 24, 25, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124 }

F normal fail { 40, 41 }

F(-1) timedout fail { 21, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 42, 43, 44, 45, 46, 126 }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 37, 38, 39, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145 }

B grade { }

C grade { }

F normal fail { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 40, 41, 42, 43, 44, 45, 46, 55, 56, 57, 58, 59, 60, 68, 69, 70, 71, 72, 73, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130 }

F(-1) timedout fail { }

F(-2) exception fail { 47, 48, 49, 50, 51, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 125 }

2.1.6 Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 47, 48, 49, 50, 51, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 125, 132, 133, 134, 139, 140 }

B grade { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 44, 45, 46, 55, 56, 57, 58, 59, 60, 68, 69, 70, 71, 72, 73, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130, 131, 135, 136, 137, 138, 141, 142, 143, 144, 145 }

C grade { }

F normal fail { 40, 41 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145 }

C grade { }

F normal fail { }

F(-1) timedout fail { 40, 41 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 115, 118, 119, 123, 124 }

B grade { 37, 38, 39, 110, 113, 114, 116, 117, 120, 121, 122, 131 }

C grade { 132, 133, 134, 135, 142 }

F normal fail { 40 }

F(-1) timedout fail { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 125, 126, 127, 128, 129, 130, 136, 137, 138, 139, 140, 141, 143, 144, 145 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	61	60	60	68	64	62
N.S.	1	1.00	1.00	0.82	0.81	0.81	0.92	0.86	0.84
time (sec)	N/A	0.248	0.013	0.112	0.187	0.241	0.021	0.292	0.020

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	61	60	60	68	64	62
N.S.	1	1.00	1.00	0.82	0.81	0.81	0.92	0.86	0.84
time (sec)	N/A	0.232	0.010	0.104	0.204	0.265	0.020	0.288	0.016

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	65	61	59
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.94	0.88	0.86
time (sec)	N/A	0.217	0.012	0.064	0.191	0.322	0.026	0.300	0.016

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	58	55	55	63	60	57
N.S.	1	1.00	1.00	0.89	0.85	0.85	0.97	0.92	0.88
time (sec)	N/A	0.217	0.014	0.030	0.183	0.248	0.070	0.292	0.020

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	57	55	62	58	57	56
N.S.	1	1.00	1.00	0.90	0.87	0.98	0.92	0.90	0.89
time (sec)	N/A	0.230	0.018	0.034	0.191	0.243	0.078	0.315	0.021

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	58	58	55	62	61	58	56
N.S.	1	1.00	0.92	0.92	0.87	0.98	0.97	0.92	0.89
time (sec)	N/A	0.235	0.029	0.036	0.241	0.265	0.154	0.299	0.019

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	55	56	62	63	56	55
N.S.	1	1.00	0.95	0.87	0.89	0.98	1.00	0.89	0.87
time (sec)	N/A	0.240	0.032	0.035	0.244	0.247	0.303	0.355	0.018

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	62	56	56	62	63	57	56
N.S.	1	1.00	0.98	0.89	0.89	0.98	1.00	0.90	0.89
time (sec)	N/A	0.236	0.021	0.036	0.233	0.270	0.962	0.304	0.025

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	56	62	66	57	56
N.S.	1	1.00	1.00	0.89	0.89	0.98	1.05	0.90	0.89
time (sec)	N/A	0.236	0.040	0.034	0.210	0.254	2.701	0.305	7.909

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	59	59	62	70	60	60
N.S.	1	1.00	1.00	0.87	0.87	0.91	1.03	0.88	0.88
time (sec)	N/A	0.231	0.036	0.036	0.209	0.247	8.313	0.294	7.867

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	159	142	143	143	168	154	141
N.S.	1	1.00	1.00	0.89	0.90	0.90	1.06	0.97	0.89
time (sec)	N/A	0.418	0.036	0.118	0.204	0.273	0.030	0.386	7.975

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	159	142	143	143	163	154	141
N.S.	1	1.00	1.00	0.89	0.90	0.90	1.03	0.97	0.89
time (sec)	N/A	0.364	0.028	0.120	0.189	0.254	0.029	0.289	0.056

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	154	139	140	140	165	151	138
N.S.	1	1.00	1.00	0.90	0.91	0.91	1.07	0.98	0.90
time (sec)	N/A	0.321	0.023	0.142	0.187	0.252	0.032	0.291	0.055

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	150	138	138	138	156	149	135
N.S.	1	1.00	1.00	0.92	0.92	0.92	1.04	0.99	0.90
time (sec)	N/A	0.346	0.030	0.039	0.196	0.252	0.135	0.297	7.863

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	145	142	137	145	156	147	135
N.S.	1	1.00	1.00	0.98	0.94	1.00	1.08	1.01	0.93
time (sec)	N/A	0.347	0.066	0.049	0.184	0.256	0.139	0.300	0.059

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	139	140	139	145	153	148	135
N.S.	1	1.00	0.93	0.94	0.93	0.97	1.03	0.99	0.91
time (sec)	N/A	0.343	0.076	0.048	0.197	0.263	0.232	0.358	0.053

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	151	141	140	145	160	146	137
N.S.	1	1.00	1.01	0.95	0.94	0.97	1.07	0.98	0.92
time (sec)	N/A	0.351	0.061	0.046	0.179	0.257	0.402	0.282	0.034

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	130	139	139	145	153	142	134
N.S.	1	1.00	0.88	0.94	0.94	0.98	1.03	0.96	0.91
time (sec)	N/A	0.346	0.065	0.046	0.193	0.243	1.325	0.300	0.033

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	142	135	138	145	155	140	136
N.S.	1	1.00	0.99	0.94	0.97	1.01	1.08	0.98	0.95
time (sec)	N/A	0.363	0.057	0.049	0.189	0.254	3.945	0.301	0.031

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	144	136	140	145	158	141	136
N.S.	1	1.00	0.97	0.91	0.94	0.97	1.06	0.95	0.91
time (sec)	N/A	0.354	0.066	0.044	0.192	0.278	28.254	0.293	7.869

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	339	347	460	118	0	0	0	5304	2588
N.S.	1	1.02	1.36	0.35	0.00	0.00	0.00	15.65	7.63
time (sec)	N/A	0.861	0.363	0.120	0.000	0.000	0.000	1.415	7.917

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	281	377	86	0	1329593	0	3519	2696
N.S.	1	1.01	1.36	0.31	0.00	4782.71	0.00	12.66	9.70
time (sec)	N/A	0.651	0.263	0.103	0.000	61.661	0.000	1.315	8.366

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	279	360	71	0	861800	0	3843	1890
N.S.	1	1.03	1.33	0.26	0.00	3191.85	0.00	14.23	7.00
time (sec)	N/A	0.682	0.235	0.099	0.000	57.378	0.000	1.470	8.511

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	235	240	52	0	845032	0	2368	5594
N.S.	1	1.05	1.08	0.23	0.00	3789.38	0.00	10.62	25.09
time (sec)	N/A	0.500	0.222	0.073	0.000	13.964	0.000	1.460	8.370

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	234	48	0	578003	0	1714	3942
N.S.	1	1.00	1.11	0.23	0.00	2739.35	0.00	8.12	18.68
time (sec)	N/A	0.424	0.131	0.053	0.000	29.860	0.000	1.294	8.749

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	234	285	251	0	0	0	2339	2258
N.S.	1	1.02	1.24	1.10	0.00	0.00	0.00	10.21	9.86
time (sec)	N/A	0.521	0.285	0.102	0.000	0.000	0.000	1.428	8.368

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	261	315	325	0	0	0	3505	2588
N.S.	1	1.00	1.21	1.25	0.00	0.00	0.00	13.48	9.95
time (sec)	N/A	0.669	0.695	0.112	0.000	0.000	0.000	1.566	8.160

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	289	377	300	0	0	0	3353	3563
N.S.	1	1.00	1.31	1.04	0.00	0.00	0.00	11.64	12.37
time (sec)	N/A	0.678	0.550	0.142	0.000	0.000	0.000	1.490	8.141

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	397	444	249	0	0	0	5217	4754
N.S.	1	0.96	1.08	0.60	0.00	0.00	0.00	12.66	11.54
time (sec)	N/A	0.851	0.859	0.131	0.000	0.000	0.000	1.863	8.513

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	339	358	213	0	0	0	3227	3278
N.S.	1	0.98	1.03	0.61	0.00	0.00	0.00	9.30	9.45
time (sec)	N/A	0.666	0.569	0.106	0.000	0.000	0.000	1.222	8.495

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	348	378	204	0	0	0	4438	3835
N.S.	1	0.98	1.06	0.57	0.00	0.00	0.00	12.47	10.77
time (sec)	N/A	0.698	0.646	0.112	0.000	0.000	0.000	1.593	8.491

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	313	335	198	0	0	0	3014	3198
N.S.	1	0.99	1.06	0.62	0.00	0.00	0.00	9.51	10.09
time (sec)	N/A	0.593	0.795	0.273	0.000	0.000	0.000	1.240	8.311

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	356	393	232	0	0	0	5156	4707
N.S.	1	0.97	1.07	0.63	0.00	0.00	0.00	14.01	12.79
time (sec)	N/A	0.703	0.749	0.247	0.000	0.000	0.000	1.590	8.399

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	419	458	566	0	0	0	6021	8129
N.S.	1	1.04	1.14	1.40	0.00	0.00	0.00	14.94	20.17
time (sec)	N/A	0.911	0.946	0.180	0.000	0.000	0.000	1.646	8.529

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	514	534	559	667	0	0	0	9013	8684
N.S.	1	1.04	1.09	1.30	0.00	0.00	0.00	17.54	16.89
time (sec)	N/A	1.129	1.185	0.229	0.000	0.000	0.000	2.076	8.872

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	534	553	655	778	0	0	0	6939	10595
N.S.	1	1.04	1.23	1.46	0.00	0.00	0.00	12.99	19.84
time (sec)	N/A	1.246	1.495	0.253	0.000	0.000	0.000	1.718	9.009

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	399	296	5520	611	3898	47658	7808	2443
N.S.	1	1.00	0.74	13.83	1.53	9.77	119.44	19.57	6.12
time (sec)	N/A	0.756	2.096	0.413	0.260	0.379	3.032	0.404	9.254

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	185	2187	344	1603	16323	3203	1314
N.S.	1	1.00	0.71	8.41	1.32	6.17	62.78	12.32	5.05
time (sec)	N/A	0.511	0.982	0.181	0.229	0.329	1.424	0.373	8.386

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	90	136	155	444	3628	914	527
N.S.	1	1.00	0.66	0.99	1.13	3.24	26.48	6.67	3.85
time (sec)	N/A	0.324	0.326	0.081	0.218	0.300	0.626	0.381	7.950

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	368	372	438	0	0	0	0	0	0
N.S.	1	1.01	1.19	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.665	2.256	0.000	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	685	687	242	0	0	0	0	0	0
N.S.	1	1.00	0.35	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.686	2.284	0.000	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	348	378	204	0	0	0	4438	3835
N.S.	1	0.98	1.06	0.57	0.00	0.00	0.00	12.47	10.77
time (sec)	N/A	0.670	0.667	0.000	0.000	0.000	0.000	1.683	0.002

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	348	378	204	0	0	0	4438	3835
N.S.	1	0.98	1.06	0.57	0.00	0.00	0.00	12.47	10.77
time (sec)	N/A	0.591	0.109	0.094	0.000	0.000	0.000	1.689	8.213

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	348	378	204	0	0	0	4438	3835
N.S.	1	0.98	1.06	0.57	0.00	0.00	0.00	12.47	10.77
time (sec)	N/A	0.598	0.103	0.089	0.000	0.000	0.000	1.512	8.127

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	348	378	204	0	0	0	4438	3835
N.S.	1	0.98	1.06	0.57	0.00	0.00	0.00	12.47	10.77
time (sec)	N/A	0.574	0.102	0.090	0.000	0.000	0.000	1.417	8.194

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	348	378	204	0	0	0	4438	3835
N.S.	1	0.98	1.06	0.57	0.00	0.00	0.00	12.47	10.77
time (sec)	N/A	0.574	0.103	0.088	0.000	0.000	0.000	1.497	8.304

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	272	260	330	0	900	0	297	2972
N.S.	1	1.00	0.95	1.21	0.00	3.30	0.00	1.09	10.89
time (sec)	N/A	0.766	0.126	0.250	0.000	0.582	0.000	0.633	8.519

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	201	193	224	0	677	0	208	2295
N.S.	1	0.99	0.95	1.10	0.00	3.33	0.00	1.02	11.31
time (sec)	N/A	0.541	0.085	0.223	0.000	0.500	0.000	0.612	8.368

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	143	136	146	0	473	0	137	1689
N.S.	1	0.99	0.94	1.01	0.00	3.28	0.00	0.95	11.73
time (sec)	N/A	0.418	0.061	0.231	0.000	0.368	0.000	0.668	8.383

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	102	100	101	0	318	0	97	1081
N.S.	1	0.99	0.97	0.98	0.00	3.09	0.00	0.94	10.50
time (sec)	N/A	0.339	0.043	0.138	0.000	0.343	0.000	0.641	8.831

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	100	178	99	0	309	0	96	3927
N.S.	1	1.03	1.84	1.02	0.00	3.19	0.00	0.99	40.48
time (sec)	N/A	0.384	0.093	0.085	0.000	0.490	0.000	0.649	13.214

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	120	203	132	0	399	0	131	4437
N.S.	1	1.02	1.72	1.12	0.00	3.38	0.00	1.11	37.60
time (sec)	N/A	0.440	0.093	0.095	0.000	0.546	0.000	0.631	12.759

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	314	203	0	609	0	206	6187
N.S.	1	1.00	1.80	1.17	0.00	3.50	0.00	1.18	35.56
time (sec)	N/A	0.532	0.216	0.123	0.000	0.815	0.000	0.573	14.765

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	246	416	294	0	834	0	303	9141
N.S.	1	1.01	1.70	1.20	0.00	3.42	0.00	1.24	37.46
time (sec)	N/A	0.661	0.218	0.154	0.000	1.674	0.000	0.606	17.782

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	456	164	0	15467	0	7235	23332
N.S.	1	1.00	1.24	0.44	0.00	41.92	0.00	19.61	63.23
time (sec)	N/A	3.020	0.361	0.110	0.000	39.652	0.000	1.157	10.502

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	365	100	0	9364	0	5454	15674
N.S.	1	1.00	1.29	0.35	0.00	33.21	0.00	19.34	55.58
time (sec)	N/A	2.319	0.326	0.092	0.000	8.913	0.000	1.086	9.699

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	258	68	0	5788	0	4082	10209
N.S.	1	1.00	1.18	0.31	0.00	26.43	0.00	18.64	46.62
time (sec)	N/A	0.706	0.198	0.077	0.000	4.341	0.000	0.998	9.697

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	253	220	0	5930	0	3984	10170
N.S.	1	1.00	1.19	1.03	0.00	27.84	0.00	18.70	47.75
time (sec)	N/A	0.816	0.189	0.106	0.000	1.663	0.000	1.364	9.773

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	284	244	0	9850	0	3804	15505
N.S.	1	1.00	1.06	0.91	0.00	36.89	0.00	14.25	58.07
time (sec)	N/A	0.992	0.213	0.122	0.000	11.664	0.000	0.987	10.739

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	394	360	0	15830	0	6710	23019
N.S.	1	1.00	1.20	1.09	0.00	48.12	0.00	20.40	69.97
time (sec)	N/A	1.571	0.342	0.140	0.000	45.944	0.000	1.350	11.573

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	321	309	432	0	2111	0	415	3499
N.S.	1	1.00	0.97	1.35	0.00	6.60	0.00	1.30	10.93
time (sec)	N/A	0.984	0.311	0.357	0.000	0.526	0.000	0.633	0.939

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	243	236	309	0	1455	0	271	2450
N.S.	1	1.03	1.00	1.31	0.00	6.17	0.00	1.15	10.38
time (sec)	N/A	0.645	0.225	0.237	0.000	0.353	0.000	0.584	8.790

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	191	175	228	0	970	0	191	1651
N.S.	1	1.16	1.06	1.38	0.00	5.88	0.00	1.16	10.01
time (sec)	N/A	0.486	0.158	0.213	0.000	0.325	0.000	0.615	9.658

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	128	130	139	0	650	0	137	342
N.S.	1	1.04	1.06	1.13	0.00	5.28	0.00	1.11	2.78
time (sec)	N/A	0.340	0.065	0.108	0.000	0.288	0.000	0.524	0.236

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	199	268	228	0	1103	0	224	8706
N.S.	1	1.20	1.61	1.37	0.00	6.64	0.00	1.35	52.45
time (sec)	N/A	0.582	0.304	0.136	0.000	0.968	0.000	0.569	16.310

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	272	403	316	0	1764	0	280	11879
N.S.	1	1.16	1.72	1.35	0.00	7.54	0.00	1.20	50.76
time (sec)	N/A	0.871	0.422	0.168	0.000	2.081	0.000	0.587	17.156

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	370	592	466	0	2567	0	521	15905
N.S.	1	1.12	1.80	1.42	0.00	7.80	0.00	1.58	48.34
time (sec)	N/A	1.171	0.774	0.234	0.000	4.471	0.000	0.579	24.981

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	550	563	648	323	0	18909	0	8946	33799
N.S.	1	1.02	1.18	0.59	0.00	34.38	0.00	16.27	61.45
time (sec)	N/A	6.896	1.280	0.168	0.000	80.289	0.000	2.029	10.441

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	436	442	511	242	0	12597	0	7479	25862
N.S.	1	1.01	1.17	0.56	0.00	28.89	0.00	17.15	59.32
time (sec)	N/A	3.146	0.921	0.125	0.000	23.673	0.000	1.777	1.559

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	362	352	414	200	0	8951	0	6200	19494
N.S.	1	0.97	1.14	0.55	0.00	24.73	0.00	17.13	53.85
time (sec)	N/A	0.739	0.663	0.135	0.000	13.281	0.000	1.517	12.240

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	332	382	200	0	8991	0	6348	19589
N.S.	1	0.96	1.10	0.58	0.00	25.99	0.00	18.35	56.62
time (sec)	N/A	0.641	0.655	0.117	0.000	11.610	0.000	1.288	12.155

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	401	444	438	0	13111	0	7173	28164
N.S.	1	1.01	1.11	1.10	0.00	32.86	0.00	17.98	70.59
time (sec)	N/A	1.902	0.764	0.193	0.000	27.996	0.000	1.438	12.193

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	575	605	548	570	0	19333	0	8649	36097
N.S.	1	1.05	0.95	0.99	0.00	33.62	0.00	15.04	62.78
time (sec)	N/A	4.752	1.092	0.206	0.000	88.785	0.000	1.649	12.499

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	63	62	56	58	82	61	63	57
N.S.	1	0.93	0.91	0.82	0.85	1.21	0.90	0.93	0.84
time (sec)	N/A	0.363	0.022	0.086	0.189	0.241	0.080	0.274	0.036

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	51	53	77	56	58	53
N.S.	1	1.00	1.00	0.84	0.87	1.26	0.92	0.95	0.87
time (sec)	N/A	0.345	0.022	0.079	0.180	0.256	0.067	0.279	0.024

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	53	54	46	48	72	48	53	47
N.S.	1	0.98	1.00	0.85	0.89	1.33	0.89	0.98	0.87
time (sec)	N/A	0.330	0.017	0.088	0.186	0.258	0.067	0.376	0.022

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	47	49	41	43	67	44	45	43
N.S.	1	0.96	1.00	0.84	0.88	1.37	0.90	0.92	0.88
time (sec)	N/A	0.304	0.018	0.063	0.181	0.253	0.069	0.293	8.487

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	41	42	36	38	57	36	40	37
N.S.	1	0.98	1.00	0.86	0.90	1.36	0.86	0.95	0.88
time (sec)	N/A	0.252	0.014	0.065	0.185	0.252	0.065	0.304	0.029

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	46	44	38	44	71	41	47	40
N.S.	1	1.05	1.00	0.86	1.00	1.61	0.93	1.07	0.91
time (sec)	N/A	0.280	0.015	0.074	0.190	0.251	0.072	0.291	0.025

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	64	50	45	53	92	51	53	50
N.S.	1	1.16	0.91	0.82	0.96	1.67	0.93	0.96	0.91
time (sec)	N/A	0.340	0.018	0.081	0.187	0.255	0.086	0.300	8.523

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	65	56	50	56	97	56	66	55
N.S.	1	1.02	0.88	0.78	0.88	1.52	0.88	1.03	0.86
time (sec)	N/A	0.338	0.020	0.082	0.192	0.255	0.087	0.296	0.026

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	73	71	56	58	79	68	58	58
N.S.	1	1.04	1.01	0.80	0.83	1.13	0.97	0.83	0.83
time (sec)	N/A	0.312	0.034	0.110	0.274	0.257	0.092	0.311	0.035

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	62	58	49	51	74	54	51	50
N.S.	1	1.09	1.02	0.86	0.89	1.30	0.95	0.89	0.88
time (sec)	N/A	0.328	0.031	0.112	0.279	0.254	0.100	0.315	0.032

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	59	57	46	48	69	54	48	48
N.S.	1	1.05	1.02	0.82	0.86	1.23	0.96	0.86	0.86
time (sec)	N/A	0.290	0.029	0.104	0.275	0.249	0.091	0.317	0.029

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	52	50	41	43	64	48	43	42
N.S.	1	1.06	1.02	0.84	0.88	1.31	0.98	0.88	0.86
time (sec)	N/A	0.283	0.027	0.089	0.266	0.259	0.089	0.317	0.039

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	49	46	38	40	59	46	40	40
N.S.	1	1.02	0.96	0.79	0.83	1.23	0.96	0.83	0.83
time (sec)	N/A	0.199	0.028	0.088	0.273	0.246	0.087	0.294	0.045

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	54	51	43	45	68	49	45	45
N.S.	1	1.02	0.96	0.81	0.85	1.28	0.92	0.85	0.85
time (sec)	N/A	0.289	0.034	0.116	0.266	0.268	0.096	0.292	0.045

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	61	56	48	52	79	56	52	51
N.S.	1	0.98	0.90	0.77	0.84	1.27	0.90	0.84	0.82
time (sec)	N/A	0.313	0.038	0.121	0.268	0.255	0.102	0.300	8.507

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	68	61	53	57	84	61	57	57
N.S.	1	0.99	0.88	0.77	0.83	1.22	0.88	0.83	0.83
time (sec)	N/A	0.318	0.041	0.134	0.273	0.254	0.109	0.286	0.047

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	75	77	58	62	89	66	62	61
N.S.	1	0.99	1.01	0.76	0.82	1.17	0.87	0.82	0.80
time (sec)	N/A	0.333	0.039	0.139	0.281	0.253	0.116	0.386	8.742

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	91	71	61	71	114	75	61	70
N.S.	1	1.12	0.88	0.75	0.88	1.41	0.93	0.75	0.86
time (sec)	N/A	0.450	0.042	0.127	0.262	0.244	0.110	0.288	9.143

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	88	66	58	68	109	76	58	68
N.S.	1	1.10	0.82	0.72	0.85	1.36	0.95	0.72	0.85
time (sec)	N/A	0.385	0.040	0.120	0.270	0.261	0.107	0.283	0.032

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	81	60	53	63	104	70	53	63
N.S.	1	1.08	0.80	0.71	0.84	1.39	0.93	0.71	0.84
time (sec)	N/A	0.391	0.040	0.107	0.271	0.255	0.109	0.308	0.040

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	78	55	50	60	99	65	50	59
N.S.	1	1.08	0.76	0.69	0.83	1.38	0.90	0.69	0.82
time (sec)	N/A	0.306	0.042	0.101	0.267	0.260	0.111	0.277	8.562

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	78	56	50	60	99	66	50	60
N.S.	1	1.08	0.78	0.69	0.83	1.38	0.92	0.69	0.83
time (sec)	N/A	0.299	0.042	0.095	0.277	0.246	0.109	0.292	0.045

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	78	56	50	60	99	65	50	59
N.S.	1	1.08	0.78	0.69	0.83	1.38	0.90	0.69	0.82
time (sec)	N/A	0.241	0.040	0.101	0.268	0.251	0.106	0.289	0.046

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	83	63	56	65	108	71	55	65
N.S.	1	1.05	0.80	0.71	0.82	1.37	0.90	0.70	0.82
time (sec)	N/A	0.381	0.050	0.133	0.270	0.255	0.120	0.298	8.839

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	90	78	61	72	119	76	62	71
N.S.	1	1.05	0.91	0.71	0.84	1.38	0.88	0.72	0.83
time (sec)	N/A	0.414	0.040	0.134	0.272	0.258	0.124	0.286	0.048

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	97	73	66	77	124	82	67	77
N.S.	1	1.04	0.78	0.71	0.83	1.33	0.88	0.72	0.83
time (sec)	N/A	0.439	0.053	0.154	0.269	0.256	0.130	0.296	0.054

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	87	78	71	71	95	87	76	75
N.S.	1	1.01	0.91	0.83	0.83	1.10	1.01	0.88	0.87
time (sec)	N/A	0.374	0.032	0.067	0.276	0.246	0.079	0.439	0.038

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	82	73	66	66	90	80	71	69
N.S.	1	1.01	0.90	0.81	0.81	1.11	0.99	0.88	0.85
time (sec)	N/A	0.347	0.022	0.063	0.266	0.244	0.078	0.433	0.033

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	75	66	62	59	85	73	66	65
N.S.	1	1.01	0.89	0.84	0.80	1.15	0.99	0.89	0.88
time (sec)	N/A	0.338	0.021	0.079	0.267	0.251	0.076	0.523	0.027

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	68	61	56	54	80	68	54	60
N.S.	1	1.05	0.94	0.86	0.83	1.23	1.05	0.83	0.92
time (sec)	N/A	0.314	0.020	0.071	0.279	0.251	0.078	0.470	8.608

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	68	58	51	49	70	60	49	69
N.S.	1	1.17	1.00	0.88	0.84	1.21	1.03	0.84	1.19
time (sec)	N/A	0.281	0.016	0.054	0.272	0.249	0.076	0.460	0.030

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	77	72	58	55	84	65	62	59
N.S.	1	1.17	1.09	0.88	0.83	1.27	0.98	0.94	0.89
time (sec)	N/A	0.308	0.067	0.068	0.267	0.255	0.083	0.451	8.656

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	80	101	63	66	105	76	66	68
N.S.	1	1.13	1.42	0.89	0.93	1.48	1.07	0.93	0.96
time (sec)	N/A	0.347	0.045	0.073	0.270	0.246	0.097	0.438	0.037

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	81	82	68	71	110	80	79	72
N.S.	1	1.01	1.02	0.85	0.89	1.38	1.00	0.99	0.90
time (sec)	N/A	0.365	0.068	0.072	0.285	0.252	0.095	0.438	0.038

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	92	114	73	76	115	85	84	78
N.S.	1	1.06	1.31	0.84	0.87	1.32	0.98	0.97	0.90
time (sec)	N/A	0.384	0.055	0.072	0.275	0.241	0.112	0.441	8.641

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	253	145	79	0	215	71	585	171
N.S.	1	1.02	0.58	0.32	0.00	0.87	0.29	2.36	0.69
time (sec)	N/A	0.578	0.126	0.174	0.000	0.249	0.344	0.594	0.075

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	244	132	72	0	210	1205	576	164
N.S.	1	1.03	0.56	0.30	0.00	0.89	5.08	2.43	0.69
time (sec)	N/A	0.532	0.105	0.087	0.000	0.247	0.739	0.628	8.618

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	237	129	69	0	205	60	573	162
N.S.	1	1.02	0.56	0.30	0.00	0.88	0.26	2.47	0.70
time (sec)	N/A	0.497	0.105	0.081	0.000	0.251	0.329	0.593	0.072

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	230	121	64	0	222	51	566	156
N.S.	1	1.02	0.54	0.28	0.00	0.99	0.23	2.52	0.69
time (sec)	N/A	0.480	0.109	0.076	0.000	0.270	0.330	0.611	0.089

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	263	115	61	0	217	1185	565	153
N.S.	1	1.17	0.51	0.27	0.00	0.97	5.29	2.52	0.68
time (sec)	N/A	0.490	0.189	0.075	0.000	0.256	0.694	0.582	8.652

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	232	126	63	0	230	1192	572	159
N.S.	1	1.01	0.55	0.28	0.00	1.00	5.21	2.50	0.69
time (sec)	N/A	0.555	0.121	0.099	0.000	0.253	0.765	0.577	0.101

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	239	131	68	0	245	60	579	165
N.S.	1	1.00	0.55	0.29	0.00	1.03	0.25	2.43	0.69
time (sec)	N/A	0.528	0.210	0.099	0.000	0.246	0.353	0.595	0.099

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	246	140	73	0	252	1202	584	171
N.S.	1	1.00	0.57	0.30	0.00	1.03	4.91	2.38	0.70
time (sec)	N/A	0.563	0.207	0.106	0.000	0.257	0.772	0.625	8.765

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	255	156	82	0	282	1204	588	184
N.S.	1	1.05	0.64	0.34	0.00	1.16	4.95	2.42	0.76
time (sec)	N/A	0.641	0.154	0.095	0.000	0.257	0.754	0.729	0.083

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	252	155	79	0	279	82	585	182
N.S.	1	1.04	0.64	0.33	0.00	1.15	0.34	2.42	0.75
time (sec)	N/A	0.612	0.136	0.076	0.000	0.252	0.369	0.751	8.896

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	245	138	74	0	272	71	580	176
N.S.	1	1.04	0.59	0.31	0.00	1.16	0.30	2.47	0.75
time (sec)	N/A	0.583	0.222	0.077	0.000	0.248	0.344	0.722	8.853

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	292	129	71	0	267	1198	577	173
N.S.	1	1.23	0.54	0.30	0.00	1.12	5.03	2.42	0.73
time (sec)	N/A	0.574	0.204	0.075	0.000	0.256	0.697	0.874	0.127

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	290	133	71	0	277	1200	577	174
N.S.	1	1.18	0.54	0.29	0.00	1.13	4.88	2.35	0.71
time (sec)	N/A	0.581	0.201	0.079	0.000	0.250	0.721	0.754	0.111

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	294	129	71	0	277	1195	577	173
N.S.	1	1.19	0.52	0.29	0.00	1.12	4.82	2.33	0.70
time (sec)	N/A	0.540	0.192	0.070	0.000	0.251	0.701	0.720	8.700

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	140	73	0	290	75	582	179
N.S.	1	1.00	0.55	0.29	0.00	1.15	0.30	2.30	0.71
time (sec)	N/A	0.644	0.263	0.096	0.000	0.270	0.364	0.738	8.872

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	268	139	78	0	305	80	589	185
N.S.	1	1.02	0.53	0.30	0.00	1.16	0.31	2.25	0.71
time (sec)	N/A	0.632	0.233	0.098	0.000	0.261	0.367	0.768	8.752

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	148	142	151	0	486	0	144	1834
N.S.	1	0.99	0.95	1.01	0.00	3.26	0.00	0.97	12.31
time (sec)	N/A	0.443	0.075	0.256	0.000	0.335	0.000	0.638	9.001

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	594	607	721	348	0	0	0	10752	47339
N.S.	1	1.02	1.21	0.59	0.00	0.00	0.00	18.10	79.70
time (sec)	N/A	8.038	1.562	0.183	0.000	0.000	0.000	2.341	10.708

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	471	477	575	274	0	23774	0	9152	36589
N.S.	1	1.01	1.22	0.58	0.00	50.48	0.00	19.43	77.68
time (sec)	N/A	3.931	1.164	0.160	0.000	181.702	0.000	2.044	10.390

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	435	512	269	0	19375	0	8905	32587
N.S.	1	0.97	1.14	0.60	0.00	43.15	0.00	19.83	72.58
time (sec)	N/A	0.949	1.012	0.148	0.000	151.256	0.000	1.895	11.165

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	460	462	529	550	0	23991	0	9167	40860
N.S.	1	1.00	1.15	1.20	0.00	52.15	0.00	19.93	88.83
time (sec)	N/A	2.694	1.611	0.255	0.000	144.800	0.000	1.747	12.149

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	542	551	612	629	0	33432	0	10411	51386
N.S.	1	1.02	1.13	1.16	0.00	61.68	0.00	19.21	94.81
time (sec)	N/A	3.946	1.362	0.272	0.000	297.365	0.000	1.799	12.571

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	31	31	54	58	31
N.S.	1	1.00	1.00	1.05	1.55	1.55	2.70	2.90	1.55
time (sec)	N/A	0.195	0.449	0.273	0.239	0.248	158.450	0.346	8.107

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	234	149	145	295	138	367	228	287
N.S.	1	1.11	0.71	0.69	1.40	0.66	1.75	1.09	1.37
time (sec)	N/A	0.459	0.362	0.461	0.284	0.263	19.879	0.372	8.419

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	186	116	109	217	104	367	164	215
N.S.	1	1.17	0.73	0.69	1.36	0.65	2.31	1.03	1.35
time (sec)	N/A	0.404	0.271	0.423	0.281	0.261	11.903	0.324	8.289

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	139	80	73	139	71	350	102	143
N.S.	1	1.28	0.73	0.67	1.28	0.65	3.21	0.94	1.31
time (sec)	N/A	0.333	0.196	0.435	0.284	0.274	7.520	0.337	8.359

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	107	106	143	105	80	304	189	161
N.S.	1	1.15	1.14	1.54	1.13	0.86	3.27	2.03	1.73
time (sec)	N/A	0.368	0.260	0.442	0.313	0.268	18.143	0.440	9.754

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	149	85	137	123	98	0	374	422
N.S.	1	1.51	0.86	1.38	1.24	0.99	0.00	3.78	4.26
time (sec)	N/A	0.371	0.278	0.434	0.278	0.275	0.000	0.462	10.856

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	180	102	143	193	102	0	767	932
N.S.	1	1.43	0.81	1.13	1.53	0.81	0.00	6.09	7.40
time (sec)	N/A	0.393	0.304	0.444	0.290	0.268	0.000	0.607	15.498

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	226	141	179	271	137	0	1434	1621
N.S.	1	1.07	0.67	0.84	1.28	0.65	0.00	6.76	7.65
time (sec)	N/A	0.422	0.390	0.441	0.279	0.273	0.000	0.770	24.813

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	192	134	161	226	134	0	177	1132
N.S.	1	0.89	0.62	0.75	1.05	0.62	0.00	0.82	5.24
time (sec)	N/A	0.370	0.317	0.436	0.281	0.284	0.000	0.340	27.906

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	155	98	128	146	100	0	115	651
N.S.	1	1.21	0.77	1.00	1.14	0.78	0.00	0.90	5.09
time (sec)	N/A	0.307	0.230	0.444	0.287	0.266	0.000	0.334	17.263

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	126	86	116	94	90	0	239	306
N.S.	1	1.24	0.84	1.14	0.92	0.88	0.00	2.34	3.00
time (sec)	N/A	0.305	0.203	0.441	0.293	0.271	0.000	0.372	11.990

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	135	81	107	94	90	257	530	138
N.S.	1	0.86	0.52	0.68	0.60	0.57	1.64	3.38	0.88
time (sec)	N/A	0.315	0.196	0.440	0.278	0.282	18.200	0.414	8.747

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	156	87	82	148	76	0	1055	146
N.S.	1	0.98	0.54	0.51	0.92	0.48	0.00	6.59	0.91
time (sec)	N/A	0.326	0.166	0.447	0.279	0.269	0.000	0.528	8.491

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	193	124	118	226	110	0	1451	218
N.S.	1	0.85	0.55	0.52	1.00	0.49	0.00	6.42	0.96
time (sec)	N/A	0.352	0.216	0.441	0.289	0.317	0.000	0.692	8.531

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	230	158	154	304	144	0	1847	290
N.S.	1	0.79	0.54	0.53	1.04	0.49	0.00	6.33	0.99
time (sec)	N/A	0.382	0.257	0.441	0.280	0.352	0.000	0.936	8.564

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [27] had the largest ratio of [.4285709999999999980]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	26	0.077
2	A	2	2	1.00	24	0.083
3	A	2	2	1.00	23	0.087
4	A	2	2	1.00	26	0.077
5	A	2	2	1.00	26	0.077
6	A	2	2	1.00	26	0.077
7	A	2	2	1.00	26	0.077
8	A	2	2	1.00	26	0.077
9	A	2	2	1.00	26	0.077
10	A	2	2	1.00	26	0.077
11	A	2	2	1.00	28	0.071
12	A	2	2	1.00	26	0.077
13	A	2	2	1.00	25	0.080
14	A	2	2	1.00	28	0.071
15	A	2	2	1.00	28	0.071
16	A	2	2	1.00	28	0.071
17	A	2	2	1.00	28	0.071
18	A	2	2	1.00	28	0.071
19	A	2	2	1.00	28	0.071
20	A	2	2	1.00	28	0.071
21	A	12	11	1.02	28	0.393
22	A	9	8	1.01	28	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	11	10	1.03	28	0.357
24	A	10	9	1.05	26	0.346
25	A	8	7	1.00	25	0.280
26	A	8	7	1.02	28	0.250
27	A	13	12	1.00	28	0.429
28	A	10	9	1.00	28	0.321
29	A	11	10	0.96	28	0.357
30	A	10	9	0.98	28	0.321
31	A	10	9	0.98	28	0.321
32	A	10	9	0.99	26	0.346
33	A	11	10	0.97	25	0.400
34	A	12	11	1.04	28	0.393
35	A	13	12	1.04	28	0.429
36	A	13	12	1.04	28	0.429
37	A	2	2	1.00	30	0.067
38	A	2	2	1.00	30	0.067
39	A	2	2	1.00	28	0.071
40	A	8	8	1.01	30	0.267
41	A	8	8	1.00	30	0.267
42	A	10	9	0.98	28	0.321
43	A	11	10	0.98	30	0.333
44	A	11	10	0.98	31	0.323
45	A	11	10	0.98	34	0.294
46	A	11	10	0.98	34	0.294
47	A	4	3	1.00	30	0.100
48	A	4	3	0.99	30	0.100
49	A	4	3	0.99	30	0.100
50	A	4	3	0.99	28	0.107
51	A	4	3	1.03	30	0.100
52	A	4	3	1.02	30	0.100
53	A	4	3	1.00	30	0.100
54	A	4	3	1.01	30	0.100
55	A	2	2	1.00	30	0.067
56	A	2	2	1.00	30	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	2	2	1.00	27	0.074
58	A	2	2	1.00	30	0.067
59	A	2	2	1.00	30	0.067
60	A	2	2	1.00	30	0.067
61	A	7	6	1.00	30	0.200
62	A	7	6	1.03	30	0.200
63	A	8	7	1.16	30	0.233
64	A	6	5	1.04	28	0.179
65	A	7	6	1.20	30	0.200
66	A	6	5	1.16	30	0.167
67	A	6	5	1.12	30	0.167
68	A	3	3	1.02	30	0.100
69	A	3	3	1.01	30	0.100
70	A	5	5	0.97	30	0.167
71	A	4	4	0.96	27	0.148
72	A	4	4	1.01	30	0.133
73	A	4	4	1.05	30	0.133
74	A	6	5	0.93	31	0.161
75	A	5	4	1.00	31	0.129
76	A	6	5	0.98	31	0.161
77	A	5	4	0.96	31	0.129
78	A	6	5	0.98	29	0.172
79	A	6	5	1.05	31	0.161
80	A	6	5	1.16	31	0.161
81	A	6	5	1.02	31	0.161
82	A	4	4	1.04	31	0.129
83	A	4	4	1.09	31	0.129
84	A	4	4	1.05	31	0.129
85	A	4	4	1.06	31	0.129
86	A	4	4	1.02	28	0.143
87	A	4	4	1.02	31	0.129
88	A	4	4	0.98	31	0.129
89	A	4	4	0.99	31	0.129
90	A	4	4	0.99	31	0.129

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	6	6	1.12	31	0.194
92	A	6	6	1.10	31	0.194
93	A	6	6	1.08	31	0.194
94	A	6	6	1.08	31	0.194
95	A	6	6	1.08	31	0.194
96	A	5	5	1.08	28	0.179
97	A	6	6	1.05	31	0.194
98	A	6	6	1.05	31	0.194
99	A	6	6	1.04	31	0.194
100	A	6	5	1.01	31	0.161
101	A	6	5	1.01	31	0.161
102	A	6	5	1.01	31	0.161
103	A	6	5	1.05	31	0.161
104	A	9	8	1.17	29	0.276
105	A	6	5	1.17	31	0.161
106	A	6	5	1.13	31	0.161
107	A	6	5	1.01	31	0.161
108	A	6	5	1.06	31	0.161
109	A	4	4	1.02	31	0.129
110	A	4	4	1.03	31	0.129
111	A	4	4	1.02	31	0.129
112	A	4	4	1.02	31	0.129
113	A	9	8	1.17	28	0.286
114	A	4	4	1.01	31	0.129
115	A	4	4	1.00	31	0.129
116	A	4	4	1.00	31	0.129
117	A	6	6	1.05	31	0.194
118	A	6	6	1.04	31	0.194
119	A	6	6	1.04	31	0.194
120	A	11	10	1.23	31	0.323
121	A	11	10	1.18	31	0.323
122	A	11	10	1.19	28	0.357
123	A	6	6	1.00	31	0.194
124	A	6	6	1.02	31	0.194

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	4	3	0.99	33	0.091
126	A	3	3	1.02	35	0.086
127	A	4	4	1.01	35	0.114
128	A	4	4	0.97	32	0.125
129	A	4	4	1.00	35	0.114
130	A	4	4	1.02	35	0.114
131	A	1	1	1.00	42	0.024
132	A	5	4	1.11	35	0.114
133	A	5	4	1.17	35	0.114
134	A	5	4	1.28	33	0.121
135	A	7	6	1.15	35	0.171
136	A	8	7	1.51	35	0.200
137	A	9	8	1.43	35	0.229
138	A	9	8	1.07	35	0.229
139	A	8	7	0.89	35	0.200
140	A	7	6	1.21	32	0.188
141	A	8	7	1.24	35	0.200
142	A	7	6	0.86	35	0.171
143	A	5	5	0.98	35	0.143
144	A	6	6	0.85	35	0.171
145	A	7	7	0.79	35	0.200

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

Listing of integrals

3.1 $\int x^2(A+Bx+Cx^2)(a+bx^2+cx^4)\,dx$

3.1.1 Optimal result

Integrand size = 26, antiderivative size = 74

$$\int x^2(A+Bx+Cx^2)(a+bx^2+cx^4)\,dx = \frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{5}(Ab+aC)x^5+\frac{1}{6}bBx^6+\frac{1}{7}(Ac+bC)x^7+\frac{1}{8}Bcx^8+\frac{1}{9}cCx^9$$

output
```
1/3*a*A*x^3+1/4*a*B*x^4+1/5*(A*b+C*a)*x^5+1/6*b*B*x^6+1/7*(A*c+C*b)*x^7+1/
8*B*c*x^8+1/9*c*C*x^9
```

3.1.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int x^2(A+Bx+Cx^2)(a+bx^2+cx^4)\,dx = \frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{5}(Ab+aC)x^5+\frac{1}{6}bBx^6+\frac{1}{7}(Ac+bC)x^7+\frac{1}{8}Bcx^8+\frac{1}{9}cCx^9$$

input
```
Integrate[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

output
```
(a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + (b*B*x^6)/6 + ((A*c + b*
C)*x^7)/7 + (B*c*x^8)/8 + (c*C*x^9)/9
```

3.1.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^2+cx^4\right) \left(A+Bx+Cx^2\right) \, dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(x^4 (aC+Ab)+aAx^2+aBx^3+x^6 (Ac+bC)+bBx^5+Bcx^7+cCx^8\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5(aC+Ab)+\frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{7}x^7(Ac+bC)+\frac{1}{6}bBx^6+\frac{1}{8}Bcx^8+\frac{1}{9}cCx^9$$

input
```
Int[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

output
```
(a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + (b*B*x^6)/6 + ((A*c + b*
C)*x^7)/7 + (B*c*x^8)/8 + (c*C*x^9)/9
```

3.1.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.1.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{aA\,x^{3}}{3}+\frac{aB\,x^{4}}{4}+\frac{(Ab+Ca)x^{5}}{5}+\frac{bB\,x^{6}}{6}+\frac{(Ac+Cb)x^{7}}{7}+\frac{Bc\,x^{8}}{8}+\frac{cC\,x^{9}}{9}$	61
norman	$\frac{cC\,x^{9}}{9}+\frac{Bc\,x^{8}}{8}+\left(\frac{Ac}{7}+\frac{Cb}{7}\right)x^{7}+\frac{bB\,x^{6}}{6}+\left(\frac{Ab}{5}+\frac{Ca}{5}\right)x^{5}+\frac{aB\,x^{4}}{4}+\frac{aA\,x^{3}}{3}$	63
gosper	$\frac{1}{9}cC\,x^{9}+\frac{1}{8}Bc\,x^{8}+\frac{1}{7}x^{7}Ac+\frac{1}{7}x^{7}Cb+\frac{1}{6}bB\,x^{6}+\frac{1}{5}x^{5}Ab+\frac{1}{5}x^{5}Ca+\frac{1}{4}aB\,x^{4}+\frac{1}{3}aA\,x^{3}$	65
risch	$\frac{1}{9}cC\,x^{9}+\frac{1}{8}Bc\,x^{8}+\frac{1}{7}x^{7}Ac+\frac{1}{7}x^{7}Cb+\frac{1}{6}bB\,x^{6}+\frac{1}{5}x^{5}Ab+\frac{1}{5}x^{5}Ca+\frac{1}{4}aB\,x^{4}+\frac{1}{3}aA\,x^{3}$	65
parallelrisch	$\frac{1}{9}cC\,x^{9}+\frac{1}{8}Bc\,x^{8}+\frac{1}{7}x^{7}Ac+\frac{1}{7}x^{7}Cb+\frac{1}{6}bB\,x^{6}+\frac{1}{5}x^{5}Ab+\frac{1}{5}x^{5}Ca+\frac{1}{4}aB\,x^{4}+\frac{1}{3}aA\,x^{3}$	65

input
```
int(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*A*x^3+1/4*a*B*x^4+1/5*(A*b+C*a)*x^5+1/6*b*B*x^6+1/7*(A*c+C*b)*x^7+1/
8*B*c*x^8+1/9*c*C*x^9
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx=\frac{1}{9}\,Ccx^9+\frac{1}{8}\,Bcx^8+\frac{1}{6}\,Bbx^6+\frac{1}{7}\,(Cb+Ac)x^7+\frac{1}{4}\,Bax^4+\frac{1}{5}\,(Ca+Ab)x^5+\frac{1}{3}\,Aax^3$$

input
```
integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/9*C*c*x^9 + 1/8*B*c*x^8 + 1/6*B*b*x^6 + 1/7*(C*b + A*c)*x^7 + 1/4*B*a*x^
4 + 1/5*(C*a + A*b)*x^5 + 1/3*A*a*x^3
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx=\frac{Aax^3}{3}+\frac{Bax^4}{4}+\frac{Bbx^6}{6}+\frac{Bcx^8}{8}+\frac{Ccx^9}{9}+x^7\left(\frac{Ac}{7}+\frac{Cb}{7}\right)+x^5\left(\frac{Ab}{5}+\frac{Ca}{5}\right)$$

input
```
integrate(x**2*(C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
```

output
```
A*a*x**3/3 + B*a*x**4/4 + B*b*x**6/6 + B*c*x**8/8 + C*c*x**9/9 + x**7*(A*c
/7 + C*b/7) + x**5*(A*b/5 + C*a/5)
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx=\frac{1}{9}\,Ccx^9+\frac{1}{8}\,Bcx^8+\frac{1}{6}\,Bbx^6+\frac{1}{7}\,(Cb+Ac)x^7+\frac{1}{4}\,Bax^4+\frac{1}{5}\,(Ca+Ab)x^5+\frac{1}{3}\,Aax^3$$

input
```
integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/9*C*c*x^9 + 1/8*B*c*x^8 + 1/6*B*b*x^6 + 1/7*(C*b + A*c)*x^7 + 1/4*B*a*x^
4 + 1/5*(C*a + A*b)*x^5 + 1/3*A*a*x^3
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx=\frac{1}{9}\,Ccx^9+\frac{1}{8}\,Bcx^8+\frac{1}{7}\,Cbx^7+\frac{1}{7}\,Acx^7+\frac{1}{6}\,Bbx^6+\frac{1}{5}\,Cax^5+\frac{1}{5}\,Abx^5+\frac{1}{4}\,Bax^4+\frac{1}{3}\,Aax^3$$

input
```
integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/9*C*c*x^9 + 1/8*B*c*x^8 + 1/7*C*b*x^7 + 1/7*A*c*x^7 + 1/6*B*b*x^6 + 1/5*
C*a*x^5 + 1/5*A*b*x^5 + 1/4*B*a*x^4 + 1/3*A*a*x^3
```

3.1.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx=\frac{C\,c\,x^9}{9}+\frac{B\,c\,x^8}{8}+\left(\frac{A\,c}{7}+\frac{C\,b}{7}\right)x^7+\frac{B\,b\,x^6}{6}+\left(\frac{A\,b}{5}+\frac{C\,a}{5}\right)x^5+\frac{B\,a\,x^4}{4}+\frac{A\,a\,x^3}{3}$$

input
```
int(x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x)
```

output
```
x^5*((A*b)/5 + (C*a)/5) + x^7*((A*c)/7 + (C*b)/7) + (A*a*x^3)/3 + (B*a*x^4
)/4 + (B*b*x^6)/6 + (B*c*x^8)/8 + (C*c*x^9)/9
```

3.2 $\int x(A+Bx+Cx^2)(a+bx^2+cx^4)\,dx$

3.2.1 Optimal result

Integrand size = 24, antiderivative size = 74

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx=\frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{4}(Ab+aC)x^4+\frac{1}{5}bBx^5+\frac{1}{6}(Ac+bC)x^6+\frac{1}{7}Bcx^7+\frac{1}{8}cCx^8$$

output
```
1/2*a*A*x^2+1/3*a*B*x^3+1/4*(A*b+C*a)*x^4+1/5*b*B*x^5+1/6*(A*c+C*b)*x^6+1/
7*B*c*x^7+1/8*c*C*x^8
```

3.2.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx=\frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{4}(Ab+aC)x^4+\frac{1}{5}bBx^5+\frac{1}{6}(Ac+bC)x^6+\frac{1}{7}Bcx^7+\frac{1}{8}cCx^8$$

input
```
Integrate[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

output
```
(a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + (b*B*x^5)/5 + ((A*c + b*
C)*x^6)/6 + (B*c*x^7)/7 + (c*C*x^8)/8
```

3.2.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^2+cx^4\right)\left(A+Bx+Cx^2\right)dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(x^3(aC+Ab)+aAx+aBx^2+x^5(Ac+bC)+bBx^4+Bcx^6+cCx^7\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4(aC+Ab)+\frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{6}x^6(Ac+bC)+\frac{1}{5}bBx^5+\frac{1}{7}Bcx^7+\frac{1}{8}cCx^8$$

input
```
Int[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

output
```
(a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + (b*B*x^5)/5 + ((A*c + b*
C)*x^6)/6 + (B*c*x^7)/7 + (c*C*x^8)/8
```

3.2.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.2.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{aA\,x^{2}}{2}+\frac{aB\,x^{3}}{3}+\frac{(Ab+Ca)x^{4}}{4}+\frac{bB\,x^{5}}{5}+\frac{(Ac+Cb)x^{6}}{6}+\frac{Bc\,x^{7}}{7}+\frac{cC\,x^{8}}{8}$	61
norman	$\frac{cC\,x^{8}}{8}+\frac{Bc\,x^{7}}{7}+\left(\frac{Ac}{6}+\frac{Cb}{6}\right)x^{6}+\frac{bB\,x^{5}}{5}+\left(\frac{Ab}{4}+\frac{Ca}{4}\right)x^{4}+\frac{aB\,x^{3}}{3}+\frac{aA\,x^{2}}{2}$	63
gosper	$\frac{1}{8}cC\,x^{8}+\frac{1}{7}Bc\,x^{7}+\frac{1}{6}x^{6}Ac+\frac{1}{6}x^{6}Cb+\frac{1}{5}bB\,x^{5}+\frac{1}{4}x^{4}Ab+\frac{1}{4}x^{4}Ca+\frac{1}{3}aB\,x^{3}+\frac{1}{2}aA\,x^{2}$	65
risch	$\frac{1}{8}cC\,x^{8}+\frac{1}{7}Bc\,x^{7}+\frac{1}{6}x^{6}Ac+\frac{1}{6}x^{6}Cb+\frac{1}{5}bB\,x^{5}+\frac{1}{4}x^{4}Ab+\frac{1}{4}x^{4}Ca+\frac{1}{3}aB\,x^{3}+\frac{1}{2}aA\,x^{2}$	65
parallelrisch	$\frac{1}{8}cC\,x^{8}+\frac{1}{7}Bc\,x^{7}+\frac{1}{6}x^{6}Ac+\frac{1}{6}x^{6}Cb+\frac{1}{5}bB\,x^{5}+\frac{1}{4}x^{4}Ab+\frac{1}{4}x^{4}Ca+\frac{1}{3}aB\,x^{3}+\frac{1}{2}aA\,x^{2}$	65

input
```
int(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*A*x^2+1/3*a*B*x^3+1/4*(A*b+C*a)*x^4+1/5*b*B*x^5+1/6*(A*c+C*b)*x^6+1/
7*B*c*x^7+1/8*c*C*x^8
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

$$\int x\left(A+Bx+Cx^{2}\right)\left(a+bx^{2}+cx^{4}\right)\,dx=\frac{1}{8}\,Ccx^{8}+\frac{1}{7}\,Bcx^{7}+\frac{1}{5}\,Bbx^{5}+\frac{1}{6}\,(Cb+Ac)x^{6}+\frac{1}{3}\,Bax^{3}+\frac{1}{4}\,(Ca+Ab)x^{4}+\frac{1}{2}\,Aax^{2}$$

input
```
integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/8*C*c*x^8 + 1/7*B*c*x^7 + 1/5*B*b*x^5 + 1/6*(C*b + A*c)*x^6 + 1/3*B*a*x^
3 + 1/4*(C*a + A*b)*x^4 + 1/2*A*a*x^2
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx=\frac{Aax^2}{2}+\frac{Bax^3}{3}+\frac{Bbx^5}{5}+\frac{Bcx^7}{7}+\frac{Ccx^8}{8}+x^6\left(\frac{Ac}{6}+\frac{Cb}{6}\right)+x^4\left(\frac{Ab}{4}+\frac{Ca}{4}\right)$$

input
```
integrate(x*(C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
```

output
```
A*a*x**2/2 + B*a*x**3/3 + B*b*x**5/5 + B*c*x**7/7 + C*c*x**8/8 + x**6*(A*c
/6 + C*b/6) + x**4*(A*b/4 + C*a/4)
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx=\frac{1}{8}Ccx^8+\frac{1}{7}Bcx^7+\frac{1}{5}Bbx^5+\frac{1}{6}(Cb+Ac)x^6+\frac{1}{3}Bax^3+\frac{1}{4}(Ca+Ab)x^4+\frac{1}{2}Aax^2$$

input
```
integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/8*C*c*x^8 + 1/7*B*c*x^7 + 1/5*B*b*x^5 + 1/6*(C*b + A*c)*x^6 + 1/3*B*a*x^
3 + 1/4*(C*a + A*b)*x^4 + 1/2*A*a*x^2
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx=\frac{1}{8}Ccx^8+\frac{1}{7}Bcx^7+\frac{1}{6}Cbx^6+\frac{1}{6}Acx^6+\frac{1}{5}Bbx^5+\frac{1}{4}Cax^4+\frac{1}{4}Abx^4+\frac{1}{3}Bax^3+\frac{1}{2}Aax^2$$

input
```
integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/8*C*c*x^8 + 1/7*B*c*x^7 + 1/6*C*b*x^6 + 1/6*A*c*x^6 + 1/5*B*b*x^5 + 1/4*
C*a*x^4 + 1/4*A*b*x^4 + 1/3*B*a*x^3 + 1/2*A*a*x^2
```

3.2.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx=\frac{C\,c\,x^8}{8}+\frac{B\,c\,x^7}{7}+\left(\frac{A\,c}{6}+\frac{C\,b}{6}\right)x^6+\frac{B\,b\,x^5}{5}+\left(\frac{A\,b}{4}+\frac{C\,a}{4}\right)x^4+\frac{B\,a\,x^3}{3}+\frac{A\,a\,x^2}{2}$$

input
```
int(x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x)
```

output
```
x^4*((A*b)/4 + (C*a)/4) + x^6*((A*c)/6 + (C*b)/6) + (A*a*x^2)/2 + (B*a*x^3
)/3 + (B*b*x^5)/5 + (B*c*x^7)/7 + (C*c*x^8)/8
```

3.3 $\int (A+Bx+Cx^2)(a+bx^2+cx^4)\,dx$

3.3.1 Optimal result

Integrand size = 23, antiderivative size = 69

$$\int (A+Bx+Cx^2)(a+bx^2+cx^4)\,dx = aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ab+aC)x^3+\frac{1}{4}bBx^4+\frac{1}{5}(Ac+bC)x^5+\frac{1}{6}Bcx^6+\frac{1}{7}cCx^7$$

output
```
a*A*x+1/2*a*B*x^2+1/3*(A*b+C*a)*x^3+1/4*b*B*x^4+1/5*(A*c+C*b)*x^5+1/6*B*c*
x^6+1/7*c*C*x^7
```

3.3.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int (A+Bx+Cx^2)(a+bx^2+cx^4)\,dx = aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ab+aC)x^3+\frac{1}{4}bBx^4+\frac{1}{5}(Ac+bC)x^5+\frac{1}{6}Bcx^6+\frac{1}{7}cCx^7$$

input
```
Integrate[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

output
```
a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + (b*B*x^4)/4 + ((A*c + b*C)*x^5
)/5 + (B*c*x^6)/6 + (c*C*x^7)/7
```

3.3.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2+cx^4\right)\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \; 2188$$

$$\int \left(x^2(aC+Ab)+aA+aBx+x^4(Ac+bC)+bBx^3+Bcx^5+cCx^6\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}x^3(aC+Ab)+aAx+\frac{1}{2}aBx^2+\frac{1}{5}x^5(Ac+bC)+\frac{1}{4}bBx^4+\frac{1}{6}Bcx^6+\frac{1}{7}cCx^7$$

input
```
Int[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

output
```
a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + (b*B*x^4)/4 + ((A*c + b*C)*x^5
)/5 + (B*c*x^6)/6 + (c*C*x^7)/7
```

3.3.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.3.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

method	result	size
default	$aAx+\frac{Ba\,x^{2}}{2}+\frac{(Ab+Ca)x^{3}}{3}+\frac{bB\,x^{4}}{4}+\frac{(Ac+Cb)x^{5}}{5}+\frac{Bc\,x^{6}}{6}+\frac{cC\,x^{7}}{7}$	58
norman	$\frac{cC\,x^{7}}{7}+\frac{Bc\,x^{6}}{6}+\left(\frac{Ac}{5}+\frac{Cb}{5}\right)x^{5}+\frac{bB\,x^{4}}{4}+\left(\frac{Ab}{3}+\frac{Ca}{3}\right)x^{3}+\frac{Ba\,x^{2}}{2}+aAx$	60
gosper	$\frac{1}{7}cC\,x^{7}+\frac{1}{6}Bc\,x^{6}+\frac{1}{5}x^{5}Ac+\frac{1}{5}x^{5}Cb+\frac{1}{4}bB\,x^{4}+\frac{1}{3}x^{3}Ab+\frac{1}{3}x^{3}Ca+\frac{1}{2}Ba\,x^{2}+aAx$	62
risch	$\frac{1}{7}cC\,x^{7}+\frac{1}{6}Bc\,x^{6}+\frac{1}{5}x^{5}Ac+\frac{1}{5}x^{5}Cb+\frac{1}{4}bB\,x^{4}+\frac{1}{3}x^{3}Ab+\frac{1}{3}x^{3}Ca+\frac{1}{2}Ba\,x^{2}+aAx$	62
parallelrisch	$\frac{1}{7}cC\,x^{7}+\frac{1}{6}Bc\,x^{6}+\frac{1}{5}x^{5}Ac+\frac{1}{5}x^{5}Cb+\frac{1}{4}bB\,x^{4}+\frac{1}{3}x^{3}Ab+\frac{1}{3}x^{3}Ca+\frac{1}{2}Ba\,x^{2}+aAx$	62

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
a*A*x+1/2*B*a*x^2+1/3*(A*b+C*a)*x^3+1/4*b*B*x^4+1/5*(A*c+C*b)*x^5+1/6*B*c*
x^6+1/7*c*C*x^7
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)\,dx=\frac{1}{7}\,Ccx^7+\frac{1}{6}\,Bcx^6+\frac{1}{4}\,Bbx^4+\frac{1}{5}\,(Cb+Ac)x^5+\frac{1}{2}\,Bax^2+\frac{1}{3}\,(Ca+Ab)x^3+Aax$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/7*C*c*x^7 + 1/6*B*c*x^6 + 1/4*B*b*x^4 + 1/5*(C*b + A*c)*x^5 + 1/2*B*a*x^
2 + 1/3*(C*a + A*b)*x^3 + A*a*x
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int (A+Bx+Cx^2)(a+bx^2+cx^4)\,dx = Aax+\frac{Bax^2}{2}+\frac{Bbx^4}{4}+\frac{Bcx^6}{6}+\frac{Ccx^7}{7}+x^5\left(\frac{Ac}{5}+\frac{Cb}{5}\right)+x^3\left(\frac{Ab}{3}+\frac{Ca}{3}\right)$$

input
```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
```

output
```
A*a*x + B*a*x**2/2 + B*b*x**4/4 + B*c*x**6/6 + C*c*x**7/7 + x**5*(A*c/5 +
C*b/5) + x**3*(A*b/3 + C*a/3)
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int (A+Bx+Cx^2)(a+bx^2+cx^4)\,dx = \frac{1}{7}Ccx^7+\frac{1}{6}Bcx^6+\frac{1}{4}Bbx^4+\frac{1}{5}(Cb+Ac)x^5+\frac{1}{2}Bax^2+\frac{1}{3}(Ca+Ab)x^3+Aax$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/7*C*c*x^7 + 1/6*B*c*x^6 + 1/4*B*b*x^4 + 1/5*(C*b + A*c)*x^5 + 1/2*B*a*x^
2 + 1/3*(C*a + A*b)*x^3 + A*a*x
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int (A+Bx+Cx^2)(a+bx^2+cx^4)\,dx = \frac{1}{7}Ccx^7+\frac{1}{6}Bcx^6+\frac{1}{5}Cbx^5+\frac{1}{5}Acx^5+\frac{1}{4}Bbx^4+\frac{1}{3}Cax^3+\frac{1}{3}Abx^3+\frac{1}{2}Bax^2+Aax$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/7*C*c*x^7 + 1/6*B*c*x^6 + 1/5*C*b*x^5 + 1/5*A*c*x^5 + 1/4*B*b*x^4 + 1/3*
C*a*x^3 + 1/3*A*b*x^3 + 1/2*B*a*x^2 + A*a*x
```

3.3.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int (A + Bx + Cx^2) (a + bx^2 + cx^4) \, dx = \frac{C\,c\,x^7}{7} + \frac{B\,c\,x^6}{6} + \left(\frac{A\,c}{5} + \frac{C\,b}{5}\right) x^5 + \frac{B\,b\,x^4}{4} + \left(\frac{A\,b}{3} + \frac{C\,a}{3}\right) x^3 + \frac{B\,a\,x^2}{2} + A\,a\,x$$

input
```
int((A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x)
```

output
```
x^3*((A*b)/3 + (C*a)/3) + x^5*((A*c)/5 + (C*b)/5) + A*a*x + (B*a*x^2)/2 +
(B*b*x^4)/4 + (B*c*x^6)/6 + (C*c*x^7)/7
```

3.4 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x} \, dx$

3.4.1 Optimal result

Integrand size = 26, antiderivative size = 65

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x} \, dx = aBx + \frac{1}{2}(Ab+aC)x^2 + \frac{1}{3}bBx^3 + \frac{1}{4}(Ac+bC)x^4 + \frac{1}{5}Bcx^5 + \frac{1}{6}cCx^6 + aA\log(x)$$

output
```
a*B*x+1/2*(A*b+C*a)*x^2+1/3*b*B*x^3+1/4*(A*c+C*b)*x^4+1/5*B*c*x^5+1/6*c*C*
x^6+a*A*ln(x)
```

3.4.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x} \, dx = aBx + \frac{1}{2}(Ab+aC)x^2 + \frac{1}{3}bBx^3 + \frac{1}{4}(Ac+bC)x^4 + \frac{1}{5}Bcx^5 + \frac{1}{6}cCx^6 + aA\log(x)$$

input
```
Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x,x]
```

output
```
a*B*x + ((A*b + a*C)*x^2)/2 + (b*B*x^3)/3 + ((A*c + b*C)*x^4)/4 + (B*c*x^5
)/5 + (c*C*x^6)/6 + a*A*Log[x]
```

3.4.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)(A+Bx+Cx^2)}{x}\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(x(aC+Ab)+\frac{aA}{x}+aB+x^3(Ac+bC)+bBx^2+Bcx^4+cCx^5\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}x^2(aC+Ab)+aA\log(x)+aBx+\frac{1}{4}x^4(Ac+bC)+\frac{1}{3}bBx^3+\frac{1}{5}Bcx^5+\frac{1}{6}cCx^6$$

input
```
Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x,x]
```

output
```
a*B*x + ((A*b + a*C)*x^2)/2 + (b*B*x^3)/3 + ((A*c + b*C)*x^4)/4 + (B*c*x^5
)/5 + (c*C*x^6)/6 + a*A*Log[x]
```

3.4.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.4.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

method	result	size
norman	$\left(\frac{Ab}{2}+\frac{Ca}{2}\right) x^{2}+\left(\frac{Ac}{4}+\frac{Cb}{4}\right) x^{4}+Bax+\frac{Bb\,x^{3}}{3}+\frac{Bc\,x^{5}}{5}+\frac{cC\,x^{6}}{6}+aA\ln\left(x\right)$	58
default	$\frac{cC\,x^{6}}{6}+\frac{Bc\,x^{5}}{5}+\frac{Ac\,x^{4}}{4}+\frac{Cb\,x^{4}}{4}+\frac{Bb\,x^{3}}{3}+\frac{Ab\,x^{2}}{2}+\frac{Ca\,x^{2}}{2}+Bax+aA\ln\left(x\right)$	60
risch	$\frac{cC\,x^{6}}{6}+\frac{Bc\,x^{5}}{5}+\frac{Ac\,x^{4}}{4}+\frac{Cb\,x^{4}}{4}+\frac{Bb\,x^{3}}{3}+\frac{Ab\,x^{2}}{2}+\frac{Ca\,x^{2}}{2}+Bax+aA\ln\left(x\right)$	60
parallelrisch	$\frac{cC\,x^{6}}{6}+\frac{Bc\,x^{5}}{5}+\frac{Ac\,x^{4}}{4}+\frac{Cb\,x^{4}}{4}+\frac{Bb\,x^{3}}{3}+\frac{Ab\,x^{2}}{2}+\frac{Ca\,x^{2}}{2}+Bax+aA\ln\left(x\right)$	60

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x,x,method=_RETURNVERBOSE)
```

output
```
(1/2*A*b+1/2*C*a)*x^2+(1/4*A*c+1/4*C*b)*x^4+B*a*x+1/3*B*b*x^3+1/5*B*c*x^5+
1/6*c*C*x^6+a*A*ln(x)
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)}{x}\,dx = \frac{1}{6}\,Ccx^6+\frac{1}{5}\,Bcx^5+\frac{1}{3}\,Bbx^3+\frac{1}{4}\,(Cb+Ac)x^4 + Bax+\frac{1}{2}\,(Ca+Ab)x^2+Aa\log\left(x\right)$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x,x, algorithm="fricas")
```

output
```
1/6*C*c*x^6 + 1/5*B*c*x^5 + 1/3*B*b*x^3 + 1/4*(C*b + A*c)*x^4 + B*a*x + 1/
2*(C*a + A*b)*x^2 + A*a*log(x)
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x}\,dx = Aa\log(x) + Bax + \frac{Bbx^3}{3} + \frac{Bcx^5}{5} + \frac{Ccx^6}{6} + x^4\left(\frac{Ac}{4} + \frac{Cb}{4}\right) + x^2\left(\frac{Ab}{2} + \frac{Ca}{2}\right)$$

input
```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x,x)
```

output
```
A*a*log(x) + B*a*x + B*b*x**3/3 + B*c*x**5/5 + C*c*x**6/6 + x**4*(A*c/4 +
C*b/4) + x**2*(A*b/2 + C*a/2)
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x}\,dx = \frac{1}{6}Ccx^6 + \frac{1}{5}Bcx^5 + \frac{1}{3}Bbx^3 + \frac{1}{4}(Cb+Ac)x^4 + Bax + \frac{1}{2}(Ca+Ab)x^2 + Aa\log(x)$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x,x, algorithm="maxima")
```

output
```
1/6*C*c*x^6 + 1/5*B*c*x^5 + 1/3*B*b*x^3 + 1/4*(C*b + A*c)*x^4 + B*a*x + 1/
2*(C*a + A*b)*x^2 + A*a*log(x)
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x}\,dx = \frac{1}{6}Ccx^6 + \frac{1}{5}Bcx^5 + \frac{1}{4}Cbx^4 + \frac{1}{4}Acx^4 + \frac{1}{3}Bbx^3 + \frac{1}{2}Cax^2 + \frac{1}{2}Abx^2 + Bax + Aa\log(|x|)$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x,x, algorithm="giac")
```

output
```
1/6*C*c*x^6 + 1/5*B*c*x^5 + 1/4*C*b*x^4 + 1/4*A*c*x^4 + 1/3*B*b*x^3 + 1/2*
C*a*x^2 + 1/2*A*b*x^2 + B*a*x + A*a*log(abs(x))
```

3.4.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x}\,dx = x^2\left(\frac{A\,b}{2}+\frac{C\,a}{2}\right)+x^4\left(\frac{A\,c}{4}+\frac{C\,b}{4}\right)+B\,a\,x+\frac{B\,b\,x^3}{3}+\frac{B\,c\,x^5}{5}+\frac{C\,c\,x^6}{6}+A\,a\,\ln\left(x\right)$$

input
```
int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x,x)
```

output
```
x^2*((A*b)/2 + (C*a)/2) + x^4*((A*c)/4 + (C*b)/4) + B*a*x + (B*b*x^3)/3 +
(B*c*x^5)/5 + (C*c*x^6)/6 + A*a*log(x)
```

3.5 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^2}\,dx$

3.5.1 Optimal result

Integrand size = 26, antiderivative size = 63

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^2}\,dx = -\frac{aA}{x}+(Ab+aC)x+\frac{1}{2}bBx^2+\frac{1}{3}(Ac+bC)x^3 + \frac{1}{4}Bcx^4+\frac{1}{5}cCx^5+aB\log(x)$$

output
```
-a*A/x+(A*b+C*a)*x+1/2*b*B*x^2+1/3*(A*c+C*b)*x^3+1/4*B*c*x^4+1/5*c*C*x^5+a
*B*ln(x)
```

3.5.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^2}\,dx = -\frac{aA}{x}+(Ab+aC)x+\frac{1}{2}bBx^2+\frac{1}{3}(Ac+bC)x^3 + \frac{1}{4}Bcx^4+\frac{1}{5}cCx^5+aB\log(x)$$

input
```
Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^2,x]
```

output
```
-((a*A)/x) + (A*b + a*C)*x + (b*B*x^2)/2 + ((A*c + b*C)*x^3)/3 + (B*c*x^4)
/4 + (c*C*x^5)/5 + a*B*Log[x]
```

3.5.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)(A+Bx+Cx^2)}{x^2} \, dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(Ab\left(\frac{aC}{Ab}+1\right)+\frac{aA}{x^2}+\frac{aB}{x}+x^2(Ac+bC)+bBx+Bcx^3+cCx^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x(aC+Ab)-\frac{aA}{x}+aB\log(x)+\frac{1}{3}x^3(Ac+bC)+\frac{1}{2}bBx^2+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5$$

input
```
Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^2,x]
```

output
```
-((a*A)/x) + (A*b + a*C)*x + (b*B*x^2)/2 + ((A*c + b*C)*x^3)/3 + (B*c*x^4)
/4 + (c*C*x^5)/5 + a*B*Log[x]
```

3.5.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.5.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{cC\,x^{5}}{5}+\frac{Bc\,x^{4}}{4}+\frac{Ac\,x^{3}}{3}+\frac{Cb\,x^{3}}{3}+\frac{bB\,x^{2}}{2}+Abx+Cax+aB\ln\left(x\right)-\frac{aA}{x}$	57
risch	$\frac{cC\,x^{5}}{5}+\frac{Bc\,x^{4}}{4}+\frac{Ac\,x^{3}}{3}+\frac{Cb\,x^{3}}{3}+\frac{bB\,x^{2}}{2}+Abx+Cax+aB\ln\left(x\right)-\frac{aA}{x}$	57
norman	$\frac{\left(\frac{Ac}{3}+\frac{Cb}{3}\right)x^{4}+(Ab+Ca)x^{2}-Aa+\frac{Bb\,x^{3}}{2}+\frac{Bc\,x^{5}}{4}+\frac{cC\,x^{6}}{5}}{x}+aB\ln\left(x\right)$	61
parallelrisch	$\frac{12cC\,x^{6}+15Bc\,x^{5}+20Ac\,x^{4}+20Cb\,x^{4}+30Bb\,x^{3}+60Ab\,x^{2}+60Ba\ln(x)x+60Ca\,x^{2}-60Aa}{60x}$	67

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*c*C*x^5+1/4*B*c*x^4+1/3*A*c*x^3+1/3*C*b*x^3+1/2*b*B*x^2+A*b*x+C*a*x+a*
B*ln(x)-a*A/x
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int\frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^2}\,dx$$
$$=\frac{12\,Ccx^6+15\,Bcx^5+30\,Bbx^3+20\,(Cb+Ac)x^4+60\,Bax\log(x)+60\,(Ca+Ab)x^2-60\,Aa}{60\,x}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^2,x, algorithm="fricas")
```

output
```
1/60*(12*C*c*x^6 + 15*B*c*x^5 + 30*B*b*x^3 + 20*(C*b + A*c)*x^4 + 60*B*a*x
*log(x) + 60*(C*a + A*b)*x^2 - 60*A*a)/x
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)}{x^2}\,dx = -\frac{Aa}{x} + Ba\log{(x)} + \frac{Bbx^2}{2} + \frac{Bcx^4}{4} + \frac{Ccx^5}{5} + x^3\left(\frac{Ac}{3} + \frac{Cb}{3}\right) + x(Ab + Ca)$$

input
```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**2,x)
```

output
```
-A*a/x + B*a*log(x) + B*b*x**2/2 + B*c*x**4/4 + C*c*x**5/5 + x**3*(A*c/3 +
 C*b/3) + x*(A*b + C*a)
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)}{x^2}\,dx = \frac{1}{5}\,Ccx^5 + \frac{1}{4}\,Bcx^4 + \frac{1}{2}\,Bbx^2 + \frac{1}{3}\,(Cb + Ac)x^3 + Ba\log{(x)} + (Ca + Ab)x - \frac{Aa}{x}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^2,x, algorithm="maxima")
```

output
```
1/5*C*c*x^5 + 1/4*B*c*x^4 + 1/2*B*b*x^2 + 1/3*(C*b + A*c)*x^3 + B*a*log(x)
 + (C*a + A*b)*x - A*a/x
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)}{x^2}\,dx = \frac{1}{5}\,Ccx^5 + \frac{1}{4}\,Bcx^4 + \frac{1}{3}\,Cbx^3 + \frac{1}{3}\,Acx^3 + \frac{1}{2}\,Bbx^2 + Cax + Abx + Ba\log{(|x|)} - \frac{Aa}{x}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^2,x, algorithm="giac")
```

output
```
1/5*C*c*x^5 + 1/4*B*c*x^4 + 1/3*C*b*x^3 + 1/3*A*c*x^3 + 1/2*B*b*x^2 + C*a*
x + A*b*x + B*a*log(abs(x)) - A*a/x
```

3.5.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)}{x^2}\,dx = x\,(A\,b+C\,a)+x^3\left(\frac{A\,c}{3}+\frac{C\,b}{3}\right)-\frac{A\,a}{x}+\frac{B\,b\,x^2}{2}+\frac{B\,c\,x^4}{4}+\frac{C\,c\,x^5}{5}+B\,a\,\ln(x)$$

input
```
int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^2,x)
```

output
```
x*(A*b + C*a) + x^3*((A*c)/3 + (C*b)/3) - (A*a)/x + (B*b*x^2)/2 + (B*c*x^4
)/4 + (C*c*x^5)/5 + B*a*log(x)
```

3.6 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^3} \, dx$

3.6.1 Optimal result

Integrand size = 26, antiderivative size = 63

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^3} \, dx = -\frac{aA}{2x^2} - \frac{aB}{x} + bBx + \frac{1}{2}(Ac+bC)x^2 + \frac{1}{3}Bcx^3 + \frac{1}{4}cCx^4 + (Ab+aC)\log(x)$$

output
```
-1/2*a*A/x^2-a*B/x+b*B*x+1/2*(A*c+C*b)*x^2+1/3*B*c*x^3+1/4*c*C*x^4+(A*b+C*
a)*ln(x)
```

3.6.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^3} \, dx = -\frac{a(A+2Bx)}{2x^2} + \frac{1}{12}x\left(6b(2B+Cx)+cx\left(6A+4Bx+3Cx^2\right)\right) + (Ab+aC)\log(x)$$

input
```
Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^3,x]
```

output
```
-1/2*(a*(A + 2*B*x))/x^2 + (x*(6*b*(2*B + C*x) + c*x*(6*A + 4*B*x + 3*C*x^
2)))/12 + (A*b + a*C)*Log[x]
```

3.6.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)(A+Bx+Cx^2)}{x^3}\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{aC+Ab}{x}+\frac{aA}{x^3}+\frac{aB}{x^2}+x(Ac+bC)+bB+Bcx^2+cCx^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\log(x)(aC+Ab)-\frac{aA}{2x^2}-\frac{aB}{x}+\frac{1}{2}x^2(Ac+bC)+bBx+\frac{1}{3}Bcx^3+\frac{1}{4}cCx^4$$

input
```
Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^3,x]
```

output
```
-1/2*(a*A)/x^2 - (a*B)/x + b*B*x + ((A*c + b*C)*x^2)/2 + (B*c*x^3)/3 + (c*
C*x^4)/4 + (A*b + a*C)*Log[x]
```

3.6.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.6.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{cC\,x^{4}}{4}+\frac{Bc\,x^{3}}{3}+\frac{Ac\,x^{2}}{2}+\frac{Cb\,x^{2}}{2}+Bbx+\left(Ab+Ca\right)\ln\left(x\right)-\frac{aA}{2x^{2}}-\frac{aB}{x}$	58
risch	$\frac{cC\,x^{4}}{4}+\frac{Bc\,x^{3}}{3}+\frac{Ac\,x^{2}}{2}+\frac{Cb\,x^{2}}{2}+Bbx+\frac{-Bax-\frac{1}{2}Aa}{x^{2}}+A\ln\left(x\right)b+C\ln\left(x\right)a$	58
norman	$\frac{\left(\frac{Ac}{2}+\frac{Cb}{2}\right)x^{4}+Bb\,x^{3}-\frac{Aa}{2}-Bax+\frac{Bc\,x^{5}}{3}+\frac{cC\,x^{6}}{4}}{x^{2}}+\left(Ab+Ca\right)\ln\left(x\right)$	59
parallelrisch	$\frac{3cC\,x^{6}+4Bc\,x^{5}+6Ac\,x^{4}+6Cb\,x^{4}+12A\ln\left(x\right)x^{2}b+12Bb\,x^{3}+12C\ln\left(x\right)x^{2}a-12Bax-6Aa}{12x^{2}}$	69

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*c*C*x^4+1/3*B*c*x^3+1/2*A*c*x^2+1/2*C*b*x^2+B*b*x+(A*b+C*a)*ln(x)-1/2*
a*A/x^2-a*B/x
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)}{x^3}\,dx = \frac{3\,Ccx^6+4\,Bcx^5+12\,Bbx^3+6\,(Cb+Ac)x^4+12\,(Ca+Ab)x^2\log\left(x\right)-12\,Bax-6\,Aa}{12\,x^2}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^3,x, algorithm="fricas")
```

output
```
1/12*(3*C*c*x^6 + 4*B*c*x^5 + 12*B*b*x^3 + 6*(C*b + A*c)*x^4 + 12*(C*a + A
*b)*x^2*log(x) - 12*B*a*x - 6*A*a)/x^2
```

3.6.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^3}\,dx = Bbx+\frac{Bcx^3}{3}+\frac{Ccx^4}{4}+x^2\left(\frac{Ac}{2}+\frac{Cb}{2}\right) + (Ab+Ca)\log(x)+\frac{-Aa-2Bax}{2x^2}$$

input
```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**3,x)
```

output
```
B*b*x + B*c*x**3/3 + C*c*x**4/4 + x**2*(A*c/2 + C*b/2) + (A*b + C*a)*log(x
) + (-A*a - 2*B*a*x)/(2*x**2)
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^3}\,dx = \frac{1}{4}\,Ccx^4+\frac{1}{3}\,Bcx^3+Bbx+\frac{1}{2}\,(Cb+Ac)x^2 + (Ca+Ab)\log(x)-\frac{2\,Bax+Aa}{2\,x^2}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^3,x, algorithm="maxima")
```

output
```
1/4*C*c*x^4 + 1/3*B*c*x^3 + B*b*x + 1/2*(C*b + A*c)*x^2 + (C*a + A*b)*log(
x) - 1/2*(2*B*a*x + A*a)/x^2
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^3}\,dx = \frac{1}{4}\,Ccx^4+\frac{1}{3}\,Bcx^3+\frac{1}{2}\,Cbx^2+\frac{1}{2}\,Acx^2 + Bbx+(Ca+Ab)\log(|x|)-\frac{2\,Bax+Aa}{2\,x^2}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^3,x, algorithm="giac")
```

output
```
1/4*C*c*x^4 + 1/3*B*c*x^3 + 1/2*C*b*x^2 + 1/2*A*c*x^2 + B*b*x + (C*a + A*b
)*log(abs(x)) - 1/2*(2*B*a*x + A*a)/x^2
```

3.6.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^3}\,dx = x^2\left(\frac{A\,c}{2}+\frac{C\,b}{2}\right)-\frac{\frac{A\,a}{2}+B\,a\,x}{x^2}+\ln(x)\,(A\,b+C\,a)+B\,b\,x+\frac{B\,c\,x^3}{3}+\frac{C\,c\,x^4}{4}$$

input
```
int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^3,x)
```

output
```
x^2*((A*c)/2 + (C*b)/2) - ((A*a)/2 + B*a*x)/x^2 + log(x)*(A*b + C*a) + B*b
*x + (B*c*x^3)/3 + (C*c*x^4)/4
```

3.7 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4} dx$

3.7.1 Optimal result

Integrand size = 26, antiderivative size = 63

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4} dx = -\frac{aA}{3x^3} - \frac{aB}{2x^2} - \frac{Ab+aC}{x} + (Ac+bC)x + \frac{1}{2}Bcx^2 + \frac{1}{3}cCx^3 + bB\log(x)$$

output
```
-1/3*a*A/x^3-1/2*a*B/x^2+(-A*b-C*a)/x+(A*c+C*b)*x+1/2*B*c*x^2+1/3*c*C*x^3+
b*B*ln(x)
```

3.7.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4} dx = -\frac{Ab}{x} + Acx + bCx + \frac{1}{2}Bcx^2 + \frac{1}{3}cCx^3 - \frac{a(2A+3x(B+2Cx))}{6x^3} + bB\log(x)$$

input
```
Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^4,x]
```

output
```
-((A*b)/x) + A*c*x + b*C*x + (B*c*x^2)/2 + (c*C*x^3)/3 - (a*(2*A + 3*x*(B
+ 2*C*x)))/(6*x^3) + b*B*Log[x]
```

3.7.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)(A+Bx+Cx^2)}{x^4}\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{aC+Ab}{x^2}+\frac{aA}{x^4}+\frac{aB}{x^3}+Ac\left(\frac{bC}{Ac}+1\right)+\frac{bB}{x}+Bcx+cCx^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{aC+Ab}{x}-\frac{aA}{3x^3}-\frac{aB}{2x^2}+x(Ac+bC)+bB\log(x)+\frac{1}{2}Bcx^2+\frac{1}{3}cCx^3$$

input
```
Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^4,x]
```

output
```
-1/3*(a*A)/x^3 - (a*B)/(2*x^2) - (A*b + a*C)/x + (A*c + b*C)*x + (B*c*x^2)
/2 + (c*C*x^3)/3 + b*B*Log[x]
```

3.7.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.7.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{cC\,x^{3}}{3}+\frac{B\,x^{2}c}{2}+Acx+Cbx+bB\ln\left(x\right)-\frac{aB}{2x^{2}}-\frac{Ab+Ca}{x}-\frac{aA}{3x^{3}}$	55
risch	$\frac{cC\,x^{3}}{3}+\frac{B\,x^{2}c}{2}+Acx+Cbx+\frac{\left(-Ab-Ca\right)x^{2}-\frac{Bax}{2}-\frac{Aa}{3}}{x^{3}}+bB\ln\left(x\right)$	56
norman	$\frac{\left(-Ab-Ca\right)x^{2}+\left(Ac+Cb\right)x^{4}-\frac{Aa}{3}-\frac{Bax}{2}+\frac{Bc\,x^{5}}{2}+\frac{cC\,x^{6}}{3}}{x^{3}}+bB\ln\left(x\right)$	59
parallelrisch	$\frac{2cC\,x^{6}+3Bc\,x^{5}+6Ac\,x^{4}+6Bb\ln\left(x\right)x^{3}+6Cb\,x^{4}-6Ab\,x^{2}-6Ca\,x^{2}-3Bax-2Aa}{6x^{3}}$	67

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*c*C*x^3+1/2*B*x^2*c+A*c*x+C*b*x+b*B*ln(x)-1/2*a*B/x^2-(A*b+C*a)/x-1/3*
a*A/x^3
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)}{x^4}\,dx = \frac{2\,Ccx^{6}+3\,Bcx^{5}+6\,Bbx^{3}\log\left(x\right)+6\,(Cb+Ac)x^{4}-3\,Bax-6\,(Ca+Ab)x^{2}-2\,Aa}{6\,x^{3}}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^4,x, algorithm="fricas")
```

output
```
1/6*(2*C*c*x^6 + 3*B*c*x^5 + 6*B*b*x^3*log(x) + 6*(C*b + A*c)*x^4 - 3*B*a*
x - 6*(C*a + A*b)*x^2 - 2*A*a)/x^3
```

3.7.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4}\,dx = Bb\log(x)+\frac{Bcx^2}{2}+\frac{Ccx^3}{3}+x(Ac+Cb)+\frac{-2Aa-3Bax+x^2(-6Ab-6Ca)}{6x^3}$$

input
```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**4,x)
```

output
```
B*b*log(x) + B*c*x**2/2 + C*c*x**3/3 + x*(A*c + C*b) + (-2*A*a - 3*B*a*x +
 x**2*(-6*A*b - 6*C*a))/(6*x**3)
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4}\,dx = \frac{1}{3}\,Ccx^3+\frac{1}{2}\,Bcx^2+Bb\log(x)+(Cb+Ac)x-\frac{3\,Bax+6\,(Ca+Ab)x^2+2\,Aa}{6\,x^3}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^4,x, algorithm="maxima")
```

output
```
1/3*C*c*x^3 + 1/2*B*c*x^2 + B*b*log(x) + (C*b + A*c)*x - 1/6*(3*B*a*x + 6*
(C*a + A*b)*x^2 + 2*A*a)/x^3
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4}\,dx = \frac{1}{3}\,Ccx^3+\frac{1}{2}\,Bcx^2+Cbx+Acx+Bb\log(|x|)-\frac{3\,Bax+6\,(Ca+Ab)x^2+2\,Aa}{6\,x^3}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^4,x, algorithm="giac")
```

output
```
1/3*C*c*x^3 + 1/2*B*c*x^2 + C*b*x + A*c*x + B*b*log(abs(x)) - 1/6*(3*B*a*x
 + 6*(C*a + A*b)*x^2 + 2*A*a)/x^3
```

3.7.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)}{x^4}\,dx = x\,(A\,c+C\,b) - \frac{(A\,b+C\,a)\,x^2+\frac{B\,a\,x}{2}+\frac{A\,a}{3}}{x^3} + \frac{B\,c\,x^2}{2} + \frac{C\,c\,x^3}{3} + B\,b\,\ln{(x)}$$

input
```
int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^4,x)
```

output
```
x*(A*c + C*b) - ((A*a)/3 + x^2*(A*b + C*a) + (B*a*x)/2)/x^3 + (B*c*x^2)/2
+ (C*c*x^3)/3 + B*b*log(x)
```

3.8 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^5} dx$

3.8.1 Optimal result

Integrand size = 26, antiderivative size = 63

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^5} dx = -\frac{aA}{4x^4} - \frac{aB}{3x^3} - \frac{Ab+aC}{2x^2} - \frac{bB}{x} + Bcx + \frac{1}{2}cCx^2 + (Ac+bC)\log(x)$$

output
```
-1/4*a*A/x^4-1/3*a*B/x^3+1/2*(-A*b-C*a)/x^2-b*B/x+B*c*x+1/2*c*C*x^2+(A*c+C
*b)*ln(x)
```

3.8.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^5} dx = -\frac{a(3A+4Bx+6Cx^2)}{12x^4} + \frac{-Ab-2bBx+cx^3(2B+Cx)}{2x^2} + (Ac+bC)\log(x)$$

input
```
Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^5,x]
```

output
```
-1/12*(a*(3*A + 4*B*x + 6*C*x^2))/x^4 + (-(A*b) - 2*b*B*x + c*x^3*(2*B + C
*x))/(2*x^2) + (A*c + b*C)*Log[x]
```

3.8.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)(A+Bx+Cx^2)}{x^5}\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{aC+Ab}{x^3}+\frac{aA}{x^5}+\frac{aB}{x^4}+\frac{Ac+bC}{x}+\frac{bB}{x^2}+Bc+cCx\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{aC+Ab}{2x^2}-\frac{aA}{4x^4}-\frac{aB}{3x^3}+\log(x)(Ac+bC)-\frac{bB}{x}+Bcx+\frac{1}{2}cCx^2$$

input
```
Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^5,x]
```

output
```
-1/4*(a*A)/x^4 - (a*B)/(3*x^3) - (A*b + a*C)/(2*x^2) - (b*B)/x + B*c*x + (
c*C*x^2)/2 + (A*c + b*C)*Log[x]
```

3.8.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.8.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{cC\,x^{2}}{2}+Bcx+\left(Ac+Cb\right)\ln\left(x\right)-\frac{Ab+Ca}{2x^{2}}-\frac{bB}{x}-\frac{aA}{4x^{4}}-\frac{aB}{3x^{3}}$	56
risch	$\frac{cC\,x^{2}}{2}+Bcx+\frac{-Bb\,x^{3}+\left(-\frac{Ab}{2}-\frac{Ca}{2}\right)x^{2}-\frac{Bax}{3}-\frac{Aa}{4}}{x^{4}}+A\ln\left(x\right)c+C\ln\left(x\right)b$	57
norman	$\frac{\left(-\frac{Ab}{2}-\frac{Ca}{2}\right)x^{2}+Bc\,x^{5}-\frac{Aa}{4}-\frac{Bax}{3}-Bb\,x^{3}+\frac{cC\,x^{6}}{2}}{x^{4}}+\left(Ac+Cb\right)\ln\left(x\right)$	59
parallelrisch	$\frac{6cC\,x^{6}+12A\ln\left(x\right)x^{4}c+12Bc\,x^{5}+12C\ln\left(x\right)x^{4}b-12Bb\,x^{3}-6Ab\,x^{2}-6Ca\,x^{2}-4Bax-3Aa}{12x^{4}}$	69

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/2*c*C*x^2+B*c*x+(A*c+C*b)*ln(x)-1/2*(A*b+C*a)/x^2-b*B/x-1/4*a*A/x^4-1/3*
a*B/x^3
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int\frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)}{x^5}\,dx$$
$$=\frac{6\,Ccx^6+12\,Bcx^5+12\,(Cb+Ac)x^4\log{(x)}-12\,Bbx^3-4\,Bax-6\,(Ca+Ab)x^2-3\,Aa}{12\,x^4}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^5,x, algorithm="fricas")
```

output
```
1/12*(6*C*c*x^6 + 12*B*c*x^5 + 12*(C*b + A*c)*x^4*log(x) - 12*B*b*x^3 - 4*
B*a*x - 6*(C*a + A*b)*x^2 - 3*A*a)/x^4
```

3.8.6 Sympy [A] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^5}\,dx = Bcx+\frac{Ccx^2}{2}+(Ac+Cb)\log(x)+\frac{-3Aa-4Bax-12Bbx^3+x^2(-6Ab-6Ca)}{12x^4}$$

input
```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**5,x)
```

output
```
B*c*x + C*c*x**2/2 + (A*c + C*b)*log(x) + (-3*A*a - 4*B*a*x - 12*B*b*x**3
+ x**2*(-6*A*b - 6*C*a))/(12*x**4)
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^5}\,dx = \frac{1}{2}Ccx^2+Bcx+(Cb+Ac)\log(x)-\frac{12\,Bbx^3+4\,Bax+6\,(Ca+Ab)x^2+3\,Aa}{12\,x^4}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^5,x, algorithm="maxima")
```

output
```
1/2*C*c*x^2 + B*c*x + (C*b + A*c)*log(x) - 1/12*(12*B*b*x^3 + 4*B*a*x + 6*
(C*a + A*b)*x^2 + 3*A*a)/x^4
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^5}\,dx = \frac{1}{2}Ccx^2+Bcx+(Cb+Ac)\log(|x|)-\frac{12\,Bbx^3+4\,Bax+6\,(Ca+Ab)x^2+3\,Aa}{12\,x^4}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^5,x, algorithm="giac")
```

output
```
1/2*C*c*x^2 + B*c*x + (C*b + A*c)*log(abs(x)) - 1/12*(12*B*b*x^3 + 4*B*a*x
 + 6*(C*a + A*b)*x^2 + 3*A*a)/x^4
```

3.8.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^5}\,dx = \ln(x)\,(A\,c+C\,b) - \frac{B\,b\,x^3+\left(\frac{A\,b}{2}+\frac{C\,a}{2}\right)\,x^2+\frac{B\,a\,x}{3}+\frac{A\,a}{4}}{x^4} + B\,c\,x+\frac{C\,c\,x^2}{2}$$

input
```
int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^5,x)
```

output
```
log(x)*(A*c + C*b) - ((A*a)/4 + x^2*((A*b)/2 + (C*a)/2) + (B*a*x)/3 + B*b*
x^3)/x^4 + B*c*x + (C*c*x^2)/2
```

3.9 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^6} dx$

3.9.1 Optimal result

Integrand size = 26, antiderivative size = 63

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^6} dx = -\frac{aA}{5x^5}-\frac{aB}{4x^4}-\frac{Ab+aC}{3x^3}-\frac{bB}{2x^2} - \frac{Ac+bC}{x}+cCx+Bc\log(x)$$

output
```
-1/5*a*A/x^5-1/4*a*B/x^4+1/3*(-A*b-C*a)/x^3-1/2*b*B/x^2+(-A*c-C*b)/x+c*C*x
+B*c*ln(x)
```

3.9.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^6} dx = -\frac{12aA-60cCx^6+30bx^3(B+2Cx)+5ax(3B+4Cx)+20Ax^2(b+3cx^2)}{60x^5}+Bc\log(x)$$

input
```
Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^6,x]
```

output
```
-1/60*(12*a*A - 60*c*C*x^6 + 30*b*x^3*(B + 2*C*x) + 5*a*x*(3*B + 4*C*x) +
20*A*x^2*(b + 3*c*x^2))/x^5 + B*c*Log[x]
```

3.9.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)(A+Bx+Cx^2)}{x^6} \, dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{aC+Ab}{x^4}+\frac{aA}{x^6}+\frac{aB}{x^5}+\frac{Ac+bC}{x^2}+\frac{bB}{x^3}+\frac{Bc}{x}+cC\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{aC+Ab}{3x^3}-\frac{aA}{5x^5}-\frac{aB}{4x^4}-\frac{Ac+bC}{x}-\frac{bB}{2x^2}+Bc\log(x)+cCx$$

input
```
Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^6,x]
```

output
```
-1/5*(a*A)/x^5 - (a*B)/(4*x^4) - (A*b + a*C)/(3*x^3) - (b*B)/(2*x^2) - (A*
c + b*C)/x + c*C*x + B*c*Log[x]
```

3.9.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.9.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

method	result	size
default	$cCx+Bc\ln\left(x\right)-\frac{bB}{2x^{2}}-\frac{aA}{5x^{5}}-\frac{Ac+Cb}{x}-\frac{aB}{4x^{4}}-\frac{Ab+Ca}{3x^{3}}$	56
risch	$cCx+\frac{\left(-Ac-Cb\right)x^{4}-\frac{Bb\,x^{3}}{2}+\left(-\frac{Ab}{3}-\frac{Ca}{3}\right)x^{2}-\frac{Bax}{4}-\frac{Aa}{5}}{x^{5}}+Bc\ln\left(x\right)$	58
norman	$\frac{\left(-\frac{Ab}{3}-\frac{Ca}{3}\right)x^{2}+\left(-Ac-Cb\right)x^{4}+cC\,x^{6}-\frac{Aa}{5}-\frac{Bax}{4}-\frac{Bb\,x^{3}}{2}}{x^{5}}+Bc\ln\left(x\right)$	60
parallelrisch	$-\frac{-60Bc\ln\left(x\right)x^{5}-60cC\,x^{6}+60Ac\,x^{4}+60Cb\,x^{4}+30Bb\,x^{3}+20Ab\,x^{2}+20Ca\,x^{2}+15Bax+12Aa}{60x^{5}}$	67

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^6,x,method=_RETURNVERBOSE)
```

output
```
c*C*x+B*c*ln(x)-1/2*b*B/x^2-1/5*a*A/x^5-(A*c+C*b)/x-1/4*a*B/x^4-1/3*(A*b+C
*a)/x^3
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int\frac{\left(A+Bx+Cx^{2}\right)\left(a+bx^{2}+cx^{4}\right)}{x^{6}}\,dx$$
$$=\frac{60\,Ccx^{6}+60\,Bcx^{5}\log\left(x\right)-30\,Bbx^{3}-60\,(Cb+Ac)x^{4}-15\,Bax-20\,(Ca+Ab)x^{2}-12\,Aa}{60\,x^{5}}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^6,x, algorithm="fricas")
```

output
```
1/60*(60*C*c*x^6 + 60*B*c*x^5*log(x) - 30*B*b*x^3 - 60*(C*b + A*c)*x^4 - 1
5*B*a*x - 20*(C*a + A*b)*x^2 - 12*A*a)/x^5
```

3.9.6 Sympy [A] (verification not implemented)

Time = 2.70 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^6}\,dx = Bc\log(x) + Ccx + \frac{-12Aa - 15Bax - 30Bbx^3 + x^4(-60Ac - 60Cb) + x^2(-20Ab - 20Ca)}{60x^5}$$

input
```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**6,x)
```

output
```
B*c*log(x) + C*c*x + (-12*A*a - 15*B*a*x - 30*B*b*x**3 + x**4*(-60*A*c - 6
0*C*b) + x**2*(-20*A*b - 20*C*a))/(60*x**5)
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^6}\,dx = Ccx + Bc\log(x) - \frac{30\,Bbx^3 + 60\,(Cb+Ac)x^4 + 15\,Bax + 20\,(Ca+Ab)x^2 + 12\,Aa}{60\,x^5}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^6,x, algorithm="maxima")
```

output
```
C*c*x + B*c*log(x) - 1/60*(30*B*b*x^3 + 60*(C*b + A*c)*x^4 + 15*B*a*x + 20
*(C*a + A*b)*x^2 + 12*A*a)/x^5
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^6}\,dx = Ccx + Bc\log(|x|) - \frac{30\,Bbx^3 + 60\,(Cb+Ac)x^4 + 15\,Bax + 20\,(Ca+Ab)x^2 + 12\,Aa}{60\,x^5}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^6,x, algorithm="giac")
```

output
```
C*c*x + B*c*log(abs(x)) - 1/60*(30*B*b*x^3 + 60*(C*b + A*c)*x^4 + 15*B*a*x
 + 20*(C*a + A*b)*x^2 + 12*A*a)/x^5
```

3.9.9 Mupad [B] (verification not implemented)

Time = 7.91 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)}{x^6}\,dx$$
$$= C\,c\,x - \frac{(A\,c+C\,b)\,x^4 + \frac{B\,b\,x^3}{2} + \left(\frac{A\,b}{3}+\frac{C\,a}{3}\right)\,x^2 + \frac{B\,a\,x}{4} + \frac{A\,a}{5}}{x^5} + B\,c\,\ln(x)$$

input
```
int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^6,x)
```

output
```
C*c*x - ((A*a)/5 + x^2*((A*b)/3 + (C*a)/3) + x^4*(A*c + C*b) + (B*a*x)/4 +
 (B*b*x^3)/2)/x^5 + B*c*log(x)
```

3.10 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7}\,dx$

3.10.1 Optimal result

Integrand size = 26, antiderivative size = 68

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7}\,dx = -\frac{aA}{6x^6}-\frac{aB}{5x^5}-\frac{Ab+aC}{4x^4}-\frac{bB}{3x^3}-\frac{Ac+bC}{2x^2}-\frac{Bc}{x}+cC\log(x)$$

output
```
-1/6*a*A/x^6-1/5*a*B/x^5+1/4*(-A*b-C*a)/x^4-1/3*b*B/x^3+1/2*(-A*c-C*b)/x^2
-B*c/x+c*C*ln(x)
```

3.10.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7}\,dx = -\frac{a(10A+3x(4B+5Cx))+5x^2(3A(b+2cx^2)+2x(2bB+3bCx+6Bcx^2))}{60x^6}+cC\log(x)$$

input
```
Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^7,x]
```

output
```
-1/60*(a*(10*A + 3*x*(4*B + 5*C*x)) + 5*x^2*(3*A*(b + 2*c*x^2) + 2*x*(2*b*
B + 3*b*C*x + 6*B*c*x^2)))/x^6 + c*C*Log[x]
```

3.10.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)(A+Bx+Cx^2)}{x^7}dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{aC+Ab}{x^5}+\frac{aA}{x^7}+\frac{aB}{x^6}+\frac{Ac+bC}{x^3}+\frac{bB}{x^4}+\frac{Bc}{x^2}+\frac{cC}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{aC+Ab}{4x^4}-\frac{aA}{6x^6}-\frac{aB}{5x^5}-\frac{Ac+bC}{2x^2}-\frac{bB}{3x^3}-\frac{Bc}{x}+cC\log(x)$$

input
```
Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^7,x]
```

output
```
-1/6*(a*A)/x^6 - (a*B)/(5*x^5) - (A*b + a*C)/(4*x^4) - (b*B)/(3*x^3) - (A*
c + b*C)/(2*x^2) - (B*c)/x + c*C*Log[x]
```

3.10.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.10.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

method	result	size
default	$cC\ln\left(x\right)-\frac{Ac+Cb}{2x^{2}}-\frac{aA}{6x^{6}}-\frac{aB}{5x^{5}}-\frac{Bc}{x}-\frac{Ab+Ca}{4x^{4}}-\frac{bB}{3x^{3}}$	59
norman	$\frac{\left(-\frac{Ab}{4}-\frac{Ca}{4}\right)x^{2}+\left(-\frac{Ac}{2}-\frac{Cb}{2}\right)x^{4}-\frac{Aa}{6}-\frac{Bax}{5}-\frac{Bb\,x^{3}}{3}-Bc\,x^{5}}{x^{6}}+cC\ln\left(x\right)$	61
risch	$\frac{\left(-\frac{Ab}{4}-\frac{Ca}{4}\right)x^{2}+\left(-\frac{Ac}{2}-\frac{Cb}{2}\right)x^{4}-\frac{Aa}{6}-\frac{Bax}{5}-\frac{Bb\,x^{3}}{3}-Bc\,x^{5}}{x^{6}}+cC\ln\left(x\right)$	61
parallelrisch	$-\frac{-60Cc\ln(x)x^{6}+60Bc\,x^{5}+30Ac\,x^{4}+30Cb\,x^{4}+20Bb\,x^{3}+15Ab\,x^{2}+15Ca\,x^{2}+12Bax+10Aa}{60x^{6}}$	67

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^7,x,method=_RETURNVERBOSE)
```

output
```
c*C*ln(x)-1/2*(A*c+C*b)/x^2-1/6*a*A/x^6-1/5*a*B/x^5-B*c/x-1/4*(A*b+C*a)/x^
4-1/3*b*B/x^3
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int\frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)}{x^7}\,dx =\frac{60\,Ccx^6\log\left(x\right)-60\,Bcx^5-20\,Bbx^3-30\,(Cb+Ac)x^4-12\,Bax-15\,(Ca+Ab)x^2-10\,Aa}{60\,x^6}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^7,x, algorithm="fricas")
```

output
```
1/60*(60*C*c*x^6*log(x) - 60*B*c*x^5 - 20*B*b*x^3 - 30*(C*b + A*c)*x^4 - 1
2*B*a*x - 15*(C*a + A*b)*x^2 - 10*A*a)/x^6
```

3.10.6 Sympy [A] (verification not implemented)

Time = 8.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7}\,dx$$
$$= Cc\log(x) + \frac{-10Aa-12Bax-20Bbx^3-60Bcx^5+x^4(-30Ac-30Cb)+x^2(-15Ab-15Ca)}{60x^6}$$

input
```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**7,x)
```

output
```
C*c*log(x) + (-10*A*a - 12*B*a*x - 20*B*b*x**3 - 60*B*c*x**5 + x**4*(-30*A
*c - 30*C*b) + x**2*(-15*A*b - 15*C*a))/(60*x**6)
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7}\,dx$$
$$= Cc\log(x) - \frac{60\,Bcx^5+20\,Bbx^3+30\,(Cb+Ac)x^4+12\,Bax+15\,(Ca+Ab)x^2+10\,Aa}{60\,x^6}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^7,x, algorithm="maxima")
```

output
```
C*c*log(x) - 1/60*(60*B*c*x^5 + 20*B*b*x^3 + 30*(C*b + A*c)*x^4 + 12*B*a*x
 + 15*(C*a + A*b)*x^2 + 10*A*a)/x^6
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7}\,dx$$
$$= Cc\log(|x|) - \frac{60\,Bcx^5+20\,Bbx^3+30\,(Cb+Ac)x^4+12\,Bax+15\,(Ca+Ab)x^2+10\,Aa}{60\,x^6}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^7,x, algorithm="giac")
```

output
```
C*c*log(abs(x)) - 1/60*(60*B*c*x^5 + 20*B*b*x^3 + 30*(C*b + A*c)*x^4 + 12*
B*a*x + 15*(C*a + A*b)*x^2 + 10*A*a)/x^6
```

3.10.9 Mupad [B] (verification not implemented)

Time = 7.87 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7}\,dx$$
$$= C\,c\,\ln{(x)} - \frac{B\,c\,x^5 + \left(\frac{A\,c}{2}+\frac{C\,b}{2}\right)\,x^4 + \frac{B\,b\,x^3}{3} + \left(\frac{A\,b}{4}+\frac{C\,a}{4}\right)\,x^2 + \frac{B\,a\,x}{5} + \frac{A\,a}{6}}{x^6}$$

input
```
int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^7,x)
```

output
```
C*c*log(x) - ((A*a)/6 + x^2*((A*b)/4 + (C*a)/4) + x^4*((A*c)/2 + (C*b)/2)
+ (B*a*x)/5 + (B*b*x^3)/3 + B*c*x^5)/x^6
```

3.11 $\int x^2(A+Bx+Cx^2)(a+bx^2+cx^4)^2\,dx$

3.11.1 Optimal result

Integrand size = 28, antiderivative size = 159

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx=\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{5}a(2Ab+aC)x^5+\frac{1}{3}abBx^6+\frac{1}{7}\left(A\left(b^2+2ac\right)+2abC\right)x^7+\frac{1}{8}B\left(b^2+2ac\right)x^8+\frac{1}{9}\left(2Abc+\left(b^2+2ac\right)C\right)x^9+\frac{1}{5}bBcx^{10}+\frac{1}{11}c(Ac+2bC)x^{11}+\frac{1}{12}Bc^2x^{12}+\frac{1}{13}c^2Cx^{13}$$

output
```
1/3*a^2*A*x^3+1/4*a^2*B*x^4+1/5*a*(2*A*b+C*a)*x^5+1/3*a*b*B*x^6+1/7*(A*(2*
a*c+b^2)+2*a*b*C)*x^7+1/8*B*(2*a*c+b^2)*x^8+1/9*(2*A*b*c+(2*a*c+b^2)*C)*x^
9+1/5*b*B*c*x^10+1/11*c*(A*c+2*C*b)*x^11+1/12*B*c^2*x^12+1/13*c^2*C*x^13
```

3.11.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx=\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{5}a(2Ab+aC)x^5+\frac{1}{3}abBx^6+\frac{1}{7}\left(Ab^2+2aAc+2abC\right)x^7+\frac{1}{8}B\left(b^2+2ac\right)x^8+\frac{1}{9}\left(2Abc+b^2C+2acC\right)x^9+\frac{1}{5}bBcx^{10}+\frac{1}{11}c(Ac+2bC)x^{11}+\frac{1}{12}Bc^2x^{12}+\frac{1}{13}c^2Cx^{13}$$

input
```
Integrate[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

output
```
(a^2*A*x^3)/3 + (a^2*B*x^4)/4 + (a*(2*A*b + a*C)*x^5)/5 + (a*b*B*x^6)/3 +
((A*b^2 + 2*a*A*c + 2*a*b*C)*x^7)/7 + (B*(b^2 + 2*a*c)*x^8)/8 + ((2*A*b*c
+ b^2*C + 2*a*c*C)*x^9)/9 + (b*B*c*x^10)/5 + (c*(A*c + 2*b*C)*x^11)/11 + (
B*c^2*x^12)/12 + (c^2*C*x^13)/13
```

3.11.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+bx^2+cx^4\right)^2\left(A+Bx+Cx^2\right)dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(a^2Ax^2+a^2Bx^3+x^8\left(C\left(2ac+b^2\right)+2Abc\right)+x^6\left(A\left(2ac+b^2\right)+2abC\right)+ax^4(aC+2Ab)+Bx^7\left(2ac+b^2\right)+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{9}x^9\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{7}x^7\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{5}ax^5(aC+2Ab)+\frac{1}{8}Bx^8\left(2ac+b^2\right)+\frac{1}{3}abBx^6+\frac{1}{11}cx^{11}(Ac+2bC)+\frac{1}{5}bBcx^{10}+\frac{1}{12}Bc^2x^{12}+\frac{1}{13}c^2Cx^{13}$$

input
```
Int[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

output
```
(a^2*A*x^3)/3 + (a^2*B*x^4)/4 + (a*(2*A*b + a*C)*x^5)/5 + (a*b*B*x^6)/3 +
((A*(b^2 + 2*a*c) + 2*a*b*C)*x^7)/7 + (B*(b^2 + 2*a*c)*x^8)/8 + ((2*A*b*c
+ (b^2 + 2*a*c)*C)*x^9)/9 + (b*B*c*x^10)/5 + (c*(A*c + 2*b*C)*x^11)/11 + (
B*c^2*x^12)/12 + (c^2*C*x^13)/13
```

3.11.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.11.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.89

method	result
default	$\frac{c^2 C x^{13}}{13}+\frac{B c^2 x^{12}}{12}+\frac{(A c^2+2Cbc)x^{11}}{11}+\frac{bBc x^{10}}{5}+\frac{(2Abc+(2ac+b^2)C)x^9}{9}+\frac{B(2ac+b^2)x^8}{8}+\frac{(A(2ac+b^2)+2abC)}{7}$
norman	$\frac{c^2 C x^{13}}{13}+\frac{B c^2 x^{12}}{12}+\left(\frac{1}{11}A c^2+\frac{2}{11}Cbc\right)x^{11}+\frac{bBc x^{10}}{5}+\left(\frac{2}{9}Abc+\frac{2}{9}acC+\frac{1}{9}b^2C\right)x^9+\left(\frac{1}{4}Bac+\frac{1}{8}\right.$
gosper	$\frac{1}{13}c^2C x^{13}+\frac{1}{12}B c^2x^{12}+\frac{1}{11}x^{11}A c^2+\frac{2}{11}x^{11}Cbc+\frac{1}{5}bBc x^{10}+\frac{2}{9}x^9Abc+\frac{2}{9}x^9acC+\frac{1}{9}x^9b^2C+$
risch	$\frac{1}{13}c^2C x^{13}+\frac{1}{12}B c^2x^{12}+\frac{1}{11}x^{11}A c^2+\frac{2}{11}x^{11}Cbc+\frac{1}{5}bBc x^{10}+\frac{2}{9}x^9Abc+\frac{2}{9}x^9acC+\frac{1}{9}x^9b^2C+$
parallelrisch	$\frac{1}{13}c^2C x^{13}+\frac{1}{12}B c^2x^{12}+\frac{1}{11}x^{11}A c^2+\frac{2}{11}x^{11}Cbc+\frac{1}{5}bBc x^{10}+\frac{2}{9}x^9Abc+\frac{2}{9}x^9acC+\frac{1}{9}x^9b^2C+$

input
```
int(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/13*c^2*C*x^13+1/12*B*c^2*x^12+1/11*(A*c^2+2*C*b*c)*x^11+1/5*b*B*c*x^10+1
/9*(2*A*b*c+(2*a*c+b^2)*C)*x^9+1/8*B*(2*a*c+b^2)*x^8+1/7*(A*(2*a*c+b^2)+2*
a*b*C)*x^7+1/3*a*b*B*x^6+1/5*(2*A*a*b+C*a^2)*x^5+1/4*a^2*B*x^4+1/3*a^2*A*x
^3
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.90

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = \frac{1}{13}Cc^2x^{13}+\frac{1}{12}Bc^2x^{12}+\frac{1}{5}Bbcx^{10}+\frac{1}{11}\left(2Cbc+Ac^2\right)x^{11}+\frac{1}{9}\left(Cb^2+2(Ca+Ab)c\right)x^9+\frac{1}{3}Babx^6+\frac{1}{8}\left(Bb^2+2Bac\right)x^8+\frac{1}{7}\left(2Cab+Ab^2+2Aac\right)x^7+\frac{1}{4}Ba^2x^4+\frac{1}{3}Aa^2x^3+\frac{1}{5}\left(Ca^2+2Aab\right)x^5$$

input
```
integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/13*C*c^2*x^13 + 1/12*B*c^2*x^12 + 1/5*B*b*c*x^10 + 1/11*(2*C*b*c + A*c^2
)*x^11 + 1/9*(C*b^2 + 2*(C*a + A*b)*c)*x^9 + 1/3*B*a*b*x^6 + 1/8*(B*b^2 +
2*B*a*c)*x^8 + 1/7*(2*C*a*b + A*b^2 + 2*A*a*c)*x^7 + 1/4*B*a^2*x^4 + 1/3*A
*a^2*x^3 + 1/5*(C*a^2 + 2*A*a*b)*x^5
```

3.11.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.06

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = \frac{Aa^2x^3}{3}+\frac{Ba^2x^4}{4}+\frac{Babx^6}{3}+\frac{Bbcx^{10}}{5}+\frac{Bc^2x^{12}}{12}+\frac{Cc^2x^{13}}{13}+x^{11}\left(\frac{Ac^2}{11}+\frac{2Cbc}{11}\right)+x^9\cdot\left(\frac{2Abc}{9}+\frac{2Cac}{9}+\frac{Cb^2}{9}\right)+x^8\left(\frac{Bac}{4}+\frac{Bb^2}{8}\right)+x^7\cdot\left(\frac{2Aac}{7}+\frac{Ab^2}{7}+\frac{2Cab}{7}\right)+x^5\cdot\left(\frac{2Aab}{5}+\frac{Ca^2}{5}\right)$$

input
```
integrate(x**2*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)
```

output
```
A*a**2*x**3/3 + B*a**2*x**4/4 + B*a*b*x**6/3 + B*b*c*x**10/5 + B*c**2*x**1
2/12 + C*c**2*x**13/13 + x**11*(A*c**2/11 + 2*C*b*c/11) + x**9*(2*A*b*c/9
+ 2*C*a*c/9 + C*b**2/9) + x**8*(B*a*c/4 + B*b**2/8) + x**7*(2*A*a*c/7 + A*
b**2/7 + 2*C*a*b/7) + x**5*(2*A*a*b/5 + C*a**2/5)
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.90

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = \frac{1}{13}\,Cc^2x^{13}+\frac{1}{12}\,Bc^2x^{12} + \frac{1}{5}\,Bbcx^{10}+\frac{1}{11}\left(2\,Cbc+Ac^2\right)x^{11} + \frac{1}{9}\left(Cb^2+2\,(Ca+Ab)c\right)x^9 + \frac{1}{3}\,Babx^6+\frac{1}{8}\left(Bb^2+2\,Bac\right)x^8 + \frac{1}{7}\left(2\,Cab+Ab^2+2\,Aac\right)x^7+\frac{1}{4}\,Ba^2x^4 + \frac{1}{3}\,Aa^2x^3+\frac{1}{5}\left(Ca^2+2\,Aab\right)x^5$$

input
```
integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/13*C*c^2*x^13 + 1/12*B*c^2*x^12 + 1/5*B*b*c*x^10 + 1/11*(2*C*b*c + A*c^2
)*x^11 + 1/9*(C*b^2 + 2*(C*a + A*b)*c)*x^9 + 1/3*B*a*b*x^6 + 1/8*(B*b^2 +
2*B*a*c)*x^8 + 1/7*(2*C*a*b + A*b^2 + 2*A*a*c)*x^7 + 1/4*B*a^2*x^4 + 1/3*A
*a^2*x^3 + 1/5*(C*a^2 + 2*A*a*b)*x^5
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.97

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = \frac{1}{13}Cc^2x^{13}+\frac{1}{12}Bc^2x^{12}+\frac{2}{11}Cbcx^{11}+\frac{1}{11}Ac^2x^{11}+\frac{1}{5}Bbcx^{10}+\frac{1}{9}Cb^2x^9+\frac{2}{9}Cacx^9+\frac{2}{9}Abcx^9+\frac{1}{8}Bb^2x^8+\frac{1}{4}Bacx^8+\frac{2}{7}Cabx^7+\frac{1}{7}Ab^2x^7+\frac{2}{7}Aacx^7+\frac{1}{3}Babx^6+\frac{1}{5}Ca^2x^5+\frac{2}{5}Aabx^5+\frac{1}{4}Ba^2x^4+\frac{1}{3}Aa^2x^3$$

input
```
integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/13*C*c^2*x^13 + 1/12*B*c^2*x^12 + 2/11*C*b*c*x^11 + 1/11*A*c^2*x^11 + 1/
5*B*b*c*x^10 + 1/9*C*b^2*x^9 + 2/9*C*a*c*x^9 + 2/9*A*b*c*x^9 + 1/8*B*b^2*x
^8 + 1/4*B*a*c*x^8 + 2/7*C*a*b*x^7 + 1/7*A*b^2*x^7 + 2/7*A*a*c*x^7 + 1/3*B
*a*b*x^6 + 1/5*C*a^2*x^5 + 2/5*A*a*b*x^5 + 1/4*B*a^2*x^4 + 1/3*A*a^2*x^3
```

3.11.9 Mupad [B] (verification not implemented)

Time = 7.98 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.89

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = x^5\left(\frac{C\,a^2}{5}+\frac{2\,A\,b\,a}{5}\right)+x^{11}\left(\frac{A\,c^2}{11}+\frac{2\,C\,b\,c}{11}\right)+x^7\left(\frac{A\,b^2}{7}+\frac{2\,C\,a\,b}{7}+\frac{2\,A\,a\,c}{7}\right)+x^9\left(\frac{C\,b^2}{9}+\frac{2\,A\,c\,b}{9}+\frac{2\,C\,a\,c}{9}\right)+\frac{A\,a^2\,x^3}{3}+\frac{B\,a^2\,x^4}{4}+\frac{B\,c^2\,x^{12}}{12}+\frac{C\,c^2\,x^{13}}{13}+\frac{B\,x^8\,(b^2+2\,a\,c)}{8}+\frac{B\,a\,b\,x^6}{3}+\frac{B\,b\,c\,x^{10}}{5}$$

input
```
int(x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x)
```

output
```
x^5*((C*a^2)/5 + (2*A*a*b)/5) + x^11*((A*c^2)/11 + (2*C*b*c)/11) + x^7*((A
*b^2)/7 + (2*A*a*c)/7 + (2*C*a*b)/7) + x^9*((C*b^2)/9 + (2*A*b*c)/9 + (2*C
*a*c)/9) + (A*a^2*x^3)/3 + (B*a^2*x^4)/4 + (B*c^2*x^12)/12 + (C*c^2*x^13)/
13 + (B*x^8*(2*a*c + b^2))/8 + (B*a*b*x^6)/3 + (B*b*c*x^10)/5
```

3.11. $\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx$

3.12 $\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx$

3.12.1 Optimal result

Integrand size = 26, antiderivative size = 159

$$\int x\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx=\frac{1}{2}a^2Ax^2+\frac{1}{3}a^2Bx^3+\frac{1}{4}a(2Ab+aC)x^4+\frac{2}{5}abBx^5+\frac{1}{6}\left(A\left(b^2+2ac\right)+2abC\right)x^6+\frac{1}{7}B\left(b^2+2ac\right)x^7+\frac{1}{8}\left(2Abc+\left(b^2+2ac\right)C\right)x^8+\frac{2}{9}bBcx^9+\frac{1}{10}c(Ac+2bC)x^{10}+\frac{1}{11}Bc^2x^{11}+\frac{1}{12}c^2Cx^{12}$$

output
```
1/2*a^2*A*x^2+1/3*a^2*B*x^3+1/4*a*(2*A*b+C*a)*x^4+2/5*a*b*B*x^5+1/6*(A*(2*
a*c+b^2)+2*a*b*C)*x^6+1/7*B*(2*a*c+b^2)*x^7+1/8*(2*A*b*c+(2*a*c+b^2)*C)*x^
8+2/9*b*B*c*x^9+1/10*c*(A*c+2*C*b)*x^10+1/11*B*c^2*x^11+1/12*c^2*C*x^12
```

3.12.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00

$$\int x\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx=\frac{1}{2}a^2Ax^2+\frac{1}{3}a^2Bx^3+\frac{1}{4}a(2Ab+aC)x^4+\frac{2}{5}abBx^5+\frac{1}{6}\left(Ab^2+2aAc+2abC\right)x^6+\frac{1}{7}B\left(b^2+2ac\right)x^7+\frac{1}{8}\left(2Abc+b^2C+2acC\right)x^8+\frac{2}{9}bBcx^9+\frac{1}{10}c(Ac+2bC)x^{10}+\frac{1}{11}Bc^2x^{11}+\frac{1}{12}c^2Cx^{12}$$

input
```
Integrate[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

output
```
(a^2*A*x^2)/2 + (a^2*B*x^3)/3 + (a*(2*A*b + a*C)*x^4)/4 + (2*a*b*B*x^5)/5
+ ((A*b^2 + 2*a*A*c + 2*a*b*C)*x^6)/6 + (B*(b^2 + 2*a*c)*x^7)/7 + ((2*A*b*
c + b^2*C + 2*a*c*C)*x^8)/8 + (2*b*B*c*x^9)/9 + (c*(A*c + 2*b*C)*x^10)/10
+ (B*c^2*x^11)/11 + (c^2*C*x^12)/12
```

3.12.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^2+cx^4\right)^2\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(a^2Ax+a^2Bx^2+x^7\left(C\left(2ac+b^2\right)+2Abc\right)+x^5\left(A\left(2ac+b^2\right)+2abC\right)+ax^3(aC+2Ab)+Bx^6\left(2ac+b^2\right)+2\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}a^2Ax^2+\frac{1}{3}a^2Bx^3+\frac{1}{8}x^8\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{6}x^6\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{4}ax^4(aC+2Ab)+\frac{1}{7}Bx^7\left(2ac+b^2\right)+\frac{2}{5}abBx^5+\frac{1}{10}cx^{10}(Ac+2bC)+\frac{2}{9}bBcx^9+\frac{1}{11}Bc^2x^{11}+\frac{1}{12}c^2Cx^{12}$$

input
```
Int[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

output
```
(a^2*A*x^2)/2 + (a^2*B*x^3)/3 + (a*(2*A*b + a*C)*x^4)/4 + (2*a*b*B*x^5)/5
+ ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^6)/6 + (B*(b^2 + 2*a*c)*x^7)/7 + ((2*A*b*
c + (b^2 + 2*a*c)*C)*x^8)/8 + (2*b*B*c*x^9)/9 + (c*(A*c + 2*b*C)*x^10)/10
+ (B*c^2*x^11)/11 + (c^2*C*x^12)/12
```

3.12.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.12.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.89

method	result
default	$\frac{c^2 C x^{12}}{12}+\frac{B c^2 x^{11}}{11}+\frac{(A c^2+2Cbc)x^{10}}{10}+\frac{2bBc x^9}{9}+\frac{(2Abc+(2ac+b^2)C)x^8}{8}+\frac{B(2ac+b^2)x^7}{7}+\frac{(A(2ac+b^2)+2abC)}{6}$
norman	$\frac{c^2 C x^{12}}{12}+\frac{B c^2 x^{11}}{11}+\left(\frac{1}{10}A c^2+\frac{1}{5}Cbc\right)x^{10}+\frac{2bBc x^9}{9}+\left(\frac{1}{4}Abc+\frac{1}{4}acC+\frac{1}{8}b^2 C\right)x^8+\left(\frac{2}{7}Bac+\frac{1}{7}\right.$
gosper	$\frac{1}{12}c^2 C x^{12}+\frac{1}{11}B c^2 x^{11}+\frac{1}{10}x^{10}A c^2+\frac{1}{5}x^{10}Cbc+\frac{2}{9}bBc x^9+\frac{1}{4}x^8 Abc+\frac{1}{4}x^8 acC+\frac{1}{8}x^8 b^2 C+$
risch	$\frac{1}{12}c^2 C x^{12}+\frac{1}{11}B c^2 x^{11}+\frac{1}{10}x^{10}A c^2+\frac{1}{5}x^{10}Cbc+\frac{2}{9}bBc x^9+\frac{1}{4}x^8 Abc+\frac{1}{4}x^8 acC+\frac{1}{8}x^8 b^2 C+$
parallelrisch	$\frac{1}{12}c^2 C x^{12}+\frac{1}{11}B c^2 x^{11}+\frac{1}{10}x^{10}A c^2+\frac{1}{5}x^{10}Cbc+\frac{2}{9}bBc x^9+\frac{1}{4}x^8 Abc+\frac{1}{4}x^8 acC+\frac{1}{8}x^8 b^2 C+$

input
```
int(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*c^2*C*x^12+1/11*B*c^2*x^11+1/10*(A*c^2+2*C*b*c)*x^10+2/9*b*B*c*x^9+1/
8*(2*A*b*c+(2*a*c+b^2)*C)*x^8+1/7*B*(2*a*c+b^2)*x^7+1/6*(A*(2*a*c+b^2)+2*a
*b*C)*x^6+2/5*a*b*B*x^5+1/4*(2*A*a*b+C*a^2)*x^4+1/3*a^2*B*x^3+1/2*a^2*A*x^
2
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.90

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = \frac{1}{12}Cc^2x^{12}+\frac{1}{11}Bc^2x^{11}+\frac{2}{9}Bbcx^9+\frac{1}{10}\left(2Cbc+Ac^2\right)x^{10}+\frac{1}{8}\left(Cb^2+2(Ca+Ab)c\right)x^8+\frac{2}{5}Babx^5+\frac{1}{7}\left(Bb^2+2Bac\right)x^7+\frac{1}{6}\left(2Cab+Ab^2+2Aac\right)x^6+\frac{1}{3}Ba^2x^3+\frac{1}{2}Aa^2x^2+\frac{1}{4}\left(Ca^2+2Aab\right)x^4$$

input
```
integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/12*C*c^2*x^12 + 1/11*B*c^2*x^11 + 2/9*B*b*c*x^9 + 1/10*(2*C*b*c + A*c^2)
*x^10 + 1/8*(C*b^2 + 2*(C*a + A*b)*c)*x^8 + 2/5*B*a*b*x^5 + 1/7*(B*b^2 + 2
*B*a*c)*x^7 + 1/6*(2*C*a*b + A*b^2 + 2*A*a*c)*x^6 + 1/3*B*a^2*x^3 + 1/2*A*
a^2*x^2 + 1/4*(C*a^2 + 2*A*a*b)*x^4
```

3.12.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.03

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = \frac{Aa^2x^2}{2}+\frac{Ba^2x^3}{3}+\frac{2Babx^5}{5}+\frac{2Bbcx^9}{9}+\frac{Bc^2x^{11}}{11}+\frac{Cc^2x^{12}}{12}+x^{10}\left(\frac{Ac^2}{10}+\frac{Cbc}{5}\right)+x^8\left(\frac{Abc}{4}+\frac{Cac}{4}+\frac{Cb^2}{8}\right)+x^7\cdot\left(\frac{2Bac}{7}+\frac{Bb^2}{7}\right)+x^6\left(\frac{Aac}{3}+\frac{Ab^2}{6}+\frac{Cab}{3}\right)+x^4\left(\frac{Aab}{2}+\frac{Ca^2}{4}\right)$$

input
```
integrate(x*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)
```

output
```
A*a**2*x**2/2 + B*a**2*x**3/3 + 2*B*a*b*x**5/5 + 2*B*b*c*x**9/9 + B*c**2*x
**11/11 + C*c**2*x**12/12 + x**10*(A*c**2/10 + C*b*c/5) + x**8*(A*b*c/4 +
C*a*c/4 + C*b**2/8) + x**7*(2*B*a*c/7 + B*b**2/7) + x**6*(A*a*c/3 + A*b**2
/6 + C*a*b/3) + x**4*(A*a*b/2 + C*a**2/4)
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.90

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = \frac{1}{12}\,Cc^2x^{12}+\frac{1}{11}\,Bc^2x^{11}+\frac{2}{9}\,Bbcx^9+\frac{1}{10}\,(2\,Cbc+Ac^2)x^{10}+\frac{1}{8}\,(Cb^2+2\,(Ca+Ab)c)x^8+\frac{2}{5}\,Babx^5+\frac{1}{7}\,(Bb^2+2\,Bac)x^7+\frac{1}{6}\,(2\,Cab+Ab^2+2\,Aac)x^6+\frac{1}{3}\,Ba^2x^3+\frac{1}{2}\,Aa^2x^2+\frac{1}{4}\,(Ca^2+2\,Aab)x^4$$

input
```
integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/12*C*c^2*x^12 + 1/11*B*c^2*x^11 + 2/9*B*b*c*x^9 + 1/10*(2*C*b*c + A*c^2)
*x^10 + 1/8*(C*b^2 + 2*(C*a + A*b)*c)*x^8 + 2/5*B*a*b*x^5 + 1/7*(B*b^2 + 2
*B*a*c)*x^7 + 1/6*(2*C*a*b + A*b^2 + 2*A*a*c)*x^6 + 1/3*B*a^2*x^3 + 1/2*A*
a^2*x^2 + 1/4*(C*a^2 + 2*A*a*b)*x^4
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.97

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = \frac{1}{12}\,Cc^2x^{12}+\frac{1}{11}\,Bc^2x^{11}+\frac{1}{5}\,Cbcx^{10}+\frac{1}{10}\,Ac^2x^{10}+\frac{2}{9}\,Bbcx^9+\frac{1}{8}\,Cb^2x^8+\frac{1}{4}\,Cacx^8+\frac{1}{4}\,Abcx^8+\frac{1}{7}\,Bb^2x^7+\frac{2}{7}\,Bacx^7+\frac{1}{3}\,Cabx^6+\frac{1}{6}\,Ab^2x^6+\frac{1}{3}\,Aacx^6+\frac{2}{5}\,Babx^5+\frac{1}{4}\,Ca^2x^4+\frac{1}{2}\,Aabx^4+\frac{1}{3}\,Ba^2x^3+\frac{1}{2}\,Aa^2x^2$$

input
```
integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/12*C*c^2*x^12 + 1/11*B*c^2*x^11 + 1/5*C*b*c*x^10 + 1/10*A*c^2*x^10 + 2/9
*B*b*c*x^9 + 1/8*C*b^2*x^8 + 1/4*C*a*c*x^8 + 1/4*A*b*c*x^8 + 1/7*B*b^2*x^7
 + 2/7*B*a*c*x^7 + 1/3*C*a*b*x^6 + 1/6*A*b^2*x^6 + 1/3*A*a*c*x^6 + 2/5*B*a
*b*x^5 + 1/4*C*a^2*x^4 + 1/2*A*a*b*x^4 + 1/3*B*a^2*x^3 + 1/2*A*a^2*x^2
```

3.12.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.89

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = x^4\left(\frac{C\,a^2}{4}+\frac{A\,b\,a}{2}\right)+x^{10}\left(\frac{A\,c^2}{10}+\frac{C\,b\,c}{5}\right) + x^6\left(\frac{A\,b^2}{6}+\frac{C\,a\,b}{3}+\frac{A\,a\,c}{3}\right) + x^8\left(\frac{C\,b^2}{8}+\frac{A\,c\,b}{4}+\frac{C\,a\,c}{4}\right)+\frac{A\,a^2\,x^2}{2} + \frac{B\,a^2\,x^3}{3}+\frac{B\,c^2\,x^{11}}{11}+\frac{C\,c^2\,x^{12}}{12} + \frac{B\,x^7\,(b^2+2\,a\,c)}{7}+\frac{2\,B\,a\,b\,x^5}{5}+\frac{2\,B\,b\,c\,x^9}{9}$$

input
```
int(x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x)
```

output
```
x^4*((C*a^2)/4 + (A*a*b)/2) + x^10*((A*c^2)/10 + (C*b*c)/5) + x^6*((A*b^2)
/6 + (A*a*c)/3 + (C*a*b)/3) + x^8*((C*b^2)/8 + (A*b*c)/4 + (C*a*c)/4) + (A
*a^2*x^2)/2 + (B*a^2*x^3)/3 + (B*c^2*x^11)/11 + (C*c^2*x^12)/12 + (B*x^7*(
2*a*c + b^2))/7 + (2*B*a*b*x^5)/5 + (2*B*b*c*x^9)/9
```

3.13 $\int (A+Bx+Cx^2)(a+bx^2+cx^4)^2 \, dx$

3.13.1 Optimal result

Integrand size = 25, antiderivative size = 154

$$\int (A+Bx+Cx^2)(a+bx^2+cx^4)^2 \, dx = a^2Ax+\frac{1}{2}a^2Bx^2+\frac{1}{3}a(2Ab+aC)x^3+\frac{1}{2}abBx^4 +\frac{1}{5}\left(A\left(b^2+2ac\right)+2abC\right)x^5+\frac{1}{6}B\left(b^2+2ac\right)x^6 +\frac{1}{7}\left(2Abc+\left(b^2+2ac\right)C\right)x^7+\frac{1}{4}bBcx^8 +\frac{1}{9}c(Ac+2bC)x^9+\frac{1}{10}Bc^2x^{10}+\frac{1}{11}c^2Cx^{11}$$

output
```
a^2*A*x+1/2*a^2*B*x^2+1/3*a*(2*A*b+C*a)*x^3+1/2*a*b*B*x^4+1/5*(A*(2*a*c+b^
2)+2*a*b*C)*x^5+1/6*B*(2*a*c+b^2)*x^6+1/7*(2*A*b*c+(2*a*c+b^2)*C)*x^7+1/4*
b*B*c*x^8+1/9*c*(A*c+2*C*b)*x^9+1/10*B*c^2*x^10+1/11*c^2*C*x^11
```

3.13.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00

$$\int (A+Bx+Cx^2)(a+bx^2+cx^4)^2 \, dx = a^2Ax+\frac{1}{2}a^2Bx^2+\frac{1}{3}a(2Ab+aC)x^3+\frac{1}{2}abBx^4 +\frac{1}{5}\left(Ab^2+2aAc+2abC\right)x^5+\frac{1}{6}B\left(b^2+2ac\right)x^6 +\frac{1}{7}\left(2Abc+b^2C+2acC\right)x^7+\frac{1}{4}bBcx^8 +\frac{1}{9}c(Ac+2bC)x^9+\frac{1}{10}Bc^2x^{10}+\frac{1}{11}c^2Cx^{11}$$

input
```
Integrate[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

output
```
a^2*A*x + (a^2*B*x^2)/2 + (a*(2*A*b + a*C)*x^3)/3 + (a*b*B*x^4)/2 + ((A*b^
2 + 2*a*A*c + 2*a*b*C)*x^5)/5 + (B*(b^2 + 2*a*c)*x^6)/6 + ((2*A*b*c + b^2*
C + 2*a*c*C)*x^7)/7 + (b*B*c*x^8)/4 + (c*(A*c + 2*b*C)*x^9)/9 + (B*c^2*x^1
0)/10 + (c^2*C*x^11)/11
```

3.13.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2+cx^4\right)^2 \left(A+Bx+Cx^2\right) dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(a^2A+a^2Bx+x^6\left(C\left(2ac+b^2\right)+2Abc\right)+x^4\left(A\left(2ac+b^2\right)+2abC\right)+ax^2(aC+2Ab)+Bx^5\left(2ac+b^2\right)+2ab\right.$$

$$\Big\downarrow \text{ 2009}$$

$$a^2Ax+\frac{1}{2}a^2Bx^2+\frac{1}{7}x^7\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{5}x^5\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{3}ax^3(aC+2Ab)+\frac{1}{6}Bx^6\left(2ac+b^2\right)+\frac{1}{2}abBx^4+\frac{1}{9}cx^9(Ac+2bC)+\frac{1}{4}bBcx^8+\frac{1}{10}Bc^2x^{10}+\frac{1}{11}c^2Cx^{11}$$

input
```
Int[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

output
```
a^2*A*x + (a^2*B*x^2)/2 + (a*(2*A*b + a*C)*x^3)/3 + (a*b*B*x^4)/2 + ((A*(b
^2 + 2*a*c) + 2*a*b*C)*x^5)/5 + (B*(b^2 + 2*a*c)*x^6)/6 + ((2*A*b*c + (b^2
 + 2*a*c)*C)*x^7)/7 + (b*B*c*x^8)/4 + (c*(A*c + 2*b*C)*x^9)/9 + (B*c^2*x^1
0)/10 + (c^2*C*x^11)/11
```

3.13.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.13.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.90

method	result
default	$\frac{c^2 C x^{11}}{11}+\frac{B c^2 x^{10}}{10}+\frac{(A c^2+2Cbc)x^9}{9}+\frac{bBc x^8}{4}+\frac{(2Abc+(2ac+b^2)C)x^7}{7}+\frac{B(2ac+b^2)x^6}{6}+\frac{(A(2ac+b^2)+2abC)x}{5}$
norman	$\frac{c^2 C x^{11}}{11}+\frac{B c^2 x^{10}}{10}+\left(\frac{1}{9}A c^2+\frac{2}{9}Cbc\right)x^9+\frac{bBc x^8}{4}+\left(\frac{2}{7}Abc+\frac{2}{7}acC+\frac{1}{7}b^2 C\right)x^7+\left(\frac{1}{3}Bac+\frac{1}{6}B b\right.$
gosper	$\frac{1}{11}c^2 C x^{11}+\frac{1}{10}B c^2 x^{10}+\frac{1}{9}x^9 A c^2+\frac{2}{9}x^9 Cbc+\frac{1}{4}bBc x^8+\frac{2}{7}x^7 Abc+\frac{2}{7}x^7 acC+\frac{1}{7}x^7 b^2 C+\frac{1}{3}x$
risch	$\frac{1}{11}c^2 C x^{11}+\frac{1}{10}B c^2 x^{10}+\frac{1}{9}x^9 A c^2+\frac{2}{9}x^9 Cbc+\frac{1}{4}bBc x^8+\frac{2}{7}x^7 Abc+\frac{2}{7}x^7 acC+\frac{1}{7}x^7 b^2 C+\frac{1}{3}x$
parallelrisch	$\frac{1}{11}c^2 C x^{11}+\frac{1}{10}B c^2 x^{10}+\frac{1}{9}x^9 A c^2+\frac{2}{9}x^9 Cbc+\frac{1}{4}bBc x^8+\frac{2}{7}x^7 Abc+\frac{2}{7}x^7 acC+\frac{1}{7}x^7 b^2 C+\frac{1}{3}x$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/11*c^2*C*x^11+1/10*B*c^2*x^10+1/9*(A*c^2+2*C*b*c)*x^9+1/4*b*B*c*x^8+1/7*
(2*A*b*c+(2*a*c+b^2)*C)*x^7+1/6*B*(2*a*c+b^2)*x^6+1/5*(A*(2*a*c+b^2)+2*a*b
*C)*x^5+1/2*B*a*b*x^4+1/3*(2*A*a*b+C*a^2)*x^3+1/2*B*a^2*x^2+a^2*A*x
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.91

$$\int (A+Bx+Cx^2)(a+bx^2+cx^4)^2\,dx = \frac{1}{11}Cc^2x^{11}+\frac{1}{10}Bc^2x^{10} + \frac{1}{4}Bbcx^8+\frac{1}{9}(2Cbc+Ac^2)x^9 + \frac{1}{7}(Cb^2+2(Ca+Ab)c)x^7 + \frac{1}{2}Babx^4+\frac{1}{6}(Bb^2+2Bac)x^6 + \frac{1}{5}(2Cab+Ab^2+2Aac)x^5+\frac{1}{2}Ba^2x^2 + Aa^2x+\frac{1}{3}(Ca^2+2Aab)x^3$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/11*C*c^2*x^11 + 1/10*B*c^2*x^10 + 1/4*B*b*c*x^8 + 1/9*(2*C*b*c + A*c^2)*
x^9 + 1/7*(C*b^2 + 2*(C*a + A*b)*c)*x^7 + 1/2*B*a*b*x^4 + 1/6*(B*b^2 + 2*B
*a*c)*x^6 + 1/5*(2*C*a*b + A*b^2 + 2*A*a*c)*x^5 + 1/2*B*a^2*x^2 + A*a^2*x
+ 1/3*(C*a^2 + 2*A*a*b)*x^3
```

3.13.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.07

$$\int (A+Bx+Cx^2)(a+bx^2+cx^4)^2\,dx = Aa^2x+\frac{Ba^2x^2}{2}+\frac{Babx^4}{2}+\frac{Bbcx^8}{4}+\frac{Bc^2x^{10}}{10} + \frac{Cc^2x^{11}}{11}+x^9\left(\frac{Ac^2}{9}+\frac{2Cbc}{9}\right)+x^7 \cdot\left(\frac{2Abc}{7}+\frac{2Cac}{7}+\frac{Cb^2}{7}\right)+x^6\left(\frac{Bac}{3}+\frac{Bb^2}{6}\right)+x^5 \cdot\left(\frac{2Aac}{5}+\frac{Ab^2}{5}+\frac{2Cab}{5}\right)+x^3\cdot\left(\frac{2Aab}{3}+\frac{Ca^2}{3}\right)$$

input
```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)
```

output
```
A*a**2*x + B*a**2*x**2/2 + B*a*b*x**4/2 + B*b*c*x**8/4 + B*c**2*x**10/10 +
 C*c**2*x**11/11 + x**9*(A*c**2/9 + 2*C*b*c/9) + x**7*(2*A*b*c/7 + 2*C*a*c
/7 + C*b**2/7) + x**6*(B*a*c/3 + B*b**2/6) + x**5*(2*A*a*c/5 + A*b**2/5 +
2*C*a*b/5) + x**3*(2*A*a*b/3 + C*a**2/3)
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.91

$$\begin{aligned}\int (A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx &= \frac{1}{11}\,Cc^2x^{11}+\frac{1}{10}\,Bc^2x^{10}\\ &\quad+\frac{1}{4}\,Bbcx^8+\frac{1}{9}\left(2\,Cbc+Ac^2\right)x^9\\ &\quad+\frac{1}{7}\left(Cb^2+2\,(Ca+Ab)c\right)x^7\\ &\quad+\frac{1}{2}\,Babx^4+\frac{1}{6}\left(Bb^2+2\,Bac\right)x^6\\ &\quad+\frac{1}{5}\left(2\,Cab+Ab^2+2\,Aac\right)x^5+\frac{1}{2}\,Ba^2x^2\\ &\quad+Aa^2x+\frac{1}{3}\left(Ca^2+2\,Aab\right)x^3\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/11*C*c^2*x^11 + 1/10*B*c^2*x^10 + 1/4*B*b*c*x^8 + 1/9*(2*C*b*c + A*c^2)*
x^9 + 1/7*(C*b^2 + 2*(C*a + A*b)*c)*x^7 + 1/2*B*a*b*x^4 + 1/6*(B*b^2 + 2*B
*a*c)*x^6 + 1/5*(2*C*a*b + A*b^2 + 2*A*a*c)*x^5 + 1/2*B*a^2*x^2 + A*a^2*x
+ 1/3*(C*a^2 + 2*A*a*b)*x^3
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.98

$$\begin{aligned}\int (A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx &= \frac{1}{11}\,Cc^2x^{11}+\frac{1}{10}\,Bc^2x^{10}+\frac{2}{9}\,Cbcx^9\\ &\quad+\frac{1}{9}\,Ac^2x^9+\frac{1}{4}\,Bbcx^8+\frac{1}{7}\,Cb^2x^7+\frac{2}{7}\,Cacx^7\\ &\quad+\frac{2}{7}\,Abcx^7+\frac{1}{6}\,Bb^2x^6+\frac{1}{3}\,Bacx^6+\frac{2}{5}\,Cabx^5\\ &\quad+\frac{1}{5}\,Ab^2x^5+\frac{2}{5}\,Aacx^5+\frac{1}{2}\,Babx^4\\ &\quad+\frac{1}{3}\,Ca^2x^3+\frac{2}{3}\,Aabx^3+\frac{1}{2}\,Ba^2x^2+Aa^2x\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/11*C*c^2*x^11 + 1/10*B*c^2*x^10 + 2/9*C*b*c*x^9 + 1/9*A*c^2*x^9 + 1/4*B*
b*c*x^8 + 1/7*C*b^2*x^7 + 2/7*C*a*c*x^7 + 2/7*A*b*c*x^7 + 1/6*B*b^2*x^6 +
1/3*B*a*c*x^6 + 2/5*C*a*b*x^5 + 1/5*A*b^2*x^5 + 2/5*A*a*c*x^5 + 1/2*B*a*b*
x^4 + 1/3*C*a^2*x^3 + 2/3*A*a*b*x^3 + 1/2*B*a^2*x^2 + A*a^2*x
```

3.13.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.90

$$\int \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx = x^3\left(\frac{C\,a^2}{3}+\frac{2\,A\,b\,a}{3}\right)+x^9\left(\frac{A\,c^2}{9}+\frac{2\,C\,b\,c}{9}\right) + x^5\left(\frac{A\,b^2}{5}+\frac{2\,C\,a\,b}{5}+\frac{2\,A\,a\,c}{5}\right) + x^7\left(\frac{C\,b^2}{7}+\frac{2\,A\,c\,b}{7}+\frac{2\,C\,a\,c}{7}\right)+\frac{B\,a^2\,x^2}{2} + \frac{B\,c^2\,x^{10}}{10}+\frac{C\,c^2\,x^{11}}{11}+\frac{B\,x^6\,\left(b^2+2\,a\,c\right)}{6} + A\,a^2\,x+\frac{B\,a\,b\,x^4}{2}+\frac{B\,b\,c\,x^8}{4}$$

input
```
int((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x)
```

output
```
x^3*((C*a^2)/3 + (2*A*a*b)/3) + x^9*((A*c^2)/9 + (2*C*b*c)/9) + x^5*((A*b^
2)/5 + (2*A*a*c)/5 + (2*C*a*b)/5) + x^7*((C*b^2)/7 + (2*A*b*c)/7 + (2*C*a*
c)/7) + (B*a^2*x^2)/2 + (B*c^2*x^10)/10 + (C*c^2*x^11)/11 + (B*x^6*(2*a*c
+ b^2))/6 + A*a^2*x + (B*a*b*x^4)/2 + (B*b*c*x^8)/4
```

3.14 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x} \, dx$

3.14.1 Optimal result

Integrand size = 28, antiderivative size = 150

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x} \, dx = {} & a^2Bx + \frac{1}{2}a(2Ab+aC)x^2 + \frac{2}{3}abBx^3 \\ & + \frac{1}{4}\left(A\left(b^2+2ac\right)+2abC\right)x^4 + \frac{1}{5}B\left(b^2+2ac\right)x^5 \\ & + \frac{1}{6}\left(2Abc+\left(b^2+2ac\right)C\right)x^6 \\ & + \frac{2}{7}bBcx^7 + \frac{1}{8}c(Ac+2bC)x^8 \\ & + \frac{1}{9}Bc^2x^9 + \frac{1}{10}c^2Cx^{10} + a^2A\log(x)\end{aligned}$$

output
```
a^2*B*x+1/2*a*(2*A*b+C*a)*x^2+2/3*a*b*B*x^3+1/4*(A*(2*a*c+b^2)+2*a*b*C)*x^
4+1/5*B*(2*a*c+b^2)*x^5+1/6*(2*A*b*c+(2*a*c+b^2)*C)*x^6+2/7*b*B*c*x^7+1/8*
c*(A*c+2*C*b)*x^8+1/9*B*c^2*x^9+1/10*c^2*C*x^10+a^2*A*ln(x)
```

3.14.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2}{x}\,dx = a^2Bx+\frac{1}{2}a(2Ab+aC)x^2+\frac{2}{3}abBx^3 +\frac{1}{4}\left(Ab^2+2aAc+2abC\right)x^4 +\frac{1}{5}B\left(b^2+2ac\right)x^5+\frac{1}{6}\left(2Abc+b^2C+2acC\right)x^6 +\frac{2}{7}bBcx^7+\frac{1}{8}c(Ac+2bC)x^8 +\frac{1}{9}Bc^2x^9+\frac{1}{10}c^2Cx^{10}+a^2A\log(x)$$

input
```
Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x,x]
```

output
```
a^2*B*x + (a*(2*A*b + a*C)*x^2)/2 + (2*a*b*B*x^3)/3 + ((A*b^2 + 2*a*A*c +
2*a*b*C)*x^4)/4 + (B*(b^2 + 2*a*c)*x^5)/5 + ((2*A*b*c + b^2*C + 2*a*c*C)*x
^6)/6 + (2*b*B*c*x^7)/7 + (c*(A*c + 2*b*C)*x^8)/8 + (B*c^2*x^9)/9 + (c^2*C
*x^10)/10 + a^2*A*Log[x]
```

3.14.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2+cx^4\right)^2\left(A+Bx+Cx^2\right)}{x}\,dx$$

$$\Big\downarrow \; 2159$$

$$\int \left(\frac{a^2A}{x}+a^2B+x^5\left(C\left(2ac+b^2\right)+2Abc\right)+x^3\left(A\left(2ac+b^2\right)+2abC\right)+ax(aC+2Ab)+Bx^4\left(2ac+b^2\right)+2abB\right.$$

$$\Big\downarrow \; 2009$$

$$a^2A\log(x)+a^2Bx+\frac{1}{6}x^6\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{4}x^4\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{2}ax^2(aC+2Ab)+\frac{1}{5}Bx^5\left(2ac+b^2\right)+\frac{2}{3}abBx^3+\frac{1}{8}cx^8(Ac+2bC)+\frac{2}{7}bBcx^7+\frac{1}{9}Bc^2x^9+\frac{1}{10}c^2Cx^{10}$$

input
```
Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x,x]
```

output
```
a^2*B*x + (a*(2*A*b + a*C)*x^2)/2 + (2*a*b*B*x^3)/3 + ((A*(b^2 + 2*a*c) +
2*a*b*C)*x^4)/4 + (B*(b^2 + 2*a*c)*x^5)/5 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x
^6)/6 + (2*b*B*c*x^7)/7 + (c*(A*c + 2*b*C)*x^8)/8 + (B*c^2*x^9)/9 + (c^2*C
*x^10)/10 + a^2*A*Log[x]
```

3.14.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.14.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.92

method	result
norman	$\left(\frac{1}{8}Ac^2+\frac{1}{4}Cbc\right)x^8+\left(Aab+\frac{1}{2}Ca^2\right)x^2+\left(\frac{2}{5}Bac+\frac{1}{5}Bb^2\right)x^5+\left(\frac{1}{2}Aac+\frac{1}{4}Ab^2+\frac{1}{2}abC\right)x^4$
default	$\frac{c^2Cx^{10}}{10}+\frac{Bc^2x^9}{9}+\frac{Ac^2x^8}{8}+\frac{Cbcx^8}{4}+\frac{2bBcx^7}{7}+\frac{Abcx^6}{3}+\frac{Cacx^6}{3}+\frac{Cb^2x^6}{6}+\frac{2Bacx^5}{5}+\frac{Bb^2x^5}{5}+\frac{Aacx^4}{2}$
risch	$\frac{c^2Cx^{10}}{10}+\frac{Bc^2x^9}{9}+\frac{Ac^2x^8}{8}+\frac{Cbcx^8}{4}+\frac{2bBcx^7}{7}+\frac{Abcx^6}{3}+\frac{Cacx^6}{3}+\frac{Cb^2x^6}{6}+\frac{2Bacx^5}{5}+\frac{Bb^2x^5}{5}+\frac{Aacx^4}{2}$
parallelrisch	$\frac{c^2Cx^{10}}{10}+\frac{Bc^2x^9}{9}+\frac{Ac^2x^8}{8}+\frac{Cbcx^8}{4}+\frac{2bBcx^7}{7}+\frac{Abcx^6}{3}+\frac{Cacx^6}{3}+\frac{Cb^2x^6}{6}+\frac{2Bacx^5}{5}+\frac{Bb^2x^5}{5}+\frac{Aacx^4}{2}$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x,x,method=_RETURNVERBOSE)
```

output
```
(1/8*A*c^2+1/4*C*b*c)*x^8+(A*a*b+1/2*C*a^2)*x^2+(2/5*B*a*c+1/5*B*b^2)*x^5+
(1/2*A*a*c+1/4*A*b^2+1/2*a*b*C)*x^4+(1/3*A*b*c+1/3*a*c*C+1/6*b^2*C)*x^6+B*
a^2*x+1/9*B*c^2*x^9+1/10*c^2*C*x^10+2/3*B*a*b*x^3+2/7*b*B*c*x^7+a^2*A*ln(x
)
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x}\,dx = \frac{1}{10}\,Cc^2x^{10} + \frac{1}{9}\,Bc^2x^9 + \frac{2}{7}\,Bbcx^7 + \frac{1}{8}\,\big(2\,Cbc + Ac^2\big)x^8 + \frac{1}{6}\,\big(Cb^2 + 2\,(Ca + Ab)c\big)x^6 + \frac{2}{3}\,Babx^3 + \frac{1}{5}\,\big(Bb^2 + 2\,Bac\big)x^5 + \frac{1}{4}\,\big(2\,Cab + Ab^2 + 2\,Aac\big)x^4 + Ba^2x + Aa^2\log{(x)} + \frac{1}{2}\,\big(Ca^2 + 2\,Aab\big)x^2$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x,x, algorithm="fricas")
```

output
```
1/10*C*c^2*x^10 + 1/9*B*c^2*x^9 + 2/7*B*b*c*x^7 + 1/8*(2*C*b*c + A*c^2)*x^
8 + 1/6*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 2/3*B*a*b*x^3 + 1/5*(B*b^2 + 2*B*a
*c)*x^5 + 1/4*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + B*a^2*x + A*a^2*log(x) + 1
/2*(C*a^2 + 2*A*a*b)*x^2
```

3.14.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x}\,dx = Aa^2\log{(x)} + Ba^2x + \frac{2Babx^3}{3} + \frac{2Bbcx^7}{7} + \frac{Bc^2x^9}{9} + \frac{Cc^2x^{10}}{10} + x^8\left(\frac{Ac^2}{8} + \frac{Cbc}{4}\right) + x^6\left(\frac{Abc}{3} + \frac{Cac}{3} + \frac{Cb^2}{6}\right) + x^5\cdot\left(\frac{2Bac}{5} + \frac{Bb^2}{5}\right) + x^4\left(\frac{Aac}{2} + \frac{Ab^2}{4} + \frac{Cab}{2}\right) + x^2\left(Aab + \frac{Ca^2}{2}\right)$$

input
```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x,x)
```

output
```
A*a**2*log(x) + B*a**2*x + 2*B*a*b*x**3/3 + 2*B*b*c*x**7/7 + B*c**2*x**9/9
 + C*c**2*x**10/10 + x**8*(A*c**2/8 + C*b*c/4) + x**6*(A*b*c/3 + C*a*c/3 +
 C*b**2/6) + x**5*(2*B*a*c/5 + B*b**2/5) + x**4*(A*a*c/2 + A*b**2/4 + C*a*
b/2) + x**2*(A*a*b + C*a**2/2)
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.92

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x}\,dx =& \frac{1}{10}\,Cc^2x^{10}+\frac{1}{9}\,Bc^2x^9+\frac{2}{7}\,Bbcx^7 \\ &+\frac{1}{8}\,\left(2\,Cbc+Ac^2\right)x^8+\frac{1}{6}\,\left(Cb^2+2\,(Ca+Ab)c\right)x^6 \\ &+\frac{2}{3}\,Babx^3+\frac{1}{5}\,\left(Bb^2+2\,Bac\right)x^5 \\ &+\frac{1}{4}\,\left(2\,Cab+Ab^2+2\,Aac\right)x^4+Ba^2x \\ &+Aa^2\log\left(x\right)+\frac{1}{2}\,\left(Ca^2+2\,Aab\right)x^2\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x,x, algorithm="maxima")
```

output
```
1/10*C*c^2*x^10 + 1/9*B*c^2*x^9 + 2/7*B*b*c*x^7 + 1/8*(2*C*b*c + A*c^2)*x^
8 + 1/6*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 2/3*B*a*b*x^3 + 1/5*(B*b^2 + 2*B*a
*c)*x^5 + 1/4*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + B*a^2*x + A*a^2*log(x) + 1
/2*(C*a^2 + 2*A*a*b)*x^2
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x}\,dx = \frac{1}{10}Cc^2x^{10} + \frac{1}{9}Bc^2x^9 + \frac{1}{4}Cbcx^8 + \frac{1}{8}Ac^2x^8 + \frac{2}{7}Bbcx^7 + \frac{1}{6}Cb^2x^6 + \frac{1}{3}Cacx^6 + \frac{1}{3}Abcx^6 + \frac{1}{5}Bb^2x^5 + \frac{2}{5}Bacx^5 + \frac{1}{2}Cabx^4 + \frac{1}{4}Ab^2x^4 + \frac{1}{2}Aacx^4 + \frac{2}{3}Babx^3 + \frac{1}{2}Ca^2x^2 + Aabx^2 + Ba^2x + Aa^2\log(|x|)$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x,x, algorithm="giac")
```

output
```
1/10*C*c^2*x^10 + 1/9*B*c^2*x^9 + 1/4*C*b*c*x^8 + 1/8*A*c^2*x^8 + 2/7*B*b*
c*x^7 + 1/6*C*b^2*x^6 + 1/3*C*a*c*x^6 + 1/3*A*b*c*x^6 + 1/5*B*b^2*x^5 + 2/
5*B*a*c*x^5 + 1/2*C*a*b*x^4 + 1/4*A*b^2*x^4 + 1/2*A*a*c*x^4 + 2/3*B*a*b*x^
3 + 1/2*C*a^2*x^2 + A*a*b*x^2 + B*a^2*x + A*a^2*log(abs(x))
```

3.14.9 Mupad [B] (verification not implemented)

Time = 7.86 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x}\,dx = x^2\left(\frac{C\,a^2}{2}+A\,b\,a\right) + x^8\left(\frac{A\,c^2}{8}+\frac{C\,b\,c}{4}\right) + x^4\left(\frac{A\,b^2}{4}+\frac{C\,a\,b}{2}+\frac{A\,a\,c}{2}\right) + x^6\left(\frac{C\,b^2}{6}+\frac{A\,c\,b}{3}+\frac{C\,a\,c}{3}\right) + \frac{B\,c^2\,x^9}{9} + \frac{C\,c^2\,x^{10}}{10} + A\,a^2\,\ln(x) + \frac{B\,x^5\,(b^2+2\,a\,c)}{5} + B\,a^2\,x + \frac{2\,B\,a\,b\,x^3}{3} + \frac{2\,B\,b\,c\,x^7}{7}$$

input
```
int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x,x)
```

output
```
x^2*((C*a^2)/2 + A*a*b) + x^8*((A*c^2)/8 + (C*b*c)/4) + x^4*((A*b^2)/4 + (
A*a*c)/2 + (C*a*b)/2) + x^6*((C*b^2)/6 + (A*b*c)/3 + (C*a*c)/3) + (B*c^2*x
^9)/9 + (C*c^2*x^10)/10 + A*a^2*log(x) + (B*x^5*(2*a*c + b^2))/5 + B*a^2*x
 + (2*B*a*b*x^3)/3 + (2*B*b*c*x^7)/7
```

3.14. $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x}\,dx$

3.15 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^2}\,dx$

3.15.1 Optimal result

Integrand size = 28, antiderivative size = 145

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^2}\,dx = & -\frac{a^2A}{x}+a(2Ab+aC)x+abBx^2 \\ & +\frac{1}{3}\left(A\left(b^2+2ac\right)+2abC\right)x^3+\frac{1}{4}B\left(b^2+2ac\right)x^4 \\ & +\frac{1}{5}\left(2Abc+\left(b^2+2ac\right)C\right)x^5 \\ & +\frac{1}{3}bBcx^6+\frac{1}{7}c(Ac+2bC)x^7 \\ & +\frac{1}{8}Bc^2x^8+\frac{1}{9}c^2Cx^9+a^2B\log(x)\end{aligned}$$

output
```
-a^2*A/x+a*(2*A*b+C*a)*x+a*b*B*x^2+1/3*(A*(2*a*c+b^2)+2*a*b*C)*x^3+1/4*B*(
2*a*c+b^2)*x^4+1/5*(2*A*b*c+(2*a*c+b^2)*C)*x^5+1/3*b*B*c*x^6+1/7*c*(A*c+2*
C*b)*x^7+1/8*B*c^2*x^8+1/9*c^2*C*x^9+a^2*B*ln(x)
```

3.15.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2}{x^2}\,dx = -\frac{a^2A}{x}+a(2Ab+aC)x+abBx^2+\frac{1}{3}\left(Ab^2+2aAc+2abC\right)x^3+\frac{1}{4}B\left(b^2+2ac\right)x^4+\frac{1}{5}\left(2Abc+b^2C+2acC\right)x^5+\frac{1}{3}bBcx^6+\frac{1}{7}c(Ac+2bC)x^7+\frac{1}{8}Bc^2x^8+\frac{1}{9}c^2Cx^9+a^2B\log(x)$$

input
```
Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^2,x]
```

output
```
-((a^2*A)/x) + a*(2*A*b + a*C)*x + a*b*B*x^2 + ((A*b^2 + 2*a*A*c + 2*a*b*C
)*x^3)/3 + (B*(b^2 + 2*a*c)*x^4)/4 + ((2*A*b*c + b^2*C + 2*a*c*C)*x^5)/5 +
 (b*B*c*x^6)/3 + (c*(A*c + 2*b*C)*x^7)/7 + (B*c^2*x^8)/8 + (c^2*C*x^9)/9 +
 a^2*B*Log[x]
```

3.15.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2+cx^4\right)^2\left(A+Bx+Cx^2\right)}{x^2}\,dx$$

$$\Big\downarrow \text{ 2159}$$

$$\int \left(\frac{a^2A}{x^2}+\frac{a^2B}{x}+x^4\left(C\left(2ac+b^2\right)+2Abc\right)+x^2\left(A\left(2ac+b^2\right)+2abC\right)+a(aC+2Ab)+Bx^3\left(2ac+b^2\right)+2abB\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^2A}{x}+a^2B\log(x)+\frac{1}{5}x^5\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{3}x^3\left(A\left(2ac+b^2\right)+2abC\right)+ax(aC+2Ab)+\frac{1}{4}Bx^4\left(2ac+b^2\right)+abBx^2+\frac{1}{7}cx^7(Ac+2bC)+\frac{1}{3}bBcx^6+\frac{1}{8}Bc^2x^8+\frac{1}{9}c^2Cx^9$$

input
```
Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^2,x]
```

output
```
-((a^2*A)/x) + a*(2*A*b + a*C)*x + a*b*B*x^2 + ((A*(b^2 + 2*a*c) + 2*a*b*C
)*x^3)/3 + (B*(b^2 + 2*a*c)*x^4)/4 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^5)/5 +
 (b*B*c*x^6)/3 + (c*(A*c + 2*b*C)*x^7)/7 + (B*c^2*x^8)/8 + (c^2*C*x^9)/9 +
 a^2*B*Log[x]
```

3.15.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.15.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.98

method	result
norman	$\frac{\left(\frac{1}{7}Ac^2+\frac{2}{7}Cbc\right)x^8+\left(\frac{1}{2}Bac+\frac{1}{4}Bb^2\right)x^5+\left(\frac{2}{3}Aac+\frac{1}{3}Ab^2+\frac{2}{3}abC\right)x^4+\left(\frac{2}{5}Abc+\frac{2}{5}acC+\frac{1}{5}b^2C\right)x^6+\left(2Aab+Ca^2\right)x^2+Bab\,x^3-A\,a^2}{x}$
default	$\frac{c^2Cx^9}{9}+\frac{Bc^2x^8}{8}+\frac{Ac^2x^7}{7}+\frac{2Cbcx^7}{7}+\frac{bBcx^6}{3}+\frac{2Abcx^5}{5}+\frac{2Cacx^5}{5}+\frac{Cb^2x^5}{5}+\frac{Bacx^4}{2}+\frac{Bb^2x^4}{4}+\frac{2Aacx}{3}$
risch	$\frac{c^2Cx^9}{9}+\frac{Bc^2x^8}{8}+\frac{Ac^2x^7}{7}+\frac{2Cbcx^7}{7}+\frac{bBcx^6}{3}+\frac{2Abcx^5}{5}+\frac{2Cacx^5}{5}+\frac{Cb^2x^5}{5}+\frac{Bacx^4}{2}+\frac{Bb^2x^4}{4}+\frac{2Aacx}{3}$
parallelrisch	$\frac{280c^2Cx^{10}+315Bc^2x^9+360Ac^2x^8+720Cbcx^8+840bBcx^7+1008Abcx^6+1008Cacx^6+504Cb^2x^6+1260Bacx^5+630Bb^2x^5}{2520x}$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
((1/7*A*c^2+2/7*C*b*c)*x^8+(1/2*B*a*c+1/4*B*b^2)*x^5+(2/3*A*a*c+1/3*A*b^2+
2/3*a*b*C)*x^4+(2/5*A*b*c+2/5*a*c*C+1/5*b^2*C)*x^6+(2*A*a*b+C*a^2)*x^2+B*a
*b*x^3-A*a^2+1/8*B*c^2*x^9+1/9*c^2*C*x^10+1/3*b*B*c*x^7)/x+a^2*B*ln(x)
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^2}\,dx = \frac{280\,Cc^2x^{10}+315\,Bc^2x^9+840\,Bbcx^7+360\,(2\,Cbc+Ac^2)x^8+504\,(Cb^2+2\,(Ca+Ab)c)x^6+2520\,Babx^3}{}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^2,x, algorithm="fricas")
```

output
```
1/2520*(280*C*c^2*x^10 + 315*B*c^2*x^9 + 840*B*b*c*x^7 + 360*(2*C*b*c + A*
c^2)*x^8 + 504*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 2520*B*a*b*x^3 + 630*(B*b^2
 + 2*B*a*c)*x^5 + 840*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 2520*B*a^2*x*log(x
) - 2520*A*a^2 + 2520*(C*a^2 + 2*A*a*b)*x^2)/x
```

3.15.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.08

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^2}\,dx = &-\frac{Aa^2}{x}+Ba^2\log(x)+Babx^2+\frac{Bbcx^6}{3}\\&+\frac{Bc^2x^8}{8}+\frac{Cc^2x^9}{9}+x^7\left(\frac{Ac^2}{7}+\frac{2Cbc}{7}\right)+x^5\\&\cdot\left(\frac{2Abc}{5}+\frac{2Cac}{5}+\frac{Cb^2}{5}\right)+x^4\left(\frac{Bac}{2}+\frac{Bb^2}{4}\right)\\&+x^3\cdot\left(\frac{2Aac}{3}+\frac{Ab^2}{3}+\frac{2Cab}{3}\right)+x\left(2Aab+Ca^2\right)\end{aligned}$$

input
```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**2,x)
```

output
```
-A*a**2/x + B*a**2*log(x) + B*a*b*x**2 + B*b*c*x**6/3 + B*c**2*x**8/8 + C*
c**2*x**9/9 + x**7*(A*c**2/7 + 2*C*b*c/7) + x**5*(2*A*b*c/5 + 2*C*a*c/5 +
C*b**2/5) + x**4*(B*a*c/2 + B*b**2/4) + x**3*(2*A*a*c/3 + A*b**2/3 + 2*C*a
*b/3) + x*(2*A*a*b + C*a**2)
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^2}\,dx = \frac{1}{9}Cc^2x^9 + \frac{1}{8}Bc^2x^8 + \frac{1}{3}Bbcx^6 + \frac{1}{7}(2Cbc+Ac^2)x^7 + \frac{1}{5}(Cb^2+2(Ca+Ab)c)x^5 + Babx^2 + \frac{1}{4}(Bb^2+2Bac)x^4 + \frac{1}{3}(2Cab+Ab^2+2Aac)x^3 + Ba^2\log(x) - \frac{Aa^2}{x} + (Ca^2+2Aab)x$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^2,x, algorithm="maxima")
```

output
```
1/9*C*c^2*x^9 + 1/8*B*c^2*x^8 + 1/3*B*b*c*x^6 + 1/7*(2*C*b*c + A*c^2)*x^7
+ 1/5*(C*b^2 + 2*(C*a + A*b)*c)*x^5 + B*a*b*x^2 + 1/4*(B*b^2 + 2*B*a*c)*x^
4 + 1/3*(2*C*a*b + A*b^2 + 2*A*a*c)*x^3 + B*a^2*log(x) - A*a^2/x + (C*a^2
+ 2*A*a*b)*x
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^2}\,dx = \frac{1}{9}Cc^2x^9 + \frac{1}{8}Bc^2x^8 + \frac{2}{7}Cbcx^7 + \frac{1}{7}Ac^2x^7 + \frac{1}{3}Bbcx^6 + \frac{1}{5}Cb^2x^5 + \frac{2}{5}Cacx^5 + \frac{2}{5}Abcx^5 + \frac{1}{4}Bb^2x^4 + \frac{1}{2}Bacx^4 + \frac{2}{3}Cabx^3 + \frac{1}{3}Ab^2x^3 + \frac{2}{3}Aacx^3 + Babx^2 + Ca^2x + 2Aabx + Ba^2\log(|x|) - \frac{Aa^2}{x}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^2,x, algorithm="giac")
```

output
```
1/9*C*c^2*x^9 + 1/8*B*c^2*x^8 + 2/7*C*b*c*x^7 + 1/7*A*c^2*x^7 + 1/3*B*b*c*
x^6 + 1/5*C*b^2*x^5 + 2/5*C*a*c*x^5 + 2/5*A*b*c*x^5 + 1/4*B*b^2*x^4 + 1/2*
B*a*c*x^4 + 2/3*C*a*b*x^3 + 1/3*A*b^2*x^3 + 2/3*A*a*c*x^3 + B*a*b*x^2 + C*
a^2*x + 2*A*a*b*x + B*a^2*log(abs(x)) - A*a^2/x
```

3.15.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^2}\,dx = x^7\left(\frac{A\,c^2}{7}+\frac{2\,C\,b\,c}{7}\right) + x^3\left(\frac{A\,b^2}{3}+\frac{2\,C\,a\,b}{3}+\frac{2\,A\,a\,c}{3}\right) + x^5\left(\frac{C\,b^2}{5}+\frac{2\,A\,c\,b}{5}+\frac{2\,C\,a\,c}{5}\right) + x\left(C\,a^2+2\,A\,b\,a\right) - \frac{A\,a^2}{x} + \frac{B\,c^2\,x^8}{8}+\frac{C\,c^2\,x^9}{9}+B\,a^2\,\ln(x) + \frac{B\,x^4\,(b^2+2\,a\,c)}{4}+B\,a\,b\,x^2+\frac{B\,b\,c\,x^6}{3}$$

input
```
int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^2,x)
```

output
```
x^7*((A*c^2)/7 + (2*C*b*c)/7) + x^3*((A*b^2)/3 + (2*A*a*c)/3 + (2*C*a*b)/3
) + x^5*((C*b^2)/5 + (2*A*b*c)/5 + (2*C*a*c)/5) + x*(C*a^2 + 2*A*a*b) - (A
*a^2)/x + (B*c^2*x^8)/8 + (C*c^2*x^9)/9 + B*a^2*log(x) + (B*x^4*(2*a*c + b
^2))/4 + B*a*b*x^2 + (B*b*c*x^6)/3
```

3.16 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^3}\,dx$

3.16.1 Optimal result

Integrand size = 28, antiderivative size = 149

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^3}\,dx = -\frac{a^2A}{2x^2}-\frac{a^2B}{x}+2abBx+\frac{1}{2}\left(A\left(b^2+2ac\right)+2abC\right)x^2 + \frac{1}{3}B\left(b^2+2ac\right)x^3+\frac{1}{4}\left(2Abc+\left(b^2+2ac\right)C\right)x^4 + \frac{2}{5}bBcx^5+\frac{1}{6}c(Ac+2bC)x^6+\frac{1}{7}Bc^2x^7 + \frac{1}{8}c^2Cx^8+a(2Ab+aC)\log(x)$$

output
```
-1/2*a^2*A/x^2-a^2*B/x+2*a*b*B*x+1/2*(A*(2*a*c+b^2)+2*a*b*C)*x^2+1/3*B*(2*
a*c+b^2)*x^3+1/4*(2*A*b*c+(2*a*c+b^2)*C)*x^4+2/5*b*B*c*x^5+1/6*c*(A*c+2*C*
b)*x^6+1/7*B*c^2*x^7+1/8*c^2*C*x^8+a*(2*A*b+C*a)*ln(x)
```

3.16.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^3}\,dx = -\frac{a^2(A+2Bx)}{2x^2} + \frac{1}{6}ax\left(6b(2B+Cx)+cx\left(6A+4Bx+3Cx^2\right)\right) + \frac{1}{840}x^2\left(70b^2x(4B+3Cx)+56bcx^3(6B+5Cx) + 15c^2x^5(8B+7Cx)+140A\left(3b^2+3bcx^2+c^2x^4\right)\right) + a(2Ab+aC)\log(x)$$

input
```
Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^3,x]
```

output
```
-1/2*(a^2*(A + 2*B*x))/x^2 + (a*x*(6*b*(2*B + C*x) + c*x*(6*A + 4*B*x + 3*
C*x^2)))/6 + (x^2*(70*b^2*x*(4*B + 3*C*x) + 56*b*c*x^3*(6*B + 5*C*x) + 15*
c^2*x^5*(8*B + 7*C*x) + 140*A*(3*b^2 + 3*b*c*x^2 + c^2*x^4)))/840 + a*(2*A
*b + a*C)*Log[x]
```

3.16.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^2 (A+Bx+Cx^2)}{x^3} \, dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{a^2A}{x^3}+\frac{a^2B}{x^2}+x^3\left(C\left(2ac+b^2\right)+2Abc\right)+x\left(A\left(2ac+b^2\right)+2abC\right)+\frac{a(aC+2Ab)}{x}+Bx^2\left(2ac+b^2\right)+2abB\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{2x^2}-\frac{a^2B}{x}+\frac{1}{4}x^4\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{2}x^2\left(A\left(2ac+b^2\right)+2abC\right)+a\log(x)(aC+2Ab)+\frac{1}{3}Bx^3\left(2ac+b^2\right)+2abBx+\frac{1}{6}cx^6(Ac+2bC)+\frac{2}{5}bBcx^5+\frac{1}{7}Bc^2x^7+\frac{1}{8}c^2Cx^8$$

input
```
Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^3,x]
```

output
```
-1/2*(a^2*A)/x^2 - (a^2*B)/x + 2*a*b*B*x + ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^
2)/2 + (B*(b^2 + 2*a*c)*x^3)/3 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^4)/4 + (2*
b*B*c*x^5)/5 + (c*(A*c + 2*b*C)*x^6)/6 + (B*c^2*x^7)/7 + (c^2*C*x^8)/8 + a
*(2*A*b + a*C)*Log[x]
```

3.16.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.16.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.94

method	result
norman	$\frac{\left(\frac{1}{6} A\,c^{2}+\frac{1}{3} C b c \right) x^{8}+\left(\frac{2}{3} B a c +\frac{1}{3} B\,b^{2}\right) x^{5}+\left(A a c +\frac{1}{2} A\,b^{2}+a b C \right) x^{4}+\left(\frac{1}{2} A b c +\frac{1}{2} a c C +\frac{1}{4} b^{2} C \right) x^{6}-\frac{A\,a^{2}}{2}-B\,a^{2} x +\frac{B\,c^{2} x^{9}}{7}+\frac{c^{2} C\,x^{10}}{8}+2}{x^{2}}$
default	$\frac{c^{2} C\,x^{8}}{8}+\frac{B\,c^{2} x^{7}}{7}+\frac{A\,c^{2} x^{6}}{6}+\frac{C b c\,x^{6}}{3}+\frac{2 b B c\,x^{5}}{5}+\frac{A b c\,x^{4}}{2}+\frac{C a c\,x^{4}}{2}+\frac{C\,b^{2} x^{4}}{4}+\frac{2 B a c\,x^{3}}{3}+\frac{B\,b^{2} x^{3}}{3}+A a c\,x$
risch	$\frac{c^{2} C\,x^{8}}{8}+\frac{B\,c^{2} x^{7}}{7}+\frac{A\,c^{2} x^{6}}{6}+\frac{C b c\,x^{6}}{3}+\frac{2 b B c\,x^{5}}{5}+\frac{A b c\,x^{4}}{2}+\frac{C a c\,x^{4}}{2}+\frac{C\,b^{2} x^{4}}{4}+\frac{2 B a c\,x^{3}}{3}+\frac{B\,b^{2} x^{3}}{3}+A a c\,x$
parallelrisch	$\frac{105 c^{2} C\,x^{10}+120 B\,c^{2} x^{9}+140 A\,c^{2} x^{8}+280 C b c\,x^{8}+336 b B c\,x^{7}+420 A b c\,x^{6}+420 C a c\,x^{6}+210 C\,b^{2} x^{6}+560 B a c\,x^{5}+280 B\,b^{2} x^{5}+84}{840 x^{2}}$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
((1/6*A*c^2+1/3*C*b*c)*x^8+(2/3*B*a*c+1/3*B*b^2)*x^5+(A*a*c+1/2*A*b^2+a*b*
C)*x^4+(1/2*A*b*c+1/2*a*c*C+1/4*b^2*C)*x^6-1/2*A*a^2-B*a^2*x+1/7*B*c^2*x^9
+1/8*c^2*C*x^10+2*B*a*b*x^3+2/5*b*B*c*x^7)/x^2+(2*A*a*b+C*a^2)*ln(x)
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x^3}\,dx$$
$$=\frac{105\,Cc^2x^{10}+120\,Bc^2x^9+336\,Bbcx^7+140\,(2\,Cbc+Ac^2)x^8+210\,(Cb^2+2\,(Ca+Ab)c)x^6+1680\,Babx^3}{8}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^3,x, algorithm="fricas")
```

output
```
1/840*(105*C*c^2*x^10 + 120*B*c^2*x^9 + 336*B*b*c*x^7 + 140*(2*C*b*c + A*c
^2)*x^8 + 210*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 1680*B*a*b*x^3 + 280*(B*b^2
+ 2*B*a*c)*x^5 + 420*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 - 840*B*a^2*x + 840*(
C*a^2 + 2*A*a*b)*x^2*log(x) - 420*A*a^2)/x^2
```

3.16.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^3}\,dx = 2Babx + \frac{2Bbcx^5}{5} + \frac{Bc^2x^7}{7} + \frac{Cc^2x^8}{8} + a(2Ab+Ca)\log(x) + x^6\left(\frac{Ac^2}{6} + \frac{Cbc}{3}\right) + x^4\left(\frac{Abc}{2} + \frac{Cac}{2} + \frac{Cb^2}{4}\right) + x^3\cdot\left(\frac{2Bac}{3} + \frac{Bb^2}{3}\right) + x^2\left(Aac + \frac{Ab^2}{2} + Cab\right) + \frac{-Aa^2 - 2Ba^2x}{2x^2}$$

input
```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**3,x)
```

output
```
2*B*a*b*x + 2*B*b*c*x**5/5 + B*c**2*x**7/7 + C*c**2*x**8/8 + a*(2*A*b + C*
a)*log(x) + x**6*(A*c**2/6 + C*b*c/3) + x**4*(A*b*c/2 + C*a*c/2 + C*b**2/4
) + x**3*(2*B*a*c/3 + B*b**2/3) + x**2*(A*a*c + A*b**2/2 + C*a*b) + (-A*a*
*2 - 2*B*a**2*x)/(2*x**2)
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^3}\,dx = \frac{1}{8}Cc^2x^8 + \frac{1}{7}Bc^2x^7 + \frac{2}{5}Bbcx^5 + \frac{1}{6}(2Cbc + Ac^2)x^6 + \frac{1}{4}(Cb^2 + 2(Ca + Ab)c)x^4 + 2Babx + \frac{1}{3}(Bb^2 + 2Bac)x^3 + \frac{1}{2}(2Cab + Ab^2 + 2Aac)x^2 + (Ca^2 + 2Aab)\log(x) - \frac{2Ba^2x + Aa^2}{2x^2}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^3,x, algorithm="maxima")
```

output
```
1/8*C*c^2*x^8 + 1/7*B*c^2*x^7 + 2/5*B*b*c*x^5 + 1/6*(2*C*b*c + A*c^2)*x^6
+ 1/4*(C*b^2 + 2*(C*a + A*b)*c)*x^4 + 2*B*a*b*x + 1/3*(B*b^2 + 2*B*a*c)*x^
3 + 1/2*(2*C*a*b + A*b^2 + 2*A*a*c)*x^2 + (C*a^2 + 2*A*a*b)*log(x) - 1/2*(
2*B*a^2*x + A*a^2)/x^2
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99

$$\int \frac{(A + Bx + Cx^2)(a + bx^2 + cx^4)^2}{x^3}\,dx = \frac{1}{8}Cc^2x^8 + \frac{1}{7}Bc^2x^7 + \frac{1}{3}Cbcx^6 + \frac{1}{6}Ac^2x^6 + \frac{2}{5}Bbcx^5 + \frac{1}{4}Cb^2x^4 + \frac{1}{2}Cacx^4 + \frac{1}{2}Abcx^4 + \frac{1}{3}Bb^2x^3 + \frac{2}{3}Bacx^3 + Cabx^2 + \frac{1}{2}Ab^2x^2 + Aacx^2 + 2Babx + (Ca^2 + 2Aab)\log(|x|) - \frac{2Ba^2x + Aa^2}{2x^2}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^3,x, algorithm="giac")
```

output
```
1/8*C*c^2*x^8 + 1/7*B*c^2*x^7 + 1/3*C*b*c*x^6 + 1/6*A*c^2*x^6 + 2/5*B*b*c*
x^5 + 1/4*C*b^2*x^4 + 1/2*C*a*c*x^4 + 1/2*A*b*c*x^4 + 1/3*B*b^2*x^3 + 2/3*
B*a*c*x^3 + C*a*b*x^2 + 1/2*A*b^2*x^2 + A*a*c*x^2 + 2*B*a*b*x + (C*a^2 + 2
*A*a*b)*log(abs(x)) - 1/2*(2*B*a^2*x + A*a^2)/x^2
```

3.16.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^3}\,dx = x^6\left(\frac{A\,c^2}{6}+\frac{C\,b\,c}{3}\right)+\ln(x)\,\left(C\,a^2+2\,A\,b\,a\right)+x^2\left(\frac{A\,b^2}{2}+C\,a\,b+A\,a\,c\right)+x^4\left(\frac{C\,b^2}{4}+\frac{A\,c\,b}{2}+\frac{C\,a\,c}{2}\right)-\frac{\frac{A\,a^2}{2}+B\,a^2\,x}{x^2}+\frac{B\,c^2\,x^7}{7}+\frac{C\,c^2\,x^8}{8}+\frac{B\,x^3\,(b^2+2\,a\,c)}{3}+\frac{2\,B\,b\,c\,x^5}{5}+2\,B\,a\,b\,x$$

input
```
int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^3,x)
```

output
```
x^6*((A*c^2)/6 + (C*b*c)/3) + log(x)*(C*a^2 + 2*A*a*b) + x^2*((A*b^2)/2 +
A*a*c + C*a*b) + x^4*((C*b^2)/4 + (A*b*c)/2 + (C*a*c)/2) - ((A*a^2)/2 + B*
a^2*x)/x^2 + (B*c^2*x^7)/7 + (C*c^2*x^8)/8 + (B*x^3*(2*a*c + b^2))/3 + (2*
B*b*c*x^5)/5 + 2*B*a*b*x
```

3.17 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^4}\,dx$

3.17.1 Optimal result

Integrand size = 28, antiderivative size = 149

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^4}\,dx = -\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}-\frac{a(2Ab+aC)}{x} + \left(A\left(b^2+2ac\right)+2abC\right)x+\frac{1}{2}B\left(b^2+2ac\right)x^2 + \frac{1}{3}\left(2Abc+\left(b^2+2ac\right)C\right)x^3 + \frac{1}{2}bBcx^4+\frac{1}{5}c(Ac+2bC)x^5 + \frac{1}{6}Bc^2x^6+\frac{1}{7}c^2Cx^7+2abB\log(x)$$

output
```
-1/3*a^2*A/x^3-1/2*a^2*B/x^2-a*(2*A*b+C*a)/x+(A*(2*a*c+b^2)+2*a*b*C)*x+1/2
*B*(2*a*c+b^2)*x^2+1/3*(2*A*b*c+(2*a*c+b^2)*C)*x^3+1/2*b*B*c*x^4+1/5*c*(A*
c+2*C*b)*x^5+1/6*B*c^2*x^6+1/7*c^2*C*x^7+2*a*b*B*ln(x)
```

3.17.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^4}\,dx = -\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}+\frac{-2aAb-a^2C}{x} + (Ab^2+2aAc+2abC)x+\frac{1}{2}B(b^2+2ac)x^2 + \frac{1}{3}(2Abc+b^2C+2acC)x^3 + \frac{1}{2}bBcx^4+\frac{1}{5}c(Ac+2bC)x^5 + \frac{1}{6}Bc^2x^6+\frac{1}{7}c^2Cx^7+2abB\log(x)$$

input
```
Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^4,x]
```

output
```
-1/3*(a^2*A)/x^3 - (a^2*B)/(2*x^2) + (-2*a*A*b - a^2*C)/x + (A*b^2 + 2*a*A
*c + 2*a*b*C)*x + (B*(b^2 + 2*a*c)*x^2)/2 + ((2*A*b*c + b^2*C + 2*a*c*C)*x
^3)/3 + (b*B*c*x^4)/2 + (c*(A*c + 2*b*C)*x^5)/5 + (B*c^2*x^6)/6 + (c^2*C*x
^7)/7 + 2*a*b*B*Log[x]
```

3.17.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^2(A+Bx+Cx^2)}{x^4}\,dx$$

$$\Big\downarrow \text{ 2159}$$

$$\int \left(\frac{a^2A}{x^4}+\frac{a^2B}{x^3}+x^2(C(2ac+b^2)+2Abc)+Ab^2\left(\frac{2a(Ac+bC)}{Ab^2}+1\right)+\frac{a(aC+2Ab)}{x^2}+Bx(2ac+b^2)+\frac{2abB}{x}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}+\frac{1}{3}x^3\left(C\left(2ac+b^2\right)+2Abc\right)+x\left(A\left(2ac+b^2\right)+2abC\right)-\frac{a(aC+2Ab)}{x}+\frac{1}{2}Bx^2\left(2ac+b^2\right)+2abB\log(x)+\frac{1}{5}cx^5(Ac+2bC)+\frac{1}{2}bBcx^4+\frac{1}{6}Bc^2x^6+\frac{1}{7}c^2Cx^7$$

input
```
Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^4,x]
```

output
```
-1/3*(a^2*A)/x^3 - (a^2*B)/(2*x^2) - (a*(2*A*b + a*C))/x + (A*(b^2 + 2*a*c
) + 2*a*b*C)*x + (B*(b^2 + 2*a*c)*x^2)/2 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^
3)/3 + (b*B*c*x^4)/2 + (c*(A*c + 2*b*C)*x^5)/5 + (B*c^2*x^6)/6 + (c^2*C*x^
7)/7 + 2*a*b*B*Log[x]
```

3.17.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.17.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.95

method	result
norman	$\frac{\left(\frac{1}{5}A\,c^2+\frac{2}{5}Cbc\right)x^8+\left(Bac+\frac{1}{2}B\,b^2\right)x^5+\left(\frac{2}{3}Abc+\frac{2}{3}acC+\frac{1}{3}b^2C\right)x^6+\left(-2Aab-C\,a^2\right)x^2+\left(2Aac+A\,b^2+2abC\right)x^4-\frac{A\,a^2}{3}-\frac{B\,a^2x}{2}+}{x^3}$
default	$\frac{c^2C\,x^7}{7}+\frac{B\,c^2x^6}{6}+\frac{A\,c^2x^5}{5}+\frac{2Cbc\,x^5}{5}+\frac{bBc\,x^4}{2}+\frac{2Abc\,x^3}{3}+\frac{2Cac\,x^3}{3}+\frac{C\,b^2x^3}{3}+Bac\,x^2+\frac{B\,b^2x^2}{2}+2Aa$
risch	$\frac{c^2C\,x^7}{7}+\frac{B\,c^2x^6}{6}+\frac{A\,c^2x^5}{5}+\frac{2Cbc\,x^5}{5}+\frac{bBc\,x^4}{2}+\frac{2Abc\,x^3}{3}+\frac{2Cac\,x^3}{3}+\frac{C\,b^2x^3}{3}+Bac\,x^2+\frac{B\,b^2x^2}{2}+2Aa$
parallelrisch	$\frac{30c^2C\,x^{10}+35B\,c^2x^9+42A\,c^2x^8+84Cbc\,x^8+105bBc\,x^7+140Abc\,x^6+140Cac\,x^6+70C\,b^2x^6+210Bac\,x^5+105B\,b^2x^5+420Aac}{210x^3}$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
((1/5*A*c^2+2/5*C*b*c)*x^8+(B*a*c+1/2*B*b^2)*x^5+(2/3*A*b*c+2/3*a*c*C+1/3*
b^2*C)*x^6+(-2*A*a*b-C*a^2)*x^2+(2*A*a*c+A*b^2+2*C*a*b)*x^4-1/3*A*a^2-1/2*
B*a^2*x+1/6*B*c^2*x^9+1/7*c^2*C*x^10+1/2*b*B*c*x^7)/x^3+2*a*b*B*ln(x)
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^4}\,dx = \frac{30\,Cc^2x^{10}+35\,Bc^2x^9+105\,Bbcx^7+42\,(2\,Cbc+Ac^2)x^8+70\,(Cb^2+2\,(Ca+Ab)c)x^6+420\,Babx^3\log(x}{210}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^4,x, algorithm="fricas")
```

output
```
1/210*(30*C*c^2*x^10 + 35*B*c^2*x^9 + 105*B*b*c*x^7 + 42*(2*C*b*c + A*c^2)
*x^8 + 70*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 420*B*a*b*x^3*log(x) + 105*(B*b^
2 + 2*B*a*c)*x^5 + 210*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 - 105*B*a^2*x - 70*
A*a^2 - 210*(C*a^2 + 2*A*a*b)*x^2)/x^3
```

3.17.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.07

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^4}\,dx &= 2Bab\log(x)+\frac{Bbcx^4}{2}+\frac{Bc^2x^6}{6}+\frac{Cc^2x^7}{7}\\ &\quad+x^5\left(\frac{Ac^2}{5}+\frac{2Cbc}{5}\right)+x^3\cdot\left(\frac{2Abc}{3}+\frac{2Cac}{3}+\frac{Cb^2}{3}\right)\\ &\quad+x^2\left(Bac+\frac{Bb^2}{2}\right)+x\left(2Aac+Ab^2+2Cab\right)\\ &\quad+\frac{-2Aa^2-3Ba^2x+x^2(-12Aab-6Ca^2)}{6x^3}\end{aligned}$$

input
```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**4,x)
```

output
```
2*B*a*b*log(x) + B*b*c*x**4/2 + B*c**2*x**6/6 + C*c**2*x**7/7 + x**5*(A*c*
*2/5 + 2*C*b*c/5) + x**3*(2*A*b*c/3 + 2*C*a*c/3 + C*b**2/3) + x**2*(B*a*c
+ B*b**2/2) + x*(2*A*a*c + A*b**2 + 2*C*a*b) + (-2*A*a**2 - 3*B*a**2*x + x
**2*(-12*A*a*b - 6*C*a**2))/(6*x**3)
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^4}\,dx = \frac{1}{7}Cc^2x^7+\frac{1}{6}Bc^2x^6+\frac{1}{2}Bbcx^4+\frac{1}{5}(2Cbc+Ac^2)x^5 + \frac{1}{3}(Cb^2+2(Ca+Ab)c)x^3+2Bab\log(x) + \frac{1}{2}(Bb^2+2Bac)x^2+(2Cab+Ab^2+2Aac)x - \frac{3Ba^2x+2Aa^2+6(Ca^2+2Aab)x^2}{6x^3}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^4,x, algorithm="maxima")
```

output
```
1/7*C*c^2*x^7 + 1/6*B*c^2*x^6 + 1/2*B*b*c*x^4 + 1/5*(2*C*b*c + A*c^2)*x^5
+ 1/3*(C*b^2 + 2*(C*a + A*b)*c)*x^3 + 2*B*a*b*log(x) + 1/2*(B*b^2 + 2*B*a*
c)*x^2 + (2*C*a*b + A*b^2 + 2*A*a*c)*x - 1/6*(3*B*a^2*x + 2*A*a^2 + 6*(C*a
^2 + 2*A*a*b)*x^2)/x^3
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^4}\,dx = \frac{1}{7}Cc^2x^7+\frac{1}{6}Bc^2x^6+\frac{2}{5}Cbcx^5+\frac{1}{5}Ac^2x^5 + \frac{1}{2}Bbcx^4+\frac{1}{3}Cb^2x^3+\frac{2}{3}Cacx^3 + \frac{2}{3}Abcx^3+\frac{1}{2}Bb^2x^2+Bacx^2+2Cabx + Ab^2x+2Aacx+2Bab\log(|x|) - \frac{3Ba^2x+2Aa^2+6(Ca^2+2Aab)x^2}{6x^3}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^4,x, algorithm="giac")
```

output
```
1/7*C*c^2*x^7 + 1/6*B*c^2*x^6 + 2/5*C*b*c*x^5 + 1/5*A*c^2*x^5 + 1/2*B*b*c*
x^4 + 1/3*C*b^2*x^3 + 2/3*C*a*c*x^3 + 2/3*A*b*c*x^3 + 1/2*B*b^2*x^2 + B*a*
c*x^2 + 2*C*a*b*x + A*b^2*x + 2*A*a*c*x + 2*B*a*b*log(abs(x)) - 1/6*(3*B*a
^2*x + 2*A*a^2 + 6*(C*a^2 + 2*A*a*b)*x^2)/x^3
```

3.17.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^4}\,dx = x^5\left(\frac{A\,c^2}{5}+\frac{2\,C\,b\,c}{5}\right) - \frac{x^2\,(C\,a^2+2\,A\,b\,a)+\frac{A\,a^2}{3}+\frac{B\,a^2\,x}{2}}{x^3} + x\,\left(A\,b^2+2\,C\,a\,b+2\,A\,a\,c\right) + x^3\left(\frac{C\,b^2}{3}+\frac{2\,A\,c\,b}{3}+\frac{2\,C\,a\,c}{3}\right) + \frac{B\,c^2\,x^6}{6}+\frac{C\,c^2\,x^7}{7}+\frac{B\,x^2\,(b^2+2\,a\,c)}{2} + \frac{B\,b\,c\,x^4}{2}+2\,B\,a\,b\,\ln(x)$$

input
```
int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^4,x)
```

output
```
x^5*((A*c^2)/5 + (2*C*b*c)/5) - (x^2*(C*a^2 + 2*A*a*b) + (A*a^2)/3 + (B*a^
2*x)/2)/x^3 + x*(A*b^2 + 2*A*a*c + 2*C*a*b) + x^3*((C*b^2)/3 + (2*A*b*c)/3
 + (2*C*a*c)/3) + (B*c^2*x^6)/6 + (C*c^2*x^7)/7 + (B*x^2*(2*a*c + b^2))/2
+ (B*b*c*x^4)/2 + 2*B*a*b*log(x)
```

3.18 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^5}\,dx$

3.18.1 Optimal result

Integrand size = 28, antiderivative size = 148

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^5}\,dx = -\frac{a^2A}{4x^4}-\frac{a^2B}{3x^3}-\frac{a(2Ab+aC)}{2x^2}-\frac{2abB}{x} + B(b^2+2ac)\,x+\frac{1}{2}(2Abc+(b^2+2ac)\,C)\,x^2 + \frac{2}{3}bBcx^3+\frac{1}{4}c(Ac+2bC)x^4+\frac{1}{5}Bc^2x^5 + \frac{1}{6}c^2Cx^6+(A(b^2+2ac)+2abC)\log(x)$$

output
```
-1/4*a^2*A/x^4-1/3*a^2*B/x^3-1/2*a*(2*A*b+C*a)/x^2-2*a*b*B/x+B*(2*a*c+b^2)
*x+1/2*(2*A*b*c+(2*a*c+b^2)*C)*x^2+2/3*b*B*c*x^3+1/4*c*(A*c+2*C*b)*x^4+1/5
*B*c^2*x^5+1/6*c^2*C*x^6+(A*(2*a*c+b^2)+2*a*b*C)*ln(x)
```

3.18.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^5}\,dx = -\frac{a^2(3A+4Bx+6Cx^2)}{12x^4} + \frac{a(-Ab-2bBx+cx^3(2B+Cx))}{x^2} + \frac{1}{60}x(30b^2(2B+Cx)+10bcx(6A+x(4B+3Cx)) + c^2x^3(15A+2x(6B+5Cx))) + (A(b^2+2ac)+2abC)\log(x)$$

input
```
Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^5,x]
```

output
```
-1/12*(a^2*(3*A + 4*B*x + 6*C*x^2))/x^4 + (a*(-(A*b) - 2*b*B*x + c*x^3*(2*
B + C*x)))/x^2 + (x*(30*b^2*(2*B + C*x) + 10*b*c*x*(6*A + x*(4*B + 3*C*x))
 + c^2*x^3*(15*A + 2*x*(6*B + 5*C*x))))/60 + (A*(b^2 + 2*a*c) + 2*a*b*C)*L
og[x]
```

3.18.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^2(A+Bx+Cx^2)}{x^5}dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{a^2A}{x^5}+\frac{a^2B}{x^4}+x\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{A\left(2ac+b^2\right)+2abC}{x}+\frac{a(aC+2Ab)}{x^3}+B\left(2ac+b^2\right)+\frac{2abB}{x^2}+cx^3\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{4x^4}-\frac{a^2B}{3x^3}+\frac{1}{2}x^2\left(C\left(2ac+b^2\right)+2Abc\right)+\log(x)\left(A\left(2ac+b^2\right)+2abC\right)-\frac{a(aC+2Ab)}{2x^2}+Bx\left(2ac+b^2\right)-\frac{2abB}{x}+\frac{1}{4}cx^4(Ac+2bC)+\frac{2}{3}bBcx^3+\frac{1}{5}Bc^2x^5+\frac{1}{6}c^2Cx^6$$

input
```
Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^5,x]
```

output
```
-1/4*(a^2*A)/x^4 - (a^2*B)/(3*x^3) - (a*(2*A*b + a*C))/(2*x^2) - (2*a*b*B)
/x + B*(b^2 + 2*a*c)*x + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^2)/2 + (2*b*B*c*x^
3)/3 + (c*(A*c + 2*b*C)*x^4)/4 + (B*c^2*x^5)/5 + (c^2*C*x^6)/6 + (A*(b^2 +
 2*a*c) + 2*a*b*C)*Log[x]
```

3.18.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.18.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.94

method	result
default	$\frac{c^2 C\,x^6}{6}+\frac{B\,c^2 x^5}{5}+\frac{A\,c^2 x^4}{4}+\frac{Cbc\,x^4}{2}+\frac{2bBc\,x^3}{3}+Abc\,x^2+Cac\,x^2+\frac{C\,b^2 x^2}{2}+2Bacx+B\,b^2 x+(2A$
norman	$\frac{\left(\frac{1}{4}A\,c^2+\frac{1}{2}Cbc\right)x^8+\left(-Aab-\frac{1}{2}C\,a^2\right)x^2+\left(Abc+acC+\frac{1}{2}b^2 C\right)x^6+\left(2Bac+B\,b^2\right)x^5-\frac{A\,a^2}{4}-\frac{B\,a^2 x}{3}+\frac{B\,c^2 x^9}{5}+\frac{c^2 C\,x^{10}}{6}-2Bab\,x^3+}{x^4}$
risch	$\frac{c^2 C\,x^6}{6}+\frac{B\,c^2 x^5}{5}+\frac{A\,c^2 x^4}{4}+\frac{Cbc\,x^4}{2}+\frac{2bBc\,x^3}{3}+Abc\,x^2+Cac\,x^2+\frac{C\,b^2 x^2}{2}+2Bacx+B\,b^2 x+\frac{-2B}{}$
parallelrisch	$\frac{10c^2 C\,x^{10}+12B\,c^2 x^9+15A\,c^2 x^8+30Cbc\,x^8+40bBc\,x^7+60Abc\,x^6+60Cac\,x^6+30C\,b^2 x^6+120A\ln(x)x^4 ac+60A\ln(x)x^4 b^2+120}{60x^4}$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/6*c^2*C*x^6+1/5*B*c^2*x^5+1/4*A*c^2*x^4+1/2*C*b*c*x^4+2/3*b*B*c*x^3+A*b*
c*x^2+C*a*c*x^2+1/2*C*b^2*x^2+2*B*a*c*x+B*b^2*x+(2*A*a*c+A*b^2+2*C*a*b)*ln
(x)-1/2*a*(2*A*b+C*a)/x^2-2*a*b*B/x-1/4*a^2*A/x^4-1/3*a^2*B/x^3
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x^5}\,dx$$
$$=\frac{10\,Cc^2x^{10}+12\,Bc^2x^9+40\,Bbcx^7+15\,(2\,Cbc+Ac^2)x^8+30\,(Cb^2+2\,(Ca+Ab)c)x^6-120\,Babx^3+60\,(}{60\,x^4}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^5,x, algorithm="fricas")
```

output
```
1/60*(10*C*c^2*x^10 + 12*B*c^2*x^9 + 40*B*b*c*x^7 + 15*(2*C*b*c + A*c^2)*x
^8 + 30*(C*b^2 + 2*(C*a + A*b)*c)*x^6 - 120*B*a*b*x^3 + 60*(B*b^2 + 2*B*a*
c)*x^5 + 60*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4*log(x) - 20*B*a^2*x - 15*A*a^2
 - 30*(C*a^2 + 2*A*a*b)*x^2)/x^4
```

3.18.6 Sympy [A] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^5}\,dx = \frac{2Bbcx^3}{3}+\frac{Bc^2x^5}{5}+\frac{Cc^2x^6}{6}+x^4\left(\frac{Ac^2}{4}+\frac{Cbc}{2}\right)+x^2\left(Abc+Cac+\frac{Cb^2}{2}\right)+x\left(2Bac+Bb^2\right) + \left(2Aac+Ab^2+2Cab\right)\log(x)+\frac{-3Aa^2-4Ba^2x-24Babx^3+x^2(-12Aab-6Ca^2)}{12x^4}$$

input
```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**5,x)
```

output
```
2*B*b*c*x**3/3 + B*c**2*x**5/5 + C*c**2*x**6/6 + x**4*(A*c**2/4 + C*b*c/2)
 + x**2*(A*b*c + C*a*c + C*b**2/2) + x*(2*B*a*c + B*b**2) + (2*A*a*c + A*b
**2 + 2*C*a*b)*log(x) + (-3*A*a**2 - 4*B*a**2*x - 24*B*a*b*x**3 + x**2*(-1
2*A*a*b - 6*C*a**2))/(12*x**4)
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^5}\,dx = \frac{1}{6}\,Cc^2x^6+\frac{1}{5}\,Bc^2x^5+\frac{2}{3}\,Bbcx^3+\frac{1}{4}\,(2\,Cbc+Ac^2)x^4 + \frac{1}{2}\,(Cb^2+2\,(Ca+Ab)c)x^2+\left(Bb^2+2\,Bac\right)x+\left(2\,Cab+Ab^2+2\,Aac\right)\log(x) - \frac{24\,Babx^3+4\,Ba^2x+3\,Aa^2+6\,(Ca^2+2\,Aab)x^2}{12\,x^4}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^5,x, algorithm="maxima")
```

output
```
1/6*C*c^2*x^6 + 1/5*B*c^2*x^5 + 2/3*B*b*c*x^3 + 1/4*(2*C*b*c + A*c^2)*x^4
+ 1/2*(C*b^2 + 2*(C*a + A*b)*c)*x^2 + (B*b^2 + 2*B*a*c)*x + (2*C*a*b + A*b
^2 + 2*A*a*c)*log(x) - 1/12*(24*B*a*b*x^3 + 4*B*a^2*x + 3*A*a^2 + 6*(C*a^2
 + 2*A*a*b)*x^2)/x^4
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.96

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x^5}\,dx &= \frac{1}{6}\,Cc^2x^6+\frac{1}{5}\,Bc^2x^5+\frac{1}{2}\,Cbcx^4+\frac{1}{4}\,Ac^2x^4+\frac{2}{3}\,Bbcx^3+\frac{1}{2}\,Cb^2x^2\\&\quad+Cacx^2+Abcx^2+Bb^2x+2\,Bacx+\left(2\,Cab+Ab^2+2\,Aac\right)\log\left(|x|\right)\\&\quad-\frac{24\,Babx^3+4\,Ba^2x+3\,Aa^2+6\,(Ca^2+2\,Aab)x^2}{12\,x^4}\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^5,x, algorithm="giac")
```

output
```
1/6*C*c^2*x^6 + 1/5*B*c^2*x^5 + 1/2*C*b*c*x^4 + 1/4*A*c^2*x^4 + 2/3*B*b*c*
x^3 + 1/2*C*b^2*x^2 + C*a*c*x^2 + A*b*c*x^2 + B*b^2*x + 2*B*a*c*x + (2*C*a
*b + A*b^2 + 2*A*a*c)*log(abs(x)) - 1/12*(24*B*a*b*x^3 + 4*B*a^2*x + 3*A*a
^2 + 6*(C*a^2 + 2*A*a*b)*x^2)/x^4
```

3.18.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.91

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x^5}\,dx &= x^4\left(\frac{A\,c^2}{4}+\frac{C\,b\,c}{2}\right)\\&\quad-\frac{x^2\left(\frac{C\,a^2}{2}+A\,b\,a\right)+\frac{A\,a^2}{4}+\frac{B\,a^2\,x}{3}+2\,B\,a\,b\,x^3}{x^4}\\&\quad+x^2\left(\frac{C\,b^2}{2}+A\,c\,b+C\,a\,c\right)\\&\quad+\ln\left(x\right)\,\left(A\,b^2+2\,C\,a\,b+2\,A\,a\,c\right)+\frac{B\,c^2\,x^5}{5}\\&\quad+\frac{C\,c^2\,x^6}{6}+B\,x\,\left(b^2+2\,a\,c\right)+\frac{2\,B\,b\,c\,x^3}{3}\end{aligned}$$

input
```
int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^5,x)
```

output
```
x^4*((A*c^2)/4 + (C*b*c)/2) - (x^2*((C*a^2)/2 + A*a*b) + (A*a^2)/4 + (B*a^
2*x)/3 + 2*B*a*b*x^3)/x^4 + x^2*((C*b^2)/2 + A*b*c + C*a*c) + log(x)*(A*b^
2 + 2*A*a*c + 2*C*a*b) + (B*c^2*x^5)/5 + (C*c^2*x^6)/6 + B*x*(2*a*c + b^2)
 + (2*B*b*c*x^3)/3
```

3.18. $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^5}\,dx$

3.19 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^6} dx$

3.19.1 Optimal result

Integrand size = 28, antiderivative size = 143

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^6}\,dx = -\frac{a^2A}{5x^5}-\frac{a^2B}{4x^4}-\frac{a(2Ab+aC)}{3x^3}-\frac{abB}{x^2} -\frac{A(b^2+2ac)+2abC}{x}+\left(2Abc+\left(b^2+2ac\right)C\right)x +bBcx^2+\frac{1}{3}c(Ac+2bC)x^3+\frac{1}{4}Bc^2x^4 +\frac{1}{5}c^2Cx^5+B\left(b^2+2ac\right)\log(x)$$

output
```
-1/5*a^2*A/x^5-1/4*a^2*B/x^4-1/3*a*(2*A*b+C*a)/x^3-a*b*B/x^2+(-A*(2*a*c+b^
2)-2*a*b*C)/x+(2*A*b*c+(2*a*c+b^2)*C)*x+b*B*c*x^2+1/3*c*(A*c+2*C*b)*x^3+1/
4*B*c^2*x^4+1/5*c^2*C*x^5+B*(2*a*c+b^2)*ln(x)
```

3.19.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^6}\,dx = -\frac{a^2A}{5x^5}-\frac{a^2B}{4x^4}-\frac{a(2Ab+aC)}{3x^3}-\frac{abB}{x^2} -\frac{Ab^2+2aAc+2abC}{x}+2Abcx+\left(b^2+2ac\right)Cx +bBcx^2+\frac{1}{3}c(Ac+2bC)x^3+\frac{1}{4}Bc^2x^4 +\frac{1}{5}c^2Cx^5+B\left(b^2+2ac\right)\log(x)$$

input
```
Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^6,x]
```

output
```
-1/5*(a^2*A)/x^5 - (a^2*B)/(4*x^4) - (a*(2*A*b + a*C))/(3*x^3) - (a*b*B)/x
^2 - (A*b^2 + 2*a*A*c + 2*a*b*C)/x + 2*A*b*c*x + (b^2 + 2*a*c)*C*x + b*B*c
*x^2 + (c*(A*c + 2*b*C)*x^3)/3 + (B*c^2*x^4)/4 + (c^2*C*x^5)/5 + B*(b^2 +
2*a*c)*Log[x]
```

3.19.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2+cx^4\right)^2 \left(A+Bx+Cx^2\right)}{x^6} \, dx$$

$$\Big\downarrow \text{ 2159}$$

$$\int \left(\frac{a^2A}{x^6}+\frac{a^2B}{x^5}+\frac{A\left(2ac+b^2\right)+2abC}{x^2}+2Abc\left(\frac{bC\left(\frac{2ac}{b^2}+1\right)}{2Ac}+1\right)+\frac{a(aC+2Ab)}{x^4}+\frac{B\left(2ac+b^2\right)}{x}+\frac{2abB}{x^3}+c\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^2A}{5x^5}-\frac{a^2B}{4x^4}+x\left(C\left(2ac+b^2\right)+2Abc\right)-\frac{A\left(2ac+b^2\right)+2abC}{x}-\frac{a(aC+2Ab)}{3x^3}+B\log(x)\left(2ac+b^2\right)-\frac{abB}{x^2}+\frac{1}{3}cx^3(Ac+2bC)+bBcx^2+\frac{1}{4}Bc^2x^4+\frac{1}{5}c^2Cx^5$$

input
```
Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^6,x]
```

output
```
-1/5*(a^2*A)/x^5 - (a^2*B)/(4*x^4) - (a*(2*A*b + a*C))/(3*x^3) - (a*b*B)/x
^2 - (A*(b^2 + 2*a*c) + 2*a*b*C)/x + (2*A*b*c + (b^2 + 2*a*c)*C)*x + b*B*c
*x^2 + (c*(A*c + 2*b*C)*x^3)/3 + (B*c^2*x^4)/4 + (c^2*C*x^5)/5 + B*(b^2 +
2*a*c)*Log[x]
```

3.19.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.19.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.94

method	result
default	$\frac{c^2 C x^5}{5}+\frac{B c^2 x^4}{4}+\frac{A c^2 x^3}{3}+\frac{2 C b c x^3}{3}+B b c x^2+2 A b c x+2 C a c x+C b^2 x+B(2 a c+b^2)\ln(x)-$
risch	$\frac{c^2 C x^5}{5}+\frac{B c^2 x^4}{4}+\frac{A c^2 x^3}{3}+\frac{2 C b c x^3}{3}+B b c x^2+2 A b c x+2 C a c x+C b^2 x+\frac{(-2 A a c-A b^2-2 a b C)x^4-}{}$
norman	$\frac{(\frac{1}{3} A c^2+\frac{2}{3} C b c)x^8+(-\frac{2}{3} A a b-\frac{1}{3} C a^2)x^2+(2 A b c+2 a c C+b^2 C)x^6+(-2 A a c-A b^2-2 a b C)x^4+b B c x^7-\frac{A a^2}{5}-\frac{B a^2 x}{4}+\frac{B c^2 x^9}{4}+}{x^5}$
parallelrisch	$\frac{12 c^2 C x^{10}+15 B c^2 x^9+20 A c^2 x^8+40 C b c x^8+60 b B c x^7+120 A b c x^6+120 B\ln(x)x^5 a c+60 B\ln(x)x^5 b^2+120 C a c x^6+60 C b^2 x^6-}{60 x^5}$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/5*c^2*C*x^5+1/4*B*c^2*x^4+1/3*A*c^2*x^3+2/3*C*b*c*x^3+B*b*c*x^2+2*A*b*c*
x+2*C*a*c*x+C*b^2*x+B*(2*a*c+b^2)*ln(x)-a*b*B/x^2-1/5*a^2*A/x^5-(2*A*a*c+A
*b^2+2*C*a*b)/x-1/4*a^2*B/x^4-1/3*a*(2*A*b+C*a)/x^3
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^6}\,dx$$
$$=\frac{12\,Cc^2x^{10}+15\,Bc^2x^9+60\,Bbcx^7+20\,(2\,Cbc+Ac^2)x^8+60\,(Cb^2+2\,(Ca+Ab)c)x^6+60\,(Bb^2+2\,Bac)}{60\,x^5}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^6,x, algorithm="fricas")
```

output
```
1/60*(12*C*c^2*x^10 + 15*B*c^2*x^9 + 60*B*b*c*x^7 + 20*(2*C*b*c + A*c^2)*x
^8 + 60*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 60*(B*b^2 + 2*B*a*c)*x^5*log(x) -
60*B*a*b*x^3 - 60*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 - 15*B*a^2*x - 12*A*a^2
- 20*(C*a^2 + 2*A*a*b)*x^2)/x^5
```

3.19.6 Sympy [A] (verification not implemented)

Time = 3.95 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^6}\,dx$$
$$= Bbcx^2 + \frac{Bc^2x^4}{4} + B(2ac+b^2)\log(x) + \frac{Cc^2x^5}{5} + x^3\left(\frac{Ac^2}{3} + \frac{2Cbc}{3}\right) + x(2Abc+2Cac+Cb^2)$$
$$+ \frac{-12Aa^2 - 15Ba^2x - 60Babx^3 + x^4(-120Aac - 60Ab^2 - 120Cab) + x^2(-40Aab - 20Ca^2)}{60x^5}$$

input
```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**6,x)
```

output
```
B*b*c*x**2 + B*c**2*x**4/4 + B*(2*a*c + b**2)*log(x) + C*c**2*x**5/5 + x**
3*(A*c**2/3 + 2*C*b*c/3) + x*(2*A*b*c + 2*C*a*c + C*b**2) + (-12*A*a**2 -
15*B*a**2*x - 60*B*a*b*x**3 + x**4*(-120*A*a*c - 60*A*b**2 - 120*C*a*b) +
x**2*(-40*A*a*b - 20*C*a**2))/(60*x**5)
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^6}\,dx$$
$$= \frac{1}{5}\,Cc^2x^5 + \frac{1}{4}\,Bc^2x^4 + Bbcx^2 + \frac{1}{3}\,(2\,Cbc + Ac^2)x^3$$
$$+ (Cb^2 + 2\,(Ca + Ab)c)x + (Bb^2 + 2\,Bac)\log(x)$$
$$- \frac{60\,Babx^3 + 60\,(2\,Cab + Ab^2 + 2\,Aac)x^4 + 15\,Ba^2x + 12\,Aa^2 + 20\,(Ca^2 + 2\,Aab)x^2}{60\,x^5}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^6,x, algorithm="maxima")
```

output
```
1/5*C*c^2*x^5 + 1/4*B*c^2*x^4 + B*b*c*x^2 + 1/3*(2*C*b*c + A*c^2)*x^3 + (C
*b^2 + 2*(C*a + A*b)*c)*x + (B*b^2 + 2*B*a*c)*log(x) - 1/60*(60*B*a*b*x^3
+ 60*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 15*B*a^2*x + 12*A*a^2 + 20*(C*a^2 +
 2*A*a*b)*x^2)/x^5
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.98

$$\begin{aligned}&\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x^6}\,dx\\&=\frac{1}{5}\,Cc^2x^5+\frac{1}{4}\,Bc^2x^4+\frac{2}{3}\,Cbcx^3+\frac{1}{3}\,Ac^2x^3+Bbcx^2\\&\quad+Cb^2x+2\,Cacx+2\,Abcx+\left(Bb^2+2\,Bac\right)\log\left(|x|\right)\\&\quad-\frac{60\,Babx^3+60\,(2\,Cab+Ab^2+2\,Aac)x^4+15\,Ba^2x+12\,Aa^2+20\,(Ca^2+2\,Aab)x^2}{60\,x^5}\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^6,x, algorithm="giac")
```

output
```
1/5*C*c^2*x^5 + 1/4*B*c^2*x^4 + 2/3*C*b*c*x^3 + 1/3*A*c^2*x^3 + B*b*c*x^2
+ C*b^2*x + 2*C*a*c*x + 2*A*b*c*x + (B*b^2 + 2*B*a*c)*log(abs(x)) - 1/60*(
60*B*a*b*x^3 + 60*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 15*B*a^2*x + 12*A*a^2
+ 20*(C*a^2 + 2*A*a*b)*x^2)/x^5
```

3.19.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.95

$$\begin{aligned}&\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x^6}\,dx\\&=x^3\left(\frac{A\,c^2}{3}+\frac{2\,C\,b\,c}{3}\right)\\&\quad-\frac{x^2\left(\frac{C\,a^2}{3}+\frac{2\,A\,b\,a}{3}\right)+\frac{A\,a^2}{5}+x^4\left(A\,b^2+2\,C\,a\,b+2\,A\,a\,c\right)+\frac{B\,a^2\,x}{4}+B\,a\,b\,x^3}{x^5}\\&\quad+x\left(C\,b^2+2\,A\,c\,b+2\,C\,a\,c\right)+\ln\left(x\right)\,\left(B\,b^2+2\,B\,a\,c\right)+\frac{B\,c^2\,x^4}{4}+\frac{C\,c^2\,x^5}{5}+B\,b\,c\,x^2\end{aligned}$$

input
```
int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^6,x)
```

output
```
x^3*((A*c^2)/3 + (2*C*b*c)/3) - (x^2*((C*a^2)/3 + (2*A*a*b)/3) + (A*a^2)/5
 + x^4*(A*b^2 + 2*A*a*c + 2*C*a*b) + (B*a^2*x)/4 + B*a*b*x^3)/x^5 + x*(C*b
^2 + 2*A*b*c + 2*C*a*c) + log(x)*(B*b^2 + 2*B*a*c) + (B*c^2*x^4)/4 + (C*c^
2*x^5)/5 + B*b*c*x^2
```

3.19. $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^6}\,dx$

3.20 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^7}\,dx$

3.20.1 Optimal result

Integrand size = 28, antiderivative size = 149

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^7}\,dx = -\frac{a^2A}{6x^6}-\frac{a^2B}{5x^5}-\frac{a(2Ab+aC)}{4x^4}-\frac{2abB}{3x^3}-\frac{A(b^2+2ac)+2abC}{2x^2}-\frac{B(b^2+2ac)}{x}+2bBcx+\frac{1}{2}c(Ac+2bC)x^2+\frac{1}{3}Bc^2x^3+\frac{1}{4}c^2Cx^4+\left(2Abc+\left(b^2+2ac\right)C\right)\log(x)$$

output
```
-1/6*a^2*A/x^6-1/5*a^2*B/x^5-1/4*a*(2*A*b+C*a)/x^4-2/3*a*b*B/x^3+1/2*(-A*(
2*a*c+b^2)-2*a*b*C)/x^2-B*(2*a*c+b^2)/x+2*b*B*c*x+1/2*c*(A*c+2*C*b)*x^2+1/
3*B*c^2*x^3+1/4*c^2*C*x^4+(2*A*b*c+(2*a*c+b^2)*C)*ln(x)
```

3.20.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^7}\,dx = -\frac{b^2B}{x}+bcx(2B+Cx)+\frac{1}{12}c^2x^3(4B+3Cx)+\frac{A(-b^2+c^2x^4)}{2x^2}-\frac{a^2(10A+3x(4B+5Cx))}{60x^6}-\frac{a(3A(b+2cx^2)+2x(2bB+3bCx+6Bcx^2))}{6x^4}+\left(2Abc+\left(b^2+2ac\right)C\right)\log(x)$$

input
```
Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^7,x]
```

output
```
-((b^2*B)/x) + b*c*x*(2*B + C*x) + (c^2*x^3*(4*B + 3*C*x))/12 + (A*(-b^2 +
 c^2*x^4))/(2*x^2) - (a^2*(10*A + 3*x*(4*B + 5*C*x)))/(60*x^6) - (a*(3*A*(
b + 2*c*x^2) + 2*x*(2*b*B + 3*b*C*x + 6*B*c*x^2)))/(6*x^4) + (2*A*b*c + (b
^2 + 2*a*c)*C)*Log[x]
```

3.20.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2+cx^4\right)^2\left(A+Bx+Cx^2\right)}{x^7}dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{a^2A}{x^7}+\frac{a^2B}{x^6}+\frac{A\left(2ac+b^2\right)+2abC}{x^3}+\frac{C\left(2ac+b^2\right)+2Abc}{x}+\frac{a(aC+2Ab)}{x^5}+\frac{B\left(2ac+b^2\right)}{x^2}+\frac{2abB}{x^4}+cx(Ac$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{6x^6}-\frac{a^2B}{5x^5}-\frac{A\left(2ac+b^2\right)+2abC}{2x^2}+\log(x)\left(C\left(2ac+b^2\right)+2Abc\right)-\frac{a(aC+2Ab)}{4x^4}-\frac{B\left(2ac+b^2\right)}{x}-\frac{2abB}{3x^3}+\frac{1}{2}cx^2(Ac+2bC)+2bBcx+\frac{1}{3}Bc^2x^3+\frac{1}{4}c^2Cx^4$$

input
```
Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^7,x]
```

output
```
-1/6*(a^2*A)/x^6 - (a^2*B)/(5*x^5) - (a*(2*A*b + a*C))/(4*x^4) - (2*a*b*B)
/(3*x^3) - (A*(b^2 + 2*a*c) + 2*a*b*C)/(2*x^2) - (B*(b^2 + 2*a*c))/x + 2*b
*B*c*x + (c*(A*c + 2*b*C)*x^2)/2 + (B*c^2*x^3)/3 + (c^2*C*x^4)/4 + (2*A*b*
c + (b^2 + 2*a*c)*C)*Log[x]
```

3.20.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.20.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.91

method	result
default	$\frac{c^2 C x^4}{4}+\frac{B c^2 x^3}{3}+\frac{A c^2 x^2}{2}+Cbc x^2+2Bbcx+\left(2Abc+2acC+b^2 C\right)\ln\left(x\right)-\frac{2Aac+A b^2+2abC}{2x^2}-$
norman	$\frac{\left(\frac{1}{2}A c^2+Cbc\right)x^8+\left(-\frac{1}{2}Aab-\frac{1}{4}C a^2\right)x^2+\left(-Aac-\frac{1}{2}A b^2-abC\right)x^4+\left(-2Bac-B b^2\right)x^5-\frac{A a^2}{6}-\frac{B a^2 x}{5}+\frac{B c^2 x^9}{3}+\frac{c^2 C x^{10}}{4}-\frac{2Bab x}{3}}{x^6}$
risch	$\frac{c^2 C x^4}{4}+\frac{B c^2 x^3}{3}+\frac{A c^2 x^2}{2}+Cbc x^2+2Bbcx+\frac{\left(-2Bac-B b^2\right)x^5+\left(-Aac-\frac{1}{2}A b^2-abC\right)x^4-\frac{2Bab x^3}{3}+\left(-\frac{1}{2}Aab\right.}{x^6}$
parallelrisch	$\frac{15c^2 C x^{10}+20B c^2 x^9+30A c^2 x^8+60Cbc x^8+120A\ln(x)x^6 bc+120bBc x^7+120C\ln(x)x^6 ac+60C\ln(x)x^6 b^2-120Bac x^5-60B}{60x^6}$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/4*c^2*C*x^4+1/3*B*c^2*x^3+1/2*A*c^2*x^2+C*b*c*x^2+2*B*b*c*x+(2*A*b*c+2*C
*a*c+C*b^2)*ln(x)-1/2*(2*A*a*c+A*b^2+2*C*a*b)/x^2-1/6*a^2*A/x^6-1/5*a^2*B/
x^5-B*(2*a*c+b^2)/x-1/4*a*(2*A*b+C*a)/x^4-2/3*a*b*B/x^3
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^7}\,dx$$
$$=\frac{15\,Cc^2x^{10}+20\,Bc^2x^9+120\,Bbcx^7+30\,(2\,Cbc+Ac^2)x^8+60\,(Cb^2+2\,(Ca+Ab)c)x^6\log\left(x\right)-40\,Babx^3}{60\,x^6}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^7,x, algorithm="fricas")
```

output
```
1/60*(15*C*c^2*x^10 + 20*B*c^2*x^9 + 120*B*b*c*x^7 + 30*(2*C*b*c + A*c^2)*
x^8 + 60*(C*b^2 + 2*(C*a + A*b)*c)*x^6*log(x) - 40*B*a*b*x^3 - 60*(B*b^2 +
 2*B*a*c)*x^5 - 30*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 - 12*B*a^2*x - 10*A*a^2
 - 15*(C*a^2 + 2*A*a*b)*x^2)/x^6
```

3.20.6 Sympy [A] (verification not implemented)

Time = 28.25 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^7}\,dx$$
$$= 2Bbcx + \frac{Bc^2x^3}{3} + \frac{Cc^2x^4}{4} + x^2\left(\frac{Ac^2}{2} + Cbc\right) + \left(2Abc + 2Cac + Cb^2\right)\log(x)$$
$$+ \frac{-10Aa^2 - 12Ba^2x - 40Babx^3 + x^5(-120Bac - 60Bb^2) + x^4(-60Aac - 30Ab^2 - 60Cab) + x^2(-30A\ldots}{60x^6}$$

input
```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**7,x)
```

output
```
2*B*b*c*x + B*c**2*x**3/3 + C*c**2*x**4/4 + x**2*(A*c**2/2 + C*b*c) + (2*A
*b*c + 2*C*a*c + C*b**2)*log(x) + (-10*A*a**2 - 12*B*a**2*x - 40*B*a*b*x**
3 + x**5*(-120*B*a*c - 60*B*b**2) + x**4*(-60*A*a*c - 30*A*b**2 - 60*C*a*b
) + x**2*(-30*A*a*b - 15*C*a**2))/(60*x**6)
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^7}\,dx$$
$$= \frac{1}{4}Cc^2x^4 + \frac{1}{3}Bc^2x^3 + 2Bbcx + \frac{1}{2}\left(2Cbc + Ac^2\right)x^2 + \left(Cb^2 + 2(Ca + Ab)c\right)\log(x)$$
$$- \frac{40Babx^3 + 60(Bb^2 + 2Bac)x^5 + 30(2Cab + Ab^2 + 2Aac)x^4 + 12Ba^2x + 10Aa^2 + 15(Ca^2 + 2Aab)\ldots}{60x^6}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^7,x, algorithm="maxima")
```

output
```
1/4*C*c^2*x^4 + 1/3*B*c^2*x^3 + 2*B*b*c*x + 1/2*(2*C*b*c + A*c^2)*x^2 + (C
*b^2 + 2*(C*a + A*b)*c)*log(x) - 1/60*(40*B*a*b*x^3 + 60*(B*b^2 + 2*B*a*c)
*x^5 + 30*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 12*B*a^2*x + 10*A*a^2 + 15*(C*
a^2 + 2*A*a*b)*x^2)/x^6
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x^7}\,dx$$
$$= \frac{1}{4}\,Cc^2x^4 + \frac{1}{3}\,Bc^2x^3 + Cbcx^2 + \frac{1}{2}\,Ac^2x^2 + 2\,Bbcx + \left(Cb^2 + 2\,Cac + 2\,Abc\right)\log\left(|x|\right)$$
$$- \frac{40\,Babx^3 + 60\,(Bb^2 + 2\,Bac)x^5 + 30\,(2\,Cab + Ab^2 + 2\,Aac)x^4 + 12\,Ba^2x + 10\,Aa^2 + 15\,(Ca^2 + 2\,Aab)}{60\,x^6}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^7,x, algorithm="giac")
```

output
```
1/4*C*c^2*x^4 + 1/3*B*c^2*x^3 + C*b*c*x^2 + 1/2*A*c^2*x^2 + 2*B*b*c*x + (C
*b^2 + 2*C*a*c + 2*A*b*c)*log(abs(x)) - 1/60*(40*B*a*b*x^3 + 60*(B*b^2 + 2
*B*a*c)*x^5 + 30*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 12*B*a^2*x + 10*A*a^2 +
 15*(C*a^2 + 2*A*a*b)*x^2)/x^6
```

3.20.9 Mupad [B] (verification not implemented)

Time = 7.87 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x^7}\,dx = x^2\left(\frac{A\,c^2}{2} + C\,b\,c\right)$$
$$- \frac{x^2\left(\frac{C\,a^2}{4} + \frac{A\,b\,a}{2}\right) + x^5\,(B\,b^2 + 2\,B\,a\,c) + \frac{A\,a^2}{6} + x^4\left(\frac{A\,b^2}{2} + C\,a\,b + A\,a\,c\right) + \frac{B\,a^2\,x}{5} + \frac{2\,B\,a\,b\,x^3}{3}}{x^6}$$
$$+ \ln(x)\,\left(C\,b^2 + 2\,A\,c\,b + 2\,C\,a\,c\right) + \frac{B\,c^2\,x^3}{3} + \frac{C\,c^2\,x^4}{4} + 2\,B\,b\,c\,x$$

input
```
int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^7,x)
```

output
```
x^2*((A*c^2)/2 + C*b*c) - (x^2*((C*a^2)/4 + (A*a*b)/2) + x^5*(B*b^2 + 2*B*
a*c) + (A*a^2)/6 + x^4*((A*b^2)/2 + A*a*c + C*a*b) + (B*a^2*x)/5 + (2*B*a*
b*x^3)/3)/x^6 + log(x)*(C*b^2 + 2*A*b*c + 2*C*a*c) + (B*c^2*x^3)/3 + (C*c^
2*x^4)/4 + 2*B*b*c*x
```

3.20. $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^7} dx$

3.21 $\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$

3.21.1 Optimal result

Integrand size = 28, antiderivative size = 339

$$\begin{aligned}
&\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx \\
&= \frac{(Ac-bC)x}{c^2}+\frac{Bx^2}{2c}+\frac{Cx^3}{3c} \\
&\quad -\frac{\left(Abc-b^2C+acC-\frac{Ac(b^2-2ac)-b(b^2-3ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad -\frac{\left(Abc-b^2C+acC+\frac{Ac(b^2-2ac)-b(b^2-3ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}} \\
&\quad -\frac{B(b^2-2ac)\,\mathrm{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{bB\log(a+bx^2+cx^4)}{4c^2}
\end{aligned}$$

output
```
(A*c-C*b)*x/c^2+1/2*B*x^2/c+1/3*C*x^3/c-1/4*b*B*ln(c*x^4+b*x^2+a)/c^2-1/2*
B*(-2*a*c+b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1
/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(A*b*c-b^2*
C+a*c*C+(-A*c*(-2*a*c+b^2)+b*(-3*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2
^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*
c+b^2)^(1/2))^(1/2))*(A*b*c-b^2*C+a*c*C+(A*c*(-2*a*c+b^2)-b*(-3*a*c+b^2)*C
)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.21.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.36

$$\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$$

$$= \frac{12\sqrt{c}(Ac-bC)x+6Bc^{3/2}x^2+4c^{3/2}Cx^3+\frac{6\sqrt{2}\left(Ac\left(b^2-2ac-b\sqrt{b^2-4ac}\right)+\left(-b^3+3abc+b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}\right)C\right)\arctan\left(\right.}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}}{}$$

input
```
Integrate[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
(12*Sqrt[c]*(A*c - b*C)*x + 6*B*c^(3/2)*x^2 + 4*c^(3/2)*C*x^3 + (6*Sqrt[2]
*(A*c*(b^2 - 2*a*c - b*Sqrt[b^2 - 4*a*c]) + (-b^3 + 3*a*b*c + b^2*Sqrt[b^2
 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (6*
Sqrt[2]*(-(A*c*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])) + (b^3 - 3*a*b*c + b^2
*Sqrt[b^2 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c
]]) - (3*B*Sqrt[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[-b + Sqrt[b^2
- 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] - (3*B*Sqrt[c]*(b^2 - 2*a*c + b*Sqr
t[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(
12*c^(5/2))
```

3.21.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {2193, 27, 1434, 1143, 1602, 27, 1602, 25, 1480, 218, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \; 2193$$

$$\int \frac{x^4(Cx^2+A)}{cx^4+bx^2+a}dx+\int \frac{Bx^5}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \; 27$$

$$\int \frac{x^4\left(Cx^2+A\right)}{cx^4+bx^2+a}dx+B\int \frac{x^5}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \; 1434$$

$$\int \frac{x^4\left(Cx^2+A\right)}{cx^4+bx^2+a}dx+\frac{1}{2}B\int \frac{x^4}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \; 1143$$

$$\int \frac{x^4\left(Cx^2+A\right)}{cx^4+bx^2+a}dx+\frac{1}{2}B\int \left(\frac{1}{c}-\frac{bx^2+a}{c\left(cx^4+bx^2+a\right)}\right)dx^2$$

$$\Big\downarrow \; 1602$$

$$-\frac{\int \frac{3x^2\left(aC-(Ac-bC)x^2\right)}{cx^4+bx^2+a}dx}{3c}+\frac{1}{2}B\int \left(\frac{1}{c}-\frac{bx^2+a}{c\left(cx^4+bx^2+a\right)}\right)dx^2+\frac{Cx^3}{3c}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{x^2\left(aC-(Ac-bC)x^2\right)}{cx^4+bx^2+a}dx}{c}+\frac{1}{2}B\int \left(\frac{1}{c}-\frac{bx^2+a}{c\left(cx^4+bx^2+a\right)}\right)dx^2+\frac{Cx^3}{3c}$$

$$\Big\downarrow \; 1602$$

$$-\frac{-\frac{\int -\frac{\left(-Cb^2+Acb+acC\right)x^2+a(Ac-bC)}{cx^4+bx^2+a}dx}{c}-\frac{x(Ac-bC)}{c}}{c}+\frac{1}{2}B\int \left(\frac{1}{c}-\frac{bx^2+a}{c\left(cx^4+bx^2+a\right)}\right)dx^2+\frac{Cx^3}{3c}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\int \frac{\left(-Cb^2+Acb+acC\right)x^2+a(Ac-bC)}{cx^4+bx^2+a}dx}{c}-\frac{x(Ac-bC)}{c}}{c}+\frac{1}{2}B\int \left(\frac{1}{c}-\frac{bx^2+a}{c\left(cx^4+bx^2+a\right)}\right)dx^2+\frac{Cx^3}{3c}$$

$$\Big\downarrow \; 1480$$

$$-\frac{\frac{\frac{1}{2}\left(-\frac{Ac\left(b^2-2ac\right)-bC\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}+acC+Abc+b^2(-C)\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{Ac\left(b^2-2ac\right)-bC\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}+acC+Abc+b^2(-C)\right)\int \frac{}{cx^2+\frac{1}{2}\left(b\right.}}{c}}{c}$$

$$\frac{1}{2}B\int \left(\frac{1}{c}-\frac{bx^2+a}{c\left(cx^4+bx^2+a\right)}\right)dx^2+\frac{Cx^3}{3c}$$

$$\Big\downarrow \; 218$$

3.21. $\int \frac{x^4\left(A+Bx+Cx^2\right)}{a+bx^2+cx^4}\,dx$

$$\frac{\frac{1}{2}B\int\left(\frac{1}{c}-\frac{bx^2+a}{c\left(cx^4+bx^2+a\right)}\right)dx^2-\frac{\frac{\left(-\frac{Ac\left(b^2-2ac\right)-bC\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}+acC+Abc+b^2(-C)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{Ac\left(b^2-2ac\right)-bC\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}+acC+Abc+b^2(-C)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}}{c}$$

$$\frac{Cx^3}{3c}$$

$\downarrow$ 2009

$$-\frac{\frac{\frac{\left(-\frac{Ac\left(b^2-2ac\right)-bC\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}+acC+Abc+b^2(-C)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{Ac\left(b^2-2ac\right)-bC\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}+acC+Abc+b^2(-C)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}}{c}$$

$$\frac{1}{2}B\left(-\frac{\left(b^2-2ac\right)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^2+cx^4\right)}{2c^2}+\frac{x^2}{c}\right)+\frac{Cx^3}{3c}$$

input
```
Int[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
(C*x^3)/(3*c) - (-(((A*c - b*C)*x)/c) + (((A*b*c - b^2*C + a*c*C - (A*c*(b
^2 - 2*a*c) - b*(b^2 - 3*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*
a*c]]) + ((A*b*c - b^2*C + a*c*C + (A*c*(b^2 - 2*a*c) - b*(b^2 - 3*a*c)*C)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/c)/c + (B*(x^2/c - ((b^2
 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]
) - (b*Log[a + b*x^2 + c*x^4])/(2*c^2)))/2
```

3.21.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

3.21. $\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$

3.21.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.35

method	result
risch	$\frac{C x^3}{3c} + \frac{B x^2}{2c} + \frac{Ax}{c} - \frac{Cbx}{c^2} + \frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)} \frac{\left(-bBc\_R^3+\left(-Abc-acC+b^2C\right)\_R^2-Bac\_R-Aac+abC\right)\ln\left(x-\_\right.}{2c\_R^3+\_Rb}}{2c^2}$
default	$\frac{\frac{1}{3}cC x^3+\frac{1}{2}B x^2c+Acx-Cbx}{c^2} + \frac{-\frac{\left(2ac\sqrt{-4ac+b^2}-b^2\sqrt{-4ac+b^2}+4abc-b^3\right)\left(\frac{B\ln\left(2c x^2+\sqrt{-4ac+b^2}+b\right)}{2}+\frac{\left(2Ac-C\sqrt{-4ac+b^2}-Cb\right)\sqrt{2}\ \mathrm{arc}}{2\sqrt{\left(b+\sqrt{-4\right.}}}\right.}{2c\left(4ac-b^2\right)}}{}$

input
```
int(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*C*x^3/c+1/2*B*x^2/c+1/c*A*x-1/c^2*C*b*x+1/2/c^2*sum((-b*B*c*_R^3+(-A*b
*c-C*a*c+C*b^2)*_R^2-B*a*c*_R-A*a*c+a*b*C)/(2*_R^3*c+_R*b)*ln(x-_R),_R=Roo
tOf(_Z^4*c+_Z^2*b+a))
```

3.21.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.21.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.21.7 Maxima [F]

$$\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \int \frac{(Cx^2+Bx+A)x^4}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/6*(2*C*c*x^3 + 3*B*c*x^2 - 6*(C*b - A*c)*x)/c^2 - integrate((B*b*c*x^3 +
 B*a*c*x - C*a*b + A*a*c - (C*b^2 - (C*a + A*b)*c)*x^2)/(c*x^4 + b*x^2 + a
), x)/c^2
```

3.21.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5304 vs. 2(294) = 588.

Time = 1.42 (sec) , antiderivative size = 5304, normalized size of antiderivative = 15.65

$$\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/4*B*b*log(abs(c*x^4 + b*x^2 + a))/c^2 - 1/8*((2*b^5*c^3 - 16*a*b^3*c^4
+ 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3
*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2
 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3
- 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4
*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*A*c^2 - (2*b^6*c^2 - 18*a*b^4*c^3
 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4
*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^
3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3
+ 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 -
4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b
^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*...
```

3.21.9 Mupad [B] (verification not implemented)

Time = 7.92 (sec) , antiderivative size = 2588, normalized size of antiderivative = 7.63

$$\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x)
```

output
```
x*(A/c - (C*b)/c^2) + symsum(log((C^3*a^4*c - C^3*a^3*b^2 - A*B^2*a^3*c^2
+ A*C^2*a^2*b^3 + A^2*C*a^3*c^2 + A^3*a^2*b*c^2 + A*B^2*a^2*b^2*c - 2*A^2*
C*a^2*b^2*c - B^2*C*a^3*b*c)/c^3 - root(128*a*b^2*c^6*z^4 - 16*b^4*c^5*z^4
 - 256*a^2*c^7*z^4 - 256*B*a^2*b*c^5*z^3 + 128*B*a*b^3*c^4*z^3 - 16*B*b^5*
c^3*z^3 - 64*A*C*a*b^4*c^2*z^2 + 144*A*C*a^2*b^2*c^3*z^2 + 8*A*C*b^6*c*z^2
 + 80*C^2*a^3*b*c^3*z^2 + 32*B^2*a*b^4*c^2*z^2 - 48*A^2*a^2*b*c^4*z^2 + 28
*A^2*a*b^3*c^3*z^2 + 36*C^2*a*b^5*c*z^2 - 64*A*C*a^3*c^4*z^2 - 100*C^2*a^2
*b^3*c^2*z^2 - 56*B^2*a^2*b^2*c^3*z^2 - 4*B^2*b^6*c*z^2 - 32*B^2*a^3*c^4*z
^2 - 4*A^2*b^5*c^2*z^2 - 4*C^2*b^7*z^2 + 32*A*B*C*a^3*b*c^2*z - 8*A*B*C*a^
2*b^3*c*z - 20*B*C^2*a^3*b^2*c*z + 4*A^2*B*a^2*b^2*c^2*z - 16*B^3*a^3*b*c^
2*z + 4*B^3*a^2*b^3*c*z + 16*B*C^2*a^4*c^2*z + 4*B*C^2*a^2*b^4*z - 16*A^2*
B*a^3*c^3*z + 2*A^3*C*a^3*b*c + 4*A*B^2*C*a^4*c - 2*A^2*C^2*a^4*c + 2*A*C^
3*a^4*b - A^2*B^2*a^3*b*c - B^2*C^2*a^4*b - A^2*C^2*a^3*b^2 - A^4*a^3*c^2
- B^4*a^4*c - C^4*a^5, z, k)*(root(128*a*b^2*c^6*z^4 - 16*b^4*c^5*z^4 - 25
6*a^2*c^7*z^4 - 256*B*a^2*b*c^5*z^3 + 128*B*a*b^3*c^4*z^3 - 16*B*b^5*c^3*z
^3 - 64*A*C*a*b^4*c^2*z^2 + 144*A*C*a^2*b^2*c^3*z^2 + 8*A*C*b^6*c*z^2 + 80
*C^2*a^3*b*c^3*z^2 + 32*B^2*a*b^4*c^2*z^2 - 48*A^2*a^2*b*c^4*z^2 + 28*A^2*
a*b^3*c^3*z^2 + 36*C^2*a*b^5*c*z^2 - 64*A*C*a^3*c^4*z^2 - 100*C^2*a^2*b^3*
c^2*z^2 - 56*B^2*a^2*b^2*c^3*z^2 - 4*B^2*b^6*c*z^2 - 32*B^2*a^3*c^4*z^2 -
4*A^2*b^5*c^2*z^2 - 4*C^2*b^7*z^2 + 32*A*B*C*a^3*b*c^2*z - 8*A*B*C*a^2*...
```

3.21. $\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$

3.22 $\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$

3.22.1 Optimal result

Integrand size = 28, antiderivative size = 278

$$\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \frac{Bx}{c}+\frac{Cx^2}{2c}-\frac{B\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{B\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{(Abc-b^2C+2acC)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}} + \frac{(Ac-bC)\log\left(a+bx^2+cx^4\right)}{4c^2}$$

output
```
B*x/c+1/2*C*x^2/c+1/4*(A*c-C*b)*ln(c*x^4+b*x^2+a)/c^2+1/2*(A*b*c+2*C*a*c-C
*b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)-1/2*B
*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(2*a*c-b^2)/(-4
*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*B*arctan
(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^2)/(-4*a*c+b
^2)^(1/2))/c^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.22.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.36

$$\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$$
$$= \frac{4Bcx+2cCx^2-\frac{2\sqrt{2}B\sqrt{c}\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{2\sqrt{2}B\sqrt{c}\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{}$$

input
```
Integrate[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
(4*B*c*x + 2*c*C*x^2 - (2*Sqrt[2]*B*Sqrt[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2
- 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (2*Sqrt[2]*B*Sqrt[c]*(b^2 - 2*a*c
+ b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*
c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((A*c*(-b + Sqrt[b
^2 - 4*a*c]) + (b^2 - 2*a*c - b*Sqrt[b^2 - 4*a*c])*C)*Log[-b + Sqrt[b^2 -
4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] - ((-(A*c*(b + Sqrt[b^2 - 4*a*c])) +
(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
])/Sqrt[b^2 - 4*a*c])/(4*c^2)
```

3.22.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2193, 27, 1442, 1480, 218, 1578, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$$
$$\Big\downarrow \text{ 2193}$$
$$\int \frac{x^3(Cx^2+A)}{cx^4+bx^2+a}dx+\int \frac{Bx^4}{cx^4+bx^2+a}dx$$
$$\Big\downarrow \text{ 27}$$
$$\int \frac{x^3(Cx^2+A)}{cx^4+bx^2+a}dx+B\int \frac{x^4}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{1442}$$

$$\int \frac{x^3 (Cx^2 + A)}{cx^4 + bx^2 + a} dx + B\left(\frac{x}{c} - \frac{\int \frac{bx^2+a}{cx^4+bx^2+a} dx}{c}\right)$$

$$\Big\downarrow \text{1480}$$

$$\int \frac{x^3 (Cx^2 + A)}{cx^4 + bx^2 + a} dx + B\left(\frac{x}{c} - \frac{\frac{1}{2}\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)} dx + \frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)} dx}{c}\right)$$

$$\Big\downarrow \text{218}$$

$$\int \frac{x^3 (Cx^2 + A)}{cx^4 + bx^2 + a} dx + B\left(\frac{x}{c} - \frac{\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right) \arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}\right)$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2} \int \frac{x^2 (Cx^2 + A)}{cx^4 + bx^2 + a} dx^2 + B\left(\frac{x}{c} - \frac{\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right) \arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}\right)$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{2} \int \left(\frac{C}{c} - \frac{aC - (Ac - bC)x^2}{c\,(cx^4 + bx^2 + a)}\right) dx^2 + B\left(\frac{x}{c} - \frac{\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right) \arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}\right)$$

$$\Big\downarrow \text{2009}$$

3.22. $\int \frac{x^3 (A+Bx+Cx^2)}{a+bx^2+cx^4}\, dx$

$$\frac{1}{2}\left(\frac{(2acC+Abc+b^2(-C))\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}+\frac{(Ac-bC)\log\left(a+bx^2+cx^4\right)}{2c^2}+\frac{Cx^2}{c}\right)+B\left(\frac{x}{c}-\frac{\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}\right)$$

input
```
Int[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
B*(x/c - (((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c
]]) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))
/c) + ((C*x^2)/c + ((A*b*c - b^2*C + 2*a*c*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b
^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) + ((A*c - b*C)*Log[a + b*x^2 + c*x^4
])/(2*c^2))/2
```

3.22.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

3.22. $\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4} \, dx$

3.22.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.31

method	result
risch	$\frac{C x^2}{2c}+\frac{Bx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)}\frac{\left(\_R^3(Ac-Cb)-bB\_R^2-Ca\_R-Ba\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{2c}$
default	$\frac{\frac{1}{2}C x^2+Bx}{c}+\frac{\frac{\left(-Abc\sqrt{-4ac+b^2}+4Aa c^2-A b^2 c-2C\sqrt{-4ac+b^2}\, ac+C\sqrt{-4ac+b^2}\, b^2-4Cabc+C b^3\right)\ln\left(2c x^2+\sqrt{-4ac+b^2}+b\right)}{4c}+\frac{\left(-2Bac\sqrt{-4a\right.}{}}{}}{c(4ac-b^2)}$

input
```
int(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*C*x^2/c+B*x/c+1/2/c*sum((_R^3*(A*c-C*b)-b*B*_R^2-C*a*_R-B*a)/(2*_R^3*c
+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.22.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 61.66 (sec) , antiderivative size = 1329593, normalized size of antiderivative = 4782.71

$$\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx=\text{Too large to display}$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.22.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.22.7 Maxima [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \int \frac{(Cx^2+Bx+A)x^3}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/2*(C*x^2 + 2*B*x)/c + integrate(-(B*b*x^2 + (C*b - A*c)*x^3 + C*a*x + B*
a)/(c*x^4 + b*x^2 + a), x)/c
```

3.22.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3519 vs. $2(232) = 464$.

Time = 1.32 (sec) , antiderivative size = 3519, normalized size of antiderivative = 12.66

$$\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/4*(C*b - A*c)*log(abs(c*x^4 + b*x^2 + a))/c^2 + 1/2*(C*c*x^2 + 2*B*c*x)
/c^2 + 1/8*((2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*B
*c^2 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a
*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*B*abs(c) - (2*b^5*c^4 - 12*a*b^3*c^5 +
 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^
3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^
3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c...
```

3.22.9 Mupad [B] (verification not implemented)

Time = 8.37 (sec) , antiderivative size = 2696, normalized size of antiderivative = 9.70

$$\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x)
```

output
```
symsum(log((B^3*a^2*b*c - B*C^2*a^3*c + A^2*B*a^2*c^2 + B*C^2*a^2*b^2 - 2*
A*B*C*a^2*b*c)/c^2 - root(128*a*b^2*c^5*z^4 - 16*b^4*c^4*z^4 - 256*a^2*c^6
*z^4 - 256*C*a^2*b*c^4*z^3 + 128*C*a*b^3*c^3*z^3 - 128*A*a*b^2*c^4*z^3 - 1
6*C*b^5*c^2*z^3 + 16*A*b^4*c^3*z^3 + 256*A*a^2*c^5*z^3 + 160*A*C*a^2*b*c^3
*z^2 - 72*A*C*a*b^3*c^2*z^2 + 8*A*C*b^5*c*z^2 - 48*B^2*a^2*b*c^3*z^2 + 28*
B^2*a*b^3*c^2*z^2 + 40*A^2*a*b^2*c^3*z^2 + 32*C^2*a*b^4*c*z^2 - 56*C^2*a^2
*b^2*c^2*z^2 - 4*B^2*b^5*c*z^2 - 32*C^2*a^3*c^3*z^2 - 4*A^2*b^4*c^2*z^2 -
96*A^2*a^2*c^4*z^2 - 4*C^2*b^6*z^2 + 4*B^2*C*a^2*b^2*c*z - 32*A^2*C*a^2*b*
c^2*z + 12*A*C^2*a^2*b^2*c*z + 16*A*B^2*a^2*b*c^2*z + 8*A^2*C*a*b^3*c*z -
4*A*B^2*a*b^3*c*z - 4*A*C^2*a*b^4*z - 4*A^3*a*b^2*c^2*z - 16*B^2*C*a^3*c^2
*z + 16*A*C^2*a^3*c^2*z - 16*C^3*a^3*b*c*z + 4*C^3*a^2*b^3*z + 16*A^3*a^2*
c^3*z + 2*A^3*C*a^2*b*c + 4*A*B^2*C*a^3*c - 2*A^2*C^2*a^3*c + 2*A*C^3*a^3*
b - A^2*B^2*a^2*b*c - B^2*C^2*a^3*b - A^2*C^2*a^2*b^2 - A^4*a^2*c^2 - B^4*
a^3*c - C^4*a^4, z, k)*(root(128*a*b^2*c^5*z^4 - 16*b^4*c^4*z^4 - 256*a^2*
c^6*z^4 - 256*C*a^2*b*c^4*z^3 + 128*C*a*b^3*c^3*z^3 - 128*A*a*b^2*c^4*z^3
- 16*C*b^5*c^2*z^3 + 16*A*b^4*c^3*z^3 + 256*A*a^2*c^5*z^3 + 160*A*C*a^2*b*
c^3*z^2 - 72*A*C*a*b^3*c^2*z^2 + 8*A*C*b^5*c*z^2 - 48*B^2*a^2*b*c^3*z^2 +
28*B^2*a*b^3*c^2*z^2 + 40*A^2*a*b^2*c^3*z^2 + 32*C^2*a*b^4*c*z^2 - 56*C^2*
a^2*b^2*c^2*z^2 - 4*B^2*b^5*c*z^2 - 32*C^2*a^3*c^3*z^2 - 4*A^2*b^4*c^2*z^2
 - 96*A^2*a^2*c^4*z^2 - 4*C^2*b^6*z^2 + 4*B^2*C*a^2*b^2*c*z - 32*A^2*C*...
```

3.22. $\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$

3.23 $\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$

3.23.1 Optimal result

Integrand size = 28, antiderivative size = 270

$$\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \frac{Cx}{c} + \frac{\left(Ac-bC-\frac{Abc-(b^2-2ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(Ac-bC+\frac{Abc-b^2C+2acC}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{bB\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} + \frac{B\log\left(a+bx^2+cx^4\right)}{4c}$$

output
```
C*x/c+1/4*B*ln(c*x^4+b*x^2+a)/c+1/2*b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(
1/2))/c/(-4*a*c+b^2)^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1
/2))^(1/2))*(A*c-C*b+(-A*b*c+(-2*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2
^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*
c+b^2)^(1/2))^(1/2))*(A*c-C*b+(A*b*c+2*C*a*c-C*b^2)/(-4*a*c+b^2)^(1/2))/c^
(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.23.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.33

$$\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$$
$$= \frac{4\sqrt{c}Cx - \frac{2\sqrt{2}\left(Ac\left(b-\sqrt{b^2-4ac}\right)+\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{2\sqrt{2}\left(-Ac\left(b+\sqrt{b^2-4ac}\right)+\left(b^2-2ac+b\sqrt{b^2-4ac}\right.\right.}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4}}}}{4c^{3/2}}$$

input
```
Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
(4*Sqrt[c]*C*x - (2*Sqrt[2]*(A*c*(b - Sqrt[b^2 - 4*a*c]) + (-b^2 + 2*a*c +
 b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*
a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (2*Sqrt[2]*(-(A*
c*(b + Sqrt[b^2 - 4*a*c])) + (b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]]) + (B*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*Log[-b + Sq
rt[b^2 - 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] + (B*Sqrt[c]*(b + Sqrt[b^2 -
 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*c^(3/
2))
```

3.23.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {2193, 27, 1434, 1142, 1083, 219, 1103, 1602, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$$
$$\Big\downarrow \text{2193}$$
$$\int \frac{x^2(Cx^2+A)}{cx^4+bx^2+a}dx+\int \frac{Bx^3}{cx^4+bx^2+a}dx$$
$$\Big\downarrow \text{27}$$
$$\int \frac{x^2(Cx^2+A)}{cx^4+bx^2+a}dx+B\int \frac{x^3}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{1434}$$

$$\int \frac{x^2 \left(Cx^2+A\right)}{cx^4+bx^2+a}dx+\frac{1}{2}B\int \frac{x^2}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{1142}$$

$$\int \frac{x^2 \left(Cx^2+A\right)}{cx^4+bx^2+a}dx+\frac{1}{2}B\left(\frac{\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}-\frac{b\int \frac{1}{cx^4+bx^2+a}dx^2}{2c}\right)$$

$$\Big\downarrow \text{1083}$$

$$\int \frac{x^2 \left(Cx^2+A\right)}{cx^4+bx^2+a}dx+\frac{1}{2}B\left(\frac{b\int \frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)}{c}+\frac{\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}\right)$$

$$\Big\downarrow \text{219}$$

$$\int \frac{x^2 \left(Cx^2+A\right)}{cx^4+bx^2+a}dx+\frac{1}{2}B\left(\frac{\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}+\frac{b\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\int \frac{x^2 \left(Cx^2+A\right)}{cx^4+bx^2+a}dx+\frac{1}{2}B\left(\frac{b\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{2c}\right)$$

$$\Big\downarrow \text{1602}$$

$$-\frac{\int \frac{aC-(Ac-bC)x^2}{cx^4+bx^2+a}dx}{c}+\frac{1}{2}B\left(\frac{b\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{2c}\right)+\frac{Cx}{c}$$

$$\Big\downarrow \text{1480}$$

$$-\frac{-\frac{1}{2}\left(-\frac{2acC+Abc+b^2(-C)}{\sqrt{b^2-4ac}}+Ac-bC\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}\left(\frac{Abc-C\left(b^2-2ac\right)}{\sqrt{b^2-4ac}}+Ac-bC\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d}{c}$$

$$\frac{1}{2}B\left(\frac{b\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{2c}\right)+\frac{Cx}{c}$$

$$\Big\downarrow \text{218}$$

3.23. $\int \frac{x^2\left(A+Bx+Cx^2\right)}{a+bx^2+cx^4}\,dx$

$$-\frac{-\frac{\left(-\frac{2acC+Abc+b^2(-C)}{\sqrt{b^2-4ac}}+Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{Abc-C\left(b^2-2ac\right)}{\sqrt{b^2-4ac}}+Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}+\frac{1}{2}B\left(\frac{b\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{2c}\right)+\frac{Cx}{c}$$

input
```
Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
(C*x)/c - (-(((A*c - b*C - (A*b*c - b^2*C + 2*a*c*C)/Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sq
rt[b - Sqrt[b^2 - 4*a*c]])) - ((A*c - b*C + (A*b*c - (b^2 - 2*a*c)*C)/Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(S
qrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/c + (B*((b*ArcTanh[(b + 2*c*x
^2)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]) + Log[a + b*x^2 + c*x^4]/(2*
c)))/2
```

3.23.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

3.23.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.26

method	result
risch	$\frac{Cx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+\_Z^{2}b+a\right)}\frac{\left(Bc\_R^{3}+\_R^{2}(Ac-Cb)-Ca\right)\ln\left(x-\_R\right)}{2c\_R^{3}+\_Rb}}{2c}$
default	$\frac{Cx}{c}-\frac{\left(b^{2}-4ac+b\sqrt{-4ac+b^{2}}\right)\left(\frac{B\ln\left(2c\,x^{2}+\sqrt{-4ac+b^{2}}+b\right)}{2}+\frac{\left(2Ac-C\sqrt{-4ac+b^{2}}-Cb\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2c(4ac-b^{2})}+\frac{\left(b\sqrt{-4ac\right.}$

input
```
int(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
C*x/c+1/2/c*sum((B*c*_R^3+_R^2*(A*c-C*b)-C*a)/(2*_R^3*c+_R*b)*ln(x-_R),_R=
RootOf(_Z^4*c+_Z^2*b+a))
```

3.23.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 57.38 (sec) , antiderivative size = 861800, normalized size of antiderivative = 3191.85

$$\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.23.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.23.7 Maxima [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \int \frac{(Cx^2+Bx+A)x^2}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
C*x/c + integrate((B*c*x^3 - (C*b - A*c)*x^2 - C*a)/(c*x^4 + b*x^2 + a), x
)/c
```

3.23.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3843 vs. $2(227) = 454$.

Time = 1.47 (sec) , antiderivative size = 3843, normalized size of antiderivative = 14.23

$$\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
C*x/c + 1/4*B*log(abs(c*x^4 + b*x^2 + a))/c + 1/8*((2*b^4*c^3 - 16*a*b^2*c
^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^
2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^
2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 -
 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)
*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*A*c^2 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a
^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 +
 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^
2 + 8*(b^2 - 4*a*c)*a*b*c^3)*C*c^2 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 -
2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c + sqr...
```

3.23.9 Mupad [B] (verification not implemented)

Time = 8.51 (sec) , antiderivative size = 1890, normalized size of antiderivative = 7.00

$$\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x)
```

output

```
symsum(log(- root(128*a*b^2*c^4*z^4 - 16*b^4*c^3*z^4 - 256*a^2*c^5*z^4 - 1
28*B*a*b^2*c^3*z^3 + 16*B*b^4*c^2*z^3 + 256*B*a^2*c^4*z^3 - 48*A*C*a*b^2*c
^2*z^2 + 8*A*C*b^4*c*z^2 - 48*C^2*a^2*b*c^2*z^2 + 40*B^2*a*b^2*c^2*z^2 + 2
8*C^2*a*b^3*c*z^2 + 16*A^2*a*b*c^3*z^2 + 64*A*C*a^2*c^3*z^2 - 4*B^2*b^4*c*
z^2 - 96*B^2*a^2*c^3*z^2 - 4*A^2*b^3*c^2*z^2 - 4*C^2*b^5*z^2 + 8*A*B*C*a*b
^2*c*z + 16*B*C^2*a^2*b*c*z - 32*A*B*C*a^2*c^2*z - 4*B*C^2*a*b^3*z - 4*B^3
*a*b^2*c*z + 16*B^3*a^2*c^2*z + 4*A*B^2*C*a^2*c + 2*A^3*C*a*b*c - A^2*B^2*
a*b*c - 2*A^2*C^2*a^2*c + 2*A*C^3*a^2*b - B^2*C^2*a^2*b - A^2*C^2*a*b^2 -
B^4*a^2*c - A^4*a*c^2 - C^4*a^3, z, k)*((8*B*C*a^2*c^2 - 4*A*B*a*b*c^2)/c
- root(128*a*b^2*c^4*z^4 - 16*b^4*c^3*z^4 - 256*a^2*c^5*z^4 - 128*B*a*b^2*
c^3*z^3 + 16*B*b^4*c^2*z^3 + 256*B*a^2*c^4*z^3 - 48*A*C*a*b^2*c^2*z^2 + 8*
A*C*b^4*c*z^2 - 48*C^2*a^2*b*c^2*z^2 + 40*B^2*a*b^2*c^2*z^2 + 28*C^2*a*b^3
*c*z^2 + 16*A^2*a*b*c^3*z^2 + 64*A*C*a^2*c^3*z^2 - 4*B^2*b^4*c*z^2 - 96*B^
2*a^2*c^3*z^2 - 4*A^2*b^3*c^2*z^2 - 4*C^2*b^5*z^2 + 8*A*B*C*a*b^2*c*z + 16
*B*C^2*a^2*b*c*z - 32*A*B*C*a^2*c^2*z - 4*B*C^2*a*b^3*z - 4*B^3*a*b^2*c*z
+ 16*B^3*a^2*c^2*z + 4*A*B^2*C*a^2*c + 2*A^3*C*a*b*c - A^2*B^2*a*b*c - 2*A
^2*C^2*a^2*c + 2*A*C^3*a^2*b - B^2*C^2*a^2*b - A^2*C^2*a*b^2 - B^4*a^2*c -
 A^4*a*c^2 - C^4*a^3, z, k)*((16*C*a^2*c^3 - 4*C*a*b^2*c^2)/c + (x*(8*B*b^
3*c^2 - 32*B*a*b*c^3))/c - (root(128*a*b^2*c^4*z^4 - 16*b^4*c^3*z^4 - 256*
a^2*c^5*z^4 - 128*B*a*b^2*c^3*z^3 + 16*B*b^4*c^2*z^3 + 256*B*a^2*c^4*z^...
```

3.23. $\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$

3.24 $\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$

3.24.1 Optimal result

Integrand size = 26, antiderivative size = 223

$$\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = -\frac{B\sqrt{b-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} + \frac{B\sqrt{b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} - \frac{(2Ac-bC)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} + \frac{C\log\left(a+bx^2+cx^4\right)}{4c}$$

output
```
1/4*C*ln(c*x^4+b*x^2+a)/c-1/2*(2*A*c-C*b)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)
^(1/2))/c/(-4*a*c+b^2)^(1/2)-1/2*B*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2
)^(1/2))^(1/2))*(b-(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^
(1/2)+1/2*B*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*
a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)
```

3.24.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.08

$$\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \frac{-2\sqrt{2}B\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)+2\sqrt{2}B\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)+(2\ldots}{4\ldots}$$

input
```
Integrate[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
(-2*Sqrt[2]*B*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]] + 2*Sqrt[2]*B*Sqrt[c]*Sqrt[b + Sqrt[b^2 -
4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]] + (2*A*c +
 (-b + Sqrt[b^2 - 4*a*c])*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2] - (2*A*
c - (b + Sqrt[b^2 - 4*a*c])*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*c*
Sqrt[b^2 - 4*a*c])
```

3.24.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {2193, 27, 1450, 218, 1576, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{2193}$$

$$\int \frac{x(Cx^2+A)}{cx^4+bx^2+a}dx+\int \frac{Bx^2}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x(Cx^2+A)}{cx^4+bx^2+a}dx+B\int \frac{x^2}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{1450}$$

$$\int \frac{x(Cx^2+A)}{cx^4+bx^2+a}dx+ B\left(\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx\right)$$

$$\Big\downarrow \text{218}$$

$$\int \frac{x(Cx^2+A)}{cx^4+bx^2+a}dx+ B\left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{2}\int \frac{Cx^2+A}{cx^4+bx^2+a}dx^2+ B\left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{(2Ac-bC)\int \frac{1}{cx^4+bx^2+a}dx^2}{2c}+\frac{C\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}\right)+ B\left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{C\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}-\frac{(2Ac-bC)\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{c}\right)+ B\left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{C\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}-\frac{(2Ac-bC)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}\right)+ B\left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

3.24. $\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$

$$\big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{C\log\left(a+bx^2+cx^4\right)}{2c}-\frac{(2Ac-bC)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}\right)+$$
$$B\left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

input
```
Int[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
B*(((1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((1 + b/Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sq
rt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])) + (-(((2*A*c - b*C)*ArcTanh[(b
 + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c])) + (C*Log[a + b*x^2
+ c*x^4])/(2*c))/2
```

3.24.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

3.24.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.23

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+\_Z^{2}b+a\right)}\frac{\left(C\_R^{3}+B\_R^{2}+A\_R\right)\ln\left(x-\_R\right)}{2c\_R^{3}+\_Rb}\right)}{2}$
default	$4c\left(\frac{\frac{\left(2Ac\sqrt{-4ac+b^{2}}-C\sqrt{-4ac+b^{2}}\,b+4acC-b^{2}C\right)\ln\left(2c\,x^{2}+\sqrt{-4ac+b^{2}}+b\right)}{4c}+\frac{\left(-Bb\sqrt{-4ac+b^{2}}+4Bac-B\,b^{2}\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}}{4c(4ac-b^{2})}\right.$

input
```
int(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum((C*_R^3+B*_R^2+A*_R)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^
2*b+a))
```

3.24.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 13.96 (sec) , antiderivative size = 845032, normalized size of antiderivative = 3789.38

$$\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.24.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.24.7 Maxima [F]

$$\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \int \frac{(Cx^2+Bx+A)x}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*x/(c*x^4 + b*x^2 + a), x)
```

3.24.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2368 vs. $2(179) = 358$.

Time = 1.46 (sec) , antiderivative size = 2368, normalized size of antiderivative = 10.62

$$\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/4*C*log(abs(c*x^4 + b*x^2 + a))/c + 1/8*((2*b^4*c^2 - 16*a*b^2*c^3 + 32*
a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 +
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(
b^2 - 4*a*c)*a*c^3)*B*c^2 - (2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*B)*arctan(2*
sqrt(1/2)*x/sqrt((b*c + sqrt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^2 - 8*a^2
*b^2*c^3 - 2*a*b^3*c^3 + 16*a^3*c^4 + 8*a^2*b*c^4 + a*b^2*c^4 - 4*a^2*c^5)
*c^2) - 1/8*((2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s...
```

3.24.9 Mupad [B] (verification not implemented)

Time = 8.37 (sec) , antiderivative size = 5594, normalized size of antiderivative = 25.09

$$\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x)
```

output
```
symsum(log(A^3*c^2*x - B^3*a*c - B*C^2*a*b - 8*root(128*a*b^2*c^3*z^4 - 16
*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 +
 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^
2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c
^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^
2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2
*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A
^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2
 - C^4*a^2 - A^4*c^2, z, k)^3*b^3*c^2*x - C^3*a*b*x + A*C^2*b^2*x - 2*C^2*
root(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^
2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^
3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A
^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16
*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16
*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*
C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*
b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*b^3*x + 32*root(128
*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 +
256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2
+ 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*...
```

3.24. $\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$

3.25 $\int \frac{A+Bx+Cx^2}{a+bx^2+cx^4}\,dx$

3.25.1 Optimal result

Integrand size = 25, antiderivative size = 211

$$\int \frac{A+Bx+Cx^2}{a+bx^2+cx^4}\,dx = \frac{\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} - \frac{B\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

output
```
-B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)+1/2*arctan(x
*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(C+(2*A*c-C*b)/(-4*a*c+b^2)
^(1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*
c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(C+(-2*A*c+C*b)/(-4*a*c+b^2)^(1/2))*
2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.25.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx+Cx^2}{a+bx^2+cx^4}\,dx = \frac{\frac{\sqrt{2}\left(2Ac+\left(-b+\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(-2Ac+\left(b+\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} + B\log\left(-b+\sqrt{b^2-\right.}{2\sqrt{b^2-4ac}}$$

input
```
Integrate[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4),x]
```

output
```
((Sqrt[2]*(2*A*c + (-b + Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqr
t[2]*(-2*A*c + (b + Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + B*Log[-b
+ Sqrt[b^2 - 4*a*c] - 2*c*x^2] - B*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(
2*Sqrt[b^2 - 4*a*c])
```

3.25.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2202, 27, 1432, 1083, 219, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \text{ 2202}$$

$$\int \frac{Cx^2+A}{cx^4+bx^2+a}dx+\int \frac{Bx}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{Cx^2+A}{cx^4+bx^2+a}dx+B\int \frac{x}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{ 1432}$$

$$\int \frac{Cx^2+A}{cx^4+bx^2+a}dx+\frac{1}{2}B\int \frac{1}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{ 1083}$$

$$\int \frac{Cx^2+A}{cx^4+bx^2+a}dx-B\int \frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)$$

$$\Big\downarrow \text{ 219}$$

$$\int \frac{Cx^2+A}{cx^4+bx^2+a}dx-\frac{B\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{ 1480}$$

$$\frac{1}{2}\left(\frac{2Ac-bC}{\sqrt{b^2-4ac}}+C\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+$$
$$\frac{1}{2}\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx-\frac{B\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\left(\frac{2Ac-bC}{\sqrt{b^2-4ac}}+C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-$$
$$\frac{B\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4),x]
```

output
```
((C + (2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((C
- (2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (B*ArcTa
nh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

3.25.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

3.25.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.23

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(C\_R^2+B\_R+A\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}\right)}{2}$
default	$4c\left(\frac{\sqrt{-4ac+b^2}\left(\frac{B\ln\left(2c\,x^2+\sqrt{-4ac+b^2}+b\right)}{2}+\frac{\left(2Ac-C\sqrt{-4ac+b^2}-Cb\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4c(4ac-b^2)}-\frac{\sqrt{-4ac+b^2}\left(\frac{B\ln\left(-2c\,x\right.}{}\right.}{}\right.$

input
```
int((C*x^2+B*x+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum((C*_R^2+B*_R+A)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a
))
```

3.25.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 29.86 (sec) , antiderivative size = 578003, normalized size of antiderivative = 2739.35

$$\int \frac{A+Bx+Cx^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.25.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.25.7 Maxima [F]

$$\int \frac{A+Bx+Cx^2}{a+bx^2+cx^4}\,dx = \int \frac{Cx^2+Bx+A}{cx^4+bx^2+a}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(c*x^4 + b*x^2 + a), x)
```

3.25.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1714 vs. $2(171) = 342$.

Time = 1.29 (sec) , antiderivative size = 1714, normalized size of antiderivative = 8.12

$$\int \frac{A + Bx + Cx^2}{a + bx^2 + cx^4} \, dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*(b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*sqrt(b^2 - 4*a*c)*B*log(x^2 + 1/2
*(b - sqrt(b^2 - 4*a*c))/c)/((b^4 - 8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a
*b*c^2 + b^2*c^2 - 4*a*c^3)*c^2) + 1/4*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2
*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 -
8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3
+ 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^
2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*A - 2*(2*a*b^2*c^2 -
8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^
2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*
a*c^2)*C)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a*b^4 -
8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3
)*abs(c)) + 1/4*((sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(...
```

3.25.9 Mupad [B] (verification not implemented)

Time = 8.75 (sec) , antiderivative size = 3942, normalized size of antiderivative = 18.68

$$\int \frac{A + Bx + Cx^2}{a + bx^2 + cx^4} \, dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(a + b*x^2 + c*x^4),x)
```

output

```
symsum(log(A*B^2*c^2 - A^2*C*c^2 + B^3*c^2*x - C^3*a*c + A*C^2*b*c - 8*roo
t(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C*a*b^2*c*
z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2 + 64*A*C
*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2*z^2 + 16
*B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c^2*z - 4*
A*B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B^2*C^2*a*b + A^
2*B^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k)^3*b^3*c^2*x -
 16*A*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C
*a*b^2*c*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2
 + 64*A*C*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2
*z^2 + 16*B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c
^2*z - 4*A*B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B^2*C^2
*a*b + A^2*B^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k)^2*a*
c^3 - 4*A^2*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 -
16*A*C*a*b^2*c*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c
^2*z^2 + 64*A*C*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a
^2*c^2*z^2 + 16*B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2
*B*a*c^2*z - 4*A*B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B
^2*C^2*a*b + A^2*B^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k
)*c^3*x + 4*A*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z...
```

3.25. $\int \frac{A+Bx+Cx^2}{a+bx^2+cx^4}\,dx$

3.26 $\int \frac{A+Bx+Cx^2}{x(a+bx^2+cx^4)}\,dx$

3.26.1 Optimal result

Integrand size = 28, antiderivative size = 229

$$\int \frac{A+Bx+Cx^2}{x(a+bx^2+cx^4)}\,dx = \frac{\sqrt{2}B\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}B\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{(Ab-2aC)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}} + \frac{A\log(x)}{a} - \frac{A\log(a+bx^2+cx^4)}{4a}$$

output
```
A*ln(x)/a-1/4*A*ln(c*x^4+b*x^2+a)/a+1/2*(A*b-2*C*a)*arctanh((2*c*x^2+b)/(-
4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)+B*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a
*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(
1/2))^(1/2)-B*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/
2)*c^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.26.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)}\,dx = \frac{\sqrt{2}B\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}B\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{A\log(x)}{a} - \frac{\left(A\left(b+\sqrt{b^2-4ac}\right)-2aC\right)\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{4a\sqrt{b^2-4ac}} - \frac{\left(A\left(-b+\sqrt{b^2-4ac}\right)+2aC\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{4a\sqrt{b^2-4ac}}$$

input
```
Integrate[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)),x]
```

output
```
(Sqrt[2]*B*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*B*Sqrt[c]*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*S
qrt[b + Sqrt[b^2 - 4*a*c]]) + (A*Log[x])/a - ((A*(b + Sqrt[b^2 - 4*a*c]) -
 2*a*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(4*a*Sqrt[b^2 - 4*a*c]) - (
(A*(-b + Sqrt[b^2 - 4*a*c]) + 2*a*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])
/(4*a*Sqrt[b^2 - 4*a*c])
```

3.26.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2193, 27, 1406, 218, 1578, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)}\,dx$$

$$\Big\downarrow \; 2193$$

$$\int \frac{Cx^2+A}{x\left(cx^4+bx^2+a\right)}dx+\int \frac{B}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \; 27$$

$$\int \frac{Cx^2+A}{x\left(cx^4+bx^2+a\right)}dx+B\int \frac{1}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \; 1406$$

$$\int \frac{Cx^2+A}{x\left(cx^4+bx^2+a\right)}dx+B\left(\frac{c\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}\right)$$

$$\Big\downarrow \; 218$$

$$\int \frac{Cx^2+A}{x\left(cx^4+bx^2+a\right)}dx+B\left(\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

$$\Big\downarrow \; 1578$$

$$\frac{1}{2}\int \frac{Cx^2+A}{x^2\left(cx^4+bx^2+a\right)}dx^2+B\left(\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

$$\Big\downarrow \; 1200$$

$$\frac{1}{2}\int \left(\frac{A}{ax^2}+\frac{-Acx^2-Ab+aC}{a\left(cx^4+bx^2+a\right)}\right)dx^2+$$
$$B\left(\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{(Ab-2aC)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{A\log\left(a+bx^2+cx^4\right)}{2a}+\frac{A\log\left(x^2\right)}{a}\right)+$$
$$B\left(\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

input
```
Int[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)),x]
```

3.26. $\int \frac{A+Bx+Cx^2}{x(a+bx^2+cx^4)}\,dx$

output
```
B*((Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]
])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sq
rt[b + Sqrt[b^2 - 4*a*c]])) + (((A*b - 2*a*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b
^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + (A*Log[x^2])/a - (A*Log[a + b*x^2 +
c*x^4])/(2*a))/2
```

3.26.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

3.26.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.10

method	result
default	$\frac{A\ln(x)}{a}+\frac{4c\left(\frac{\sqrt{-4ac+b^2}\left(\frac{\left(A\sqrt{-4ac+b^2}-Ab+2Ca\right)\ln\left(2c\,x^2+\sqrt{-4ac+b^2}+b\right)}{4c}+\frac{Ba\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{16ac-4b^2}+\frac{\sqrt{-4ac+b^2}\left(-\frac{(-\cdots}{}\right.}{}\right.}{a}$
risch	$\frac{A\ln(x)}{a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(16a^4c^2-8a^3b^2c+a^2b^4\right)\_Z^4+\left(32A\,a^3c^2-16A\,a^2b^2c+2Aa\,b^4\right)\_Z^3+\left(24a^2c^2A^2-10a\,b^2c\,A^2+b^4A^2-8AC\,a^2bc+2A\cdots\right.\right.}\right.}{}$

input
```
int((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
A*ln(x)/a+4/a*c*((-4*a*c+b^2)^(1/2)/(16*a*c-4*b^2)*(1/4*(A*(-4*a*c+b^2)^(1
/2)-A*b+2*C*a)/c*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)+B*a*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
+(-4*a*c+b^2)^(1/2)/(16*a*c-4*b^2)*(-1/4*(-A*(-4*a*c+b^2)^(1/2)-A*b+2*C*a)
/c*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)+B*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

3.26.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.26.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/x/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.26.7 Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)}\,dx=\int \frac{Cx^2+Bx+A}{(cx^4+bx^2+a)x}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
A*log(x)/a - integrate((A*c*x^3 - B*a - (C*a - A*b)*x)/(c*x^4 + b*x^2 + a)
, x)/a
```

3.26.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2339 vs. $2(186) = 372$.

Time = 1.43 (sec) , antiderivative size = 2339, normalized size of antiderivative = 10.21

$$\int \frac{A + Bx + Cx^2}{x\,(a + bx^2 + cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/4*A*log(abs(c*x^4 + b*x^2 + a))/a + A*log(abs(x))/a + 1/4*((sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 2*b^4*c + 1
6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*
c^2 + 16*a*b^2*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 32*
a^2*c^3 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2)*B*abs(c) - (2*b^3
*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^
2 - 4*a*c)*b*c^3)*B)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b*c + sqrt(a^4*b^2*c^2
 - 4*a^5*c^3))/(a^2*c^2)))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2
+ 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2) + 1/4*((sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c
- 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*
b^2*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 - 2
*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2)*B*abs(c) - (2*b^3*c^3 - 8...
```

3.26.9 Mupad [B] (verification not implemented)

Time = 8.37 (sec) , antiderivative size = 2258, normalized size of antiderivative = 9.86

$$\int \frac{A + Bx + Cx^2}{x\,(a + bx^2 + cx^4)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)),x)
```

output
```
symsum(log(x*(B^4*c^3 + C^4*a*c^2 + A^2*C^2*c^3 - 3*A*B^2*C*c^3 - A*C^3*b*
c^2 + B^2*C^2*b*c^2) - root(128*a^3*b^2*c*z^4 - 256*a^4*c^2*z^4 - 16*a^2*b
^4*z^4 + 128*A*a^2*b^2*c*z^3 - 256*A*a^3*c^2*z^3 - 16*A*a*b^4*z^3 + 32*A*C
*a^2*b*c*z^2 - 8*A*C*a*b^3*z^2 + 16*B^2*a^2*b*c*z^2 + 40*A^2*a*b^2*c*z^2 -
 32*C^2*a^3*c*z^2 - 4*B^2*a*b^3*z^2 + 8*C^2*a^2*b^2*z^2 - 96*A^2*a^2*c^2*z
^2 - 4*A^2*b^4*z^2 + 16*A^2*C*a*b*c*z + 16*B^2*C*a^2*c*z - 16*A*C^2*a^2*c*
z - 4*B^2*C*a*b^2*z + 4*A*C^2*a*b^2*z + 4*A^3*b^2*c*z - 16*A^3*a*c^2*z - 4
*A^2*C*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b -
 B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z,
 k)*(x*(A*B^2*b*c^3 - 5*A^3*c^4 - 13*A*C^2*a*c^3 + 6*A^2*C*b*c^3 + 17*B^2*
C*a*c^3 + C^3*a*b*c^2 + A*C^2*b^2*c^2 - 4*B^2*C*b^2*c^2) - root(128*a^3*b^
2*c*z^4 - 256*a^4*c^2*z^4 - 16*a^2*b^4*z^4 + 128*A*a^2*b^2*c*z^3 - 256*A*a
^3*c^2*z^3 - 16*A*a*b^4*z^3 + 32*A*C*a^2*b*c*z^2 - 8*A*C*a*b^3*z^2 + 16*B^
2*a^2*b*c*z^2 + 40*A^2*a*b^2*c*z^2 - 32*C^2*a^3*c*z^2 - 4*B^2*a*b^3*z^2 +
8*C^2*a^2*b^2*z^2 - 96*A^2*a^2*c^2*z^2 - 4*A^2*b^4*z^2 + 16*A^2*C*a*b*c*z
+ 16*B^2*C*a^2*c*z - 16*A*C^2*a^2*c*z - 4*B^2*C*a*b^2*z + 4*A*C^2*a*b^2*z
+ 4*A^3*b^2*c*z - 16*A^3*a*c^2*z - 4*A^2*C*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C
^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c -
 A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*(x*(60*A^2*a*c^4 - 16*A^2*b^2*c^3
+ 4*B^2*b^3*c^2 + 36*C^2*a^2*c^3 + 8*A*C*b^3*c^2 - 14*B^2*a*b*c^3 - 10*...
```

3.26. $\int \frac{A+Bx+Cx^2}{x(a+bx^2+cx^4)}\,dx$

3.27 $\int \frac{A+Bx+Cx^2}{x^2(a+bx^2+cx^4)}\,dx$

3.27.1 Optimal result

Integrand size = 28, antiderivative size = 260

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx = -\frac{A}{ax} - \frac{\sqrt{c}\left(A+\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(A-\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}} + \frac{bB\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}} + \frac{B\log(x)}{a} - \frac{B\log\left(a+bx^2+cx^4\right)}{4a}$$

output
```
-A/a/x+B*ln(x)/a-1/4*B*ln(c*x^4+b*x^2+a)/a+1/2*b*B*arctanh((2*c*x^2+b)/(-4
*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*
a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(A+(A*b-2*C*a)/(-4*a*c+b^2)^(1/2))/a*2^(1/2
)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2
)^(1/2))^(1/2))*c^(1/2)*(A+(-A*b+2*C*a)/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b+(
-4*a*c+b^2)^(1/2))^(1/2)
```

3.27.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx = -\frac{\frac{4A}{x}+\frac{2\sqrt{2}\sqrt{c}\left(A\left(b+\sqrt{b^2-4ac}\right)-2aC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{2\sqrt{2}\sqrt{c}\left(A\left(-b+\sqrt{b^2-4ac}\right)+2aC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}-4B\log}{4a}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

output
```
-1/4*((4*A)/x + (2*Sqrt[2]*Sqrt[c]*(A*(b + Sqrt[b^2 - 4*a*c]) - 2*a*C)*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*S
qrt[b - Sqrt[b^2 - 4*a*c]]) + (2*Sqrt[2]*Sqrt[c]*(A*(-b + Sqrt[b^2 - 4*a*c
]) + 2*a*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - 4*B*Log[x] + (B*(b + Sqrt[b^2
 - 4*a*c])*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] + (B*(
-b + Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4
*a*c])/a
```

3.27.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2193, 27, 1434, 1144, 25, 1142, 1083, 219, 1103, 1604, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx$$

$$\Big\downarrow \ 2193$$

$$\int \frac{Cx^2+A}{x^2\left(cx^4+bx^2+a\right)}dx+\int \frac{B}{x\left(cx^4+bx^2+a\right)}dx$$

$$\Big\downarrow \ 27$$

$$\int \frac{Cx^2+A}{x^2\left(cx^4+bx^2+a\right)}dx+B\int \frac{1}{x\left(cx^4+bx^2+a\right)}dx$$

$$\Big\downarrow \text{1434}$$

$$\int \frac{Cx^2+A}{x^2\left(cx^4+bx^2+a\right)}dx+\frac{1}{2}B\int \frac{1}{x^2\left(cx^4+bx^2+a\right)}dx^2$$

$$\Big\downarrow \text{1144}$$

$$\int \frac{Cx^2+A}{x^2\left(cx^4+bx^2+a\right)}dx+\frac{1}{2}B\left(\frac{\int -\frac{cx^2+b}{cx^4+bx^2+a}dx^2}{a}+\frac{\log\left(x^2\right)}{a}\right)$$

$$\Big\downarrow \text{25}$$

$$\int \frac{Cx^2+A}{x^2\left(cx^4+bx^2+a\right)}dx+\frac{1}{2}B\left(\frac{\log\left(x^2\right)}{a}-\frac{\int \frac{cx^2+b}{cx^4+bx^2+a}dx^2}{a}\right)$$

$$\Big\downarrow \text{1142}$$

$$\int \frac{Cx^2+A}{x^2\left(cx^4+bx^2+a\right)}dx+\frac{1}{2}B\left(\frac{\log\left(x^2\right)}{a}-\frac{\frac{1}{2}b\int \frac{1}{cx^4+bx^2+a}dx^2+\frac{1}{2}\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{a}\right)$$

$$\Big\downarrow \text{1083}$$

$$\int \frac{Cx^2+A}{x^2\left(cx^4+bx^2+a\right)}dx+\frac{1}{2}B\left(\frac{\log\left(x^2\right)}{a}-\frac{\frac{1}{2}\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2-b\int \frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)}{a}\right)$$

$$\Big\downarrow \text{219}$$

$$\int \frac{Cx^2+A}{x^2\left(cx^4+bx^2+a\right)}dx+\frac{1}{2}B\left(\frac{\log\left(x^2\right)}{a}-\frac{\frac{1}{2}\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2-\frac{b\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}\right)$$

$$\Big\downarrow \text{1103}$$

$$\int \frac{Cx^2+A}{x^2\left(cx^4+bx^2+a\right)}dx+\frac{1}{2}B\left(\frac{\log\left(x^2\right)}{a}-\frac{\frac{1}{2}\log\left(a+bx^2+cx^4\right)-\frac{b\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}\right)$$

$$\Big\downarrow \text{1604}$$

$$-\frac{\int \frac{Acx^2+Ab-aC}{cx^4+bx^2+a}dx}{a}-\frac{A}{ax}+\frac{1}{2}B\left(\frac{\log\left(x^2\right)}{a}-\frac{\frac{1}{2}\log\left(a+bx^2+cx^4\right)-\frac{b\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}\right)$$

$$\Big\downarrow \text{1480}$$

3.27. $\int \frac{A+Bx+Cx^2}{x^2(a+bx^2+cx^4)}\,dx$

$$-\frac{\frac{1}{2}c\left(\frac{Ab-2aC}{\sqrt{b^2-4ac}}+A\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(A-\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{a}-\frac{A}{ax}+\frac{1}{2}B\left(\frac{\log\left(x^2\right)}{a}-\frac{\frac{1}{2}\log\left(a+bx^2+cx^4\right)-\frac{b\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}\right)$$

$$\Big\downarrow \ 218$$

$$-\frac{\frac{\sqrt{c}\left(\frac{Ab-2aC}{\sqrt{b^2-4ac}}+A\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(A-\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{A}{ax}+\frac{1}{2}B\left(\frac{\log\left(x^2\right)}{a}-\frac{\frac{1}{2}\log\left(a+bx^2+cx^4\right)-\frac{b\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}\right)$$

input
```
Int[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

output
```
-(A/(a*x)) - ((Sqrt[c]*(A + (A*b - 2*a*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) + (Sqrt[c]*(A - (A*b - 2*a*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c
]]))/a + (B*(Log[x^2]/a - (-((b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
Sqrt[b^2 - 4*a*c]) + Log[a + b*x^2 + c*x^4]/2)/a))/2
```

3.27.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.27. $\int \frac{A+Bx+Cx^2}{x^2(a+bx^2+cx^4)}\,dx$

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

3.27.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.25

method	result
default	$-\frac{A}{ax}+\frac{B\ln(x)}{a}+\frac{4c\left(\frac{\frac{\left(-Bb\sqrt{-4ac+b^{2}}-4Bac+Bb^{2}\right)\ln\left(2cx^{2}+\sqrt{-4ac+b^{2}}+b\right)}{4c}+\frac{\left(-Ab\sqrt{-4ac+b^{2}}-4Aac+Ab^{2}+2C\sqrt{-4ac+b^{2}}\,a\right)\sqrt{2}\,\text{arcta}}{2\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}}{16ac-4b^{2}}\right.}{}$
risch	$-\frac{A}{ax}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(16a^{5}c^{2}-8a^{4}b^{2}c+b^{4}a^{3}\right)\_Z^{4}+\left(32Ba^{4}c^{2}-16Ba^{3}b^{2}c+2Ba^{2}b^{4}\right)\_Z^{3}+\left(12A^{2}a^{2}bc^{2}-7A^{2}ab^{3}c+A^{2}b^{5}-16ACa^{3}c^{2}+12\right.}\right.}{}$

input
```
int((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-A/a/x+B*ln(x)/a+4/a*c*(1/(16*a*c-4*b^2)*(1/4*(-B*b*(-4*a*c+b^2)^(1/2)-4*B
*a*c+B*b^2)/c*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)+1/2*(-A*b*(-4*a*c+b^2)^(1/2
)-4*A*a*c+A*b^2+2*C*(-4*a*c+b^2)^(1/2)*a)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+1/(16*a*c-4
*b^2)*(-1/4*(-B*b*(-4*a*c+b^2)^(1/2)+4*B*a*c-B*b^2)/c*ln(-2*c*x^2+(-4*a*c+
b^2)^(1/2)-b)+1/2*(-A*b*(-4*a*c+b^2)^(1/2)+4*A*a*c-A*b^2+2*C*(-4*a*c+b^2)^
(1/2)*a)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

3.27.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^2\,(a+bx^2+cx^4)}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.27.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^2\,(a+bx^2+cx^4)}\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/x**2/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.27.7 Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx=\int \frac{Cx^2+Bx+A}{(cx^4+bx^2+a)x^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
B*log(x)/a - integrate((B*c*x^3 + A*c*x^2 + B*b*x - C*a + A*b)/(c*x^4 + b*
x^2 + a), x)/a - A/(a*x)
```

3.27.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3505 vs. $2(218) = 436$.

Time $= 1.57$ (sec) , antiderivative size $= 3505$, normalized size of antiderivative $= 13.48$

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx=\text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/4*B*log(abs(c*x^4 + b*x^2 + a))/a + B*log(abs(x))/a - A/(a*x) - 1/8*((2
*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*
c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 -
2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*A*c^2 + 2*(sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^
2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*
A*abs(c) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*s...
```

3.27.9 Mupad [B] (verification not implemented)

Time = 8.16 (sec) , antiderivative size = 2588, normalized size of antiderivative = 9.95

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx=\text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)),x)
```

output
```
symsum(log(root(128*a^4*b^2*c*z^4 - 256*a^5*c^2*z^4 - 16*a^3*b^4*z^4 + 128
*B*a^3*b^2*c*z^3 - 256*B*a^4*c^2*z^3 - 16*B*a^2*b^4*z^3 - 48*A*C*a^2*b^2*c
*z^2 + 8*A*C*a*b^4*z^2 + 40*B^2*a^2*b^2*c*z^2 - 48*A^2*a^2*b*c^2*z^2 + 16*
C^2*a^3*b*c*z^2 + 28*A^2*a*b^3*c*z^2 + 64*A*C*a^3*c^2*z^2 - 4*B^2*a*b^4*z^
2 - 96*B^2*a^3*c^2*z^2 - 4*C^2*a^2*b^3*z^2 - 4*A^2*b^5*z^2 - 8*A*B*C*a*b^2
*c*z - 16*A^2*B*a*b*c^2*z + 32*A*B*C*a^2*c^2*z + 4*A^2*B*b^3*c*z + 4*B^3*a
*b^2*c*z - 16*B^3*a^2*c^2*z + 4*A*B^2*C*a*c^2 + 2*A*C^3*a*b*c - B^2*C^2*a*
b*c - 2*A^2*C^2*a*c^2 + 2*A^3*C*b*c^2 - A^2*C^2*b^2*c - A^2*B^2*b*c^2 - C^
4*a^2*c - B^4*a*c^2 - A^4*c^3, z, k)*(root(128*a^4*b^2*c*z^4 - 256*a^5*c^2
*z^4 - 16*a^3*b^4*z^4 + 128*B*a^3*b^2*c*z^3 - 256*B*a^4*c^2*z^3 - 16*B*a^2
*b^4*z^3 - 48*A*C*a^2*b^2*c*z^2 + 8*A*C*a*b^4*z^2 + 40*B^2*a^2*b^2*c*z^2 -
 48*A^2*a^2*b*c^2*z^2 + 16*C^2*a^3*b*c*z^2 + 28*A^2*a*b^3*c*z^2 + 64*A*C*a
^3*c^2*z^2 - 4*B^2*a*b^4*z^2 - 96*B^2*a^3*c^2*z^2 - 4*C^2*a^2*b^3*z^2 - 4*
A^2*b^5*z^2 - 8*A*B*C*a*b^2*c*z - 16*A^2*B*a*b*c^2*z + 32*A*B*C*a^2*c^2*z
+ 4*A^2*B*b^3*c*z + 4*B^3*a*b^2*c*z - 16*B^3*a^2*c^2*z + 4*A*B^2*C*a*c^2 +
 2*A*C^3*a*b*c - B^2*C^2*a*b*c - 2*A^2*C^2*a*c^2 + 2*A^3*C*b*c^2 - A^2*C^2
*b^2*c - A^2*B^2*b*c^2 - C^4*a^2*c - B^4*a*c^2 - A^4*c^3, z, k)*(root(128*
a^4*b^2*c*z^4 - 256*a^5*c^2*z^4 - 16*a^3*b^4*z^4 + 128*B*a^3*b^2*c*z^3 - 2
56*B*a^4*c^2*z^3 - 16*B*a^2*b^4*z^3 - 48*A*C*a^2*b^2*c*z^2 + 8*A*C*a*b^4*z
^2 + 40*B^2*a^2*b^2*c*z^2 - 48*A^2*a^2*b*c^2*z^2 + 16*C^2*a^3*b*c*z^2 +...
```

3.27. $\int \frac{A+Bx+Cx^2}{x^2(a+bx^2+cx^4)}\,dx$

3.28 $\int \frac{A+Bx+Cx^2}{x^3(a+bx^2+cx^4)}\,dx$

3.28.1 Optimal result

Integrand size = 28, antiderivative size = 288

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{x^3(a+bx^2+cx^4)}\,dx &= -\frac{A}{2ax^2}-\frac{B}{ax}-\frac{B\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}\\ &\quad-\frac{B\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}\\ &\quad-\frac{(A(b^2-2ac)-abC)\,\mathrm{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}\\ &\quad-\frac{(Ab-aC)\log(x)}{a^2}+\frac{(Ab-aC)\log(a+bx^2+cx^4)}{4a^2}\end{aligned}$$

output
```
-1/2*A/a/x^2-B/a/x-(A*b-C*a)*ln(x)/a^2+1/4*(A*b-C*a)*ln(c*x^4+b*x^2+a)/a^2
-1/2*(A*(-2*a*c+b^2)-a*b*C)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-
4*a*c+b^2)^(1/2)-1/2*B*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/
2))*c^(1/2)*(1+b/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2
)-1/2*B*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(1-
b/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.28.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.31

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx$$
$$= \frac{-\frac{2aA}{x^2}-\frac{4aB}{x}-\frac{2\sqrt{2}aB\sqrt{c}\left(b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{2\sqrt{2}aB\sqrt{c}\left(-b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}+4(-Ab+}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

output
```
((-2*a*A)/x^2 - (4*a*B)/x - (2*Sqrt[2]*a*B*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*
c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (2*Sqrt[2]*a*B*Sqrt[c]*(-b + Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2
 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + 4*(-(A*b) + a*C)*Log[x] + ((A*(b^
2 - 2*a*c + b*Sqrt[b^2 - 4*a*c]) - a*(b + Sqrt[b^2 - 4*a*c])*C)*Log[-b + S
qrt[b^2 - 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((A*(-b^2 + 2*a*c + b*Sqr
t[b^2 - 4*a*c]) + a*(b - Sqrt[b^2 - 4*a*c])*C)*Log[b + Sqrt[b^2 - 4*a*c] +
 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a^2)
```

3.28.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {2193, 27, 1443, 25, 1480, 218, 1578, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx$$
$$\Big\downarrow \; 2193$$
$$\int \frac{Cx^2+A}{x^3\left(cx^4+bx^2+a\right)}dx+\int \frac{B}{x^2\left(cx^4+bx^2+a\right)}dx$$
$$\Big\downarrow \; 27$$
$$\int \frac{Cx^2+A}{x^3\left(cx^4+bx^2+a\right)}dx+B\int \frac{1}{x^2\left(cx^4+bx^2+a\right)}dx$$

$$\downarrow 1443$$

$$\int \frac{Cx^2+A}{x^3\left(cx^4+bx^2+a\right)}dx+B\left(\frac{\int -\frac{cx^2+b}{cx^4+bx^2+a}dx}{a}-\frac{1}{ax}\right)$$

$$\downarrow 25$$

$$\int \frac{Cx^2+A}{x^3\left(cx^4+bx^2+a\right)}dx+B\left(-\frac{\int \frac{cx^2+b}{cx^4+bx^2+a}dx}{a}-\frac{1}{ax}\right)$$

$$\downarrow 1480$$

$$\int \frac{Cx^2+A}{x^3\left(cx^4+bx^2+a\right)}dx+ B\left(-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{a}-\frac{1}{ax}\right)$$

$$\downarrow 218$$

$$\int \frac{Cx^2+A}{x^3\left(cx^4+bx^2+a\right)}dx+ B\left(-\frac{\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{1}{ax}\right)$$

$$\downarrow 1578$$

$$\frac{1}{2}\int \frac{Cx^2+A}{x^4\left(cx^4+bx^2+a\right)}dx^2+ B\left(-\frac{\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{1}{ax}\right)$$

$$\downarrow 1200$$

$$\frac{1}{2}\int \left(\frac{A}{ax^4}+\frac{c(Ab-aC)x^2+A\left(b^2-ac\right)-abC}{a^2\left(cx^4+bx^2+a\right)}+\frac{aC-Ab}{a^2x^2}\right)dx^2+ B\left(-\frac{\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{1}{ax}\right)$$

3.28. $\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\left(A\left(b^2-2ac\right)-abC\right)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}+\frac{(Ab-aC)\log\left(a+bx^2+cx^4\right)}{2a^2}-\frac{\log\left(x^2\right)(Ab-aC)}{a^2}-\frac{A}{ax^2}\right)+B\left(-\frac{\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{1}{ax}\right)$$

input
```
Int[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

output
```
B*(-(1/(a*x)) - ((Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
 (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sq
rt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a) + (-(A/(a*x^2
)) - ((A*(b^2 - 2*a*c) - a*b*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
(a^2*Sqrt[b^2 - 4*a*c]) - ((A*b - a*C)*Log[x^2])/a^2 + ((A*b - a*C)*Log[a
+ b*x^2 + c*x^4])/(2*a^2))/2
```

3.28.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

3.28. $\int \frac{A+Bx+Cx^2}{x^3(a+bx^2+cx^4)} \, dx$

3.28.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.04

method	result
default	$-\frac{A}{2a\,x^{2}}-\frac{B}{ax}+\frac{(-Ab+Ca)\ln(x)}{a^{2}}+\frac{4c\left(-\frac{\left(b\sqrt{-4ac+b^{2}}+4ac-b^{2}\right)\left(\frac{\left(A\sqrt{-4ac+b^{2}}-Ab+2Ca\right)\ln\left(2c\,x^{2}+\sqrt{-4ac+b^{2}}+b\right)}{4c}+\frac{Ba\sqrt{2}\,\arctan}{\sqrt{(b}}\right.}{32ac-8b^{2}}\right.}{}$
risch	Expression too large to display

input
```
int((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*A/a/x^2-B/a/x+1/a^2*(-A*b+C*a)*ln(x)+4/a^2*c*(-(b*(-4*a*c+b^2)^(1/2)+
4*a*c-b^2)/(32*a*c-8*b^2)*(1/4*(A*(-4*a*c+b^2)^(1/2)-A*b+2*C*a)/c*ln(2*c*x
^2+(-4*a*c+b^2)^(1/2)+b)+B*a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))-(b^2-4*a*c+b*(-4*a*c+b^2
)^(1/2))/(32*a*c-8*b^2)*(-1/4*(-A*(-4*a*c+b^2)^(1/2)-A*b+2*C*a)/c*ln(-2*c*
x^2+(-4*a*c+b^2)^(1/2)-b)+B*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

3.28.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.28.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/x**3/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.28.7 Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx = \int \frac{Cx^2+Bx+A}{(cx^4+bx^2+a)x^3}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
(C*a - A*b)*log(x)/a^2 + integrate(-(B*a*c*x^2 + (C*a - A*b)*c*x^3 + B*a*b
 + (C*a*b - A*b^2 + A*a*c)*x)/(c*x^4 + b*x^2 + a), x)/a^2 - 1/2*(2*B*x + A
)/(a*x^2)
```

3.28.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3353 vs. $2(240) = 480$.

Time $= 1.49$ (sec) , antiderivative size $= 3353$, normalized size of antiderivative $= 11.64$

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/4*(C*a - A*b)*log(abs(c*x^4 + b*x^2 + a))/a^2 + (C*a - A*b)*log(abs(x))
/a^2 + 1/8*((2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*B*c^2 - 2*(
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*
c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*
a*c)*a*b*c^3)*B*abs(c) + (2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*B)*arctan(2*...
```

3.28.9 Mupad [B] (verification not implemented)

Time = 8.14 (sec) , antiderivative size = 3563, normalized size of antiderivative = 12.37

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)),x)
```

output
```
symsum(log(root(128*a^5*b^2*c*z^4 - 256*a^6*c^2*z^4 - 16*a^4*b^4*z^4 + 128
*C*a^4*b^2*c*z^3 + 256*A*a^4*b*c^2*z^3 - 128*A*a^3*b^3*c*z^3 - 256*C*a^5*c
^2*z^3 - 16*C*a^3*b^4*z^3 + 16*A*a^2*b^5*z^3 + 160*A*C*a^3*b*c^2*z^2 - 72*
A*C*a^2*b^3*c*z^2 + 8*A*C*a*b^5*z^2 + 40*C^2*a^3*b^2*c*z^2 - 48*B^2*a^3*b*
c^2*z^2 + 28*B^2*a^2*b^3*c*z^2 + 32*A^2*a*b^4*c*z^2 - 56*A^2*a^2*b^2*c^2*z
^2 - 4*B^2*a*b^5*z^2 - 96*C^2*a^4*c^2*z^2 - 4*C^2*a^2*b^4*z^2 - 32*A^2*a^3
*c^3*z^2 - 4*A^2*b^6*z^2 - 16*B^2*C*a^2*b*c^2*z + 32*A*C^2*a^2*b*c^2*z - 1
2*A^2*C*a*b^2*c^2*z - 4*A*B^2*a*b^2*c^2*z + 4*B^2*C*a*b^3*c*z - 8*A*C^2*a*
b^3*c*z + 16*A^3*a*b*c^3*z + 4*A^2*C*b^4*c*z + 4*C^3*a^2*b^2*c*z - 16*A^2*
C*a^2*c^3*z + 16*A*B^2*a^2*c^3*z - 16*C^3*a^3*c^2*z - 4*A^3*b^3*c^2*z + 2*
A*C^3*a*b*c^2 + 4*A*B^2*C*a*c^3 - 2*A^2*C^2*a*c^3 + 2*A^3*C*b*c^3 - B^2*C^
2*a*b*c^2 - A^2*B^2*b*c^3 - A^2*C^2*b^2*c^2 - C^4*a^2*c^2 - B^4*a*c^3 - A^
4*c^4, z, k)*(root(128*a^5*b^2*c*z^4 - 256*a^6*c^2*z^4 - 16*a^4*b^4*z^4 +
128*C*a^4*b^2*c*z^3 + 256*A*a^4*b*c^2*z^3 - 128*A*a^3*b^3*c*z^3 - 256*C*a^
5*c^2*z^3 - 16*C*a^3*b^4*z^3 + 16*A*a^2*b^5*z^3 + 160*A*C*a^3*b*c^2*z^2 -
72*A*C*a^2*b^3*c*z^2 + 8*A*C*a*b^5*z^2 + 40*C^2*a^3*b^2*c*z^2 - 48*B^2*a^3
*b*c^2*z^2 + 28*B^2*a^2*b^3*c*z^2 + 32*A^2*a*b^4*c*z^2 - 56*A^2*a^2*b^2*c^
2*z^2 - 4*B^2*a*b^5*z^2 - 96*C^2*a^4*c^2*z^2 - 4*C^2*a^2*b^4*z^2 - 32*A^2*
a^3*c^3*z^2 - 4*A^2*b^6*z^2 - 16*B^2*C*a^2*b*c^2*z + 32*A*C^2*a^2*b*c^2*z
- 12*A^2*C*a*b^2*c^2*z - 4*A*B^2*a*b^2*c^2*z + 4*B^2*C*a*b^3*c*z - 8*A*...
```

3.28. $\int \frac{A+Bx+Cx^2}{x^3(a+bx^2+cx^4)}\,dx$

3.29 $\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

3.29.1 Optimal result

Integrand size = 28, antiderivative size = 412

$$\begin{aligned}\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx &= \frac{(2Ac-bC)x}{2c(b^2-4ac)}+\frac{Bx^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{x^3(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)}\\&+\frac{\left(Abc+(b^2-6ac)C-\frac{Ac(b^2+4ac)+b(b^2-8ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}\\&+\frac{\left(Abc+(b^2-6ac)C+\frac{Ac(b^2+4ac)+b(b^2-8ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}\\&+\frac{2aB\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}\end{aligned}$$

output
```
1/2*(2*A*c-C*b)*x/c/(-4*a*c+b^2)+1/2*B*x^2*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4
+b*x^2+a)-1/2*x^3*(A*b-2*C*a+(2*A*c-C*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)
+2*a*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)+1/4*arct
an(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(A*b*c+(-6*a*c+b^2)*C+(
-A*c*(4*a*c+b^2)-b*(-8*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2
)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4
*a*c+b^2)^(1/2))^(1/2))*(A*b*c+(-6*a*c+b^2)*C+(A*c*(4*a*c+b^2)+b*(-8*a*c+b
^2)*C)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1
/2))^(1/2)
```

3.29.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.08

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$$
$$=\frac{1}{4}\left(\frac{2(bx^2(-Acx+b(B+Cx))+a(b(B+Cx)-2cx(A+x(B+Cx))))}{c(-b^2+4ac)(a+bx^2+cx^4)}\right.$$
$$+\frac{\sqrt{2}\left(-Ac\left(b^2+4ac-b\sqrt{b^2-4ac}\right)+\left(-b^3+8abc+b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{c^{3/2}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{2}\left(Ac\left(b^2+4ac+b\sqrt{b^2-4ac}\right)+\left(b^3-8abc+b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{c^{3/2}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$
$$\left.-\frac{4aB\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}}+\frac{4aB\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

input
```
Integrate[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((2*(b*x^2*(-(A*c*x) + b*(B + C*x)) + a*(b*(B + C*x) - 2*c*x*(A + x*(B + C
*x)))))/(c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-(A*c*(b^2 + 4*
a*c - b*Sqrt[b^2 - 4*a*c])) + (-b^3 + 8*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 6*
a*c*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(c^(3/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt
[2]*(A*c*(b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c]) + (b^3 - 8*a*b*c + b^2*Sqrt[b
^2 - 4*a*c] - 6*a*c*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b + Sqrt[b^2 - 4*a*c]]])/(c^(3/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 -
4*a*c]]) - (4*a*B*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/
2) + (4*a*B*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

3.29.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 397, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {2193, 27, 1434, 1153, 1083, 219, 1598, 1602, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$\Big\downarrow \; 2193$$

$$\int \frac{x^4 \left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\int \frac{Bx^5}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \; 27$$

$$\int \frac{x^4 \left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+B\int \frac{x^5}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \; 1434$$

$$\int \frac{x^4 \left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\int \frac{x^4}{\left(cx^4+bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \; 1153$$

$$\int \frac{x^4 \left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{x^2 \left(2a+bx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2a\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}\right)$$

$$\Big\downarrow \; 1083$$

$$\int \frac{x^4 \left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{4a\int \frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)}{b^2-4ac}+\frac{x^2 \left(2a+bx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \; 219$$

$$\int \frac{x^4 \left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{4a\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{x^2 \left(2a+bx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \; 1598$$

$$\frac{\int \frac{x^2 \left((2Ac-bC)x^2+3(Ab-2aC)\right)}{cx^4+bx^2+a}dx}{2\left(b^2-4ac\right)}-\frac{x^3 \left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{4a\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{x^2 \left(2a+bx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \; 1602$$

$$\frac{\frac{x(2Ac-bC)}{c}-\frac{\int \frac{a(2Ac-bC)-\left(Abc+\left(b^2-6ac\right)C\right)x^2}{cx^4+bx^2+a}dx}{c}}{2\left(b^2-4ac\right)}-\frac{x^3 \left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{4a\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{x^2 \left(2a+bx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

3.29. $\int \frac{x^4 \left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

$$\Big\downarrow \ 1480$$

$$\frac{\frac{x(2Ac-bC)}{c}-\frac{-\frac{1}{2}\left(-\frac{Ac\left(4ac+b^2\right)+bC\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}+C\left(b^2-6ac\right)+Abc\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}\left(\frac{Ac\left(4ac+b^2\right)+bC\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}+C\left(b^2-6ac\right)+Abc\right)\cdots}{c}}{2\left(b^2-4ac\right)}$$

$$\frac{x^3\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{4a\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{x^2\left(2a+bx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{x(2Ac-bC)}{c}-\frac{-\frac{\left(-\frac{Ac\left(4ac+b^2\right)+bC\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}+C\left(b^2-6ac\right)+Abc\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{Ac\left(4ac+b^2\right)+bC\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}+C\left(b^2-6ac\right)+Abc\right)\arctan\left(\cdots\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}}{2\left(b^2-4ac\right)}$$

$$\frac{x^3\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{4a\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{x^2\left(2a+bx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

input
```
Int[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*(x^3*(A*b - 2*a*C + (2*A*c - b*C)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c
*x^4)) + (((2*A*c - b*C)*x)/c - (-(((A*b*c + (b^2 - 6*a*c)*C - (A*c*(b^2 +
 4*a*c) + b*(b^2 - 8*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]
])) - ((A*b*c + (b^2 - 6*a*c)*C + (A*c*(b^2 + 4*a*c) + b*(b^2 - 8*a*c)*C)/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/c)/(2*(b^2 - 4*a*c)) + (B
*((x^2*(2*a + b*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (4*a*ArcTanh[(
b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/2
```

3.29.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1153
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c)))   Int[(d + e*x)^(m - 2)*(a + b*x +
c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
&& LtQ[p, -1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.29. $\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

3.29.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.60

method	result
risch	$\frac{-\frac{\left(Abc+2acC-b^2C\right)x^3}{2c\left(4ac-b^2\right)}-\frac{\left(2ac-b^2\right)Bx^2}{2c\left(4ac-b^2\right)}-\frac{a(2Ac-Cb)x}{2\left(4ac-b^2\right)c}+\frac{aBb}{2c\left(4ac-b^2\right)}}{cx^4+bx^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(-\frac{\left(Abc-6acC+b^2C\right)\_R^2}{c\left(4ac-b^2\right)}+\cdots\right.}{2c\_\cdots}\right.}{4}$
default	$\frac{-\frac{\left(Abc+2acC-b^2C\right)x^3}{2c\left(4ac-b^2\right)}-\frac{\left(2ac-b^2\right)Bx^2}{2c\left(4ac-b^2\right)}-\frac{a(2Ac-Cb)x}{2\left(4ac-b^2\right)c}+\frac{aBb}{2c\left(4ac-b^2\right)}}{cx^4+bx^2+a}+\frac{2B\sqrt{-4ac+b^2}\,ac\ln\left(2cx^2+\sqrt{-4ac+b^2}+b\right)+\frac{\left(4Aac^2\sqrt{-4ac+b^2}+Ab\cdots\right.}{\cdots}}{\cdots}$

input
```
int(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*(A*b*c+2*C*a*c-C*b^2)/c/(4*a*c-b^2)*x^3-1/2*(2*a*c-b^2)*B/c/(4*a*c-b
^2)*x^2-1/2*a*(2*A*c-C*b)/(4*a*c-b^2)/c*x+1/2*a/c/(4*a*c-b^2)*B*b)/(c*x^4+
b*x^2+a)+1/4*sum((-(A*b*c-6*C*a*c+C*b^2)/c/(4*a*c-b^2)*_R^2+4/(4*a*c-b^2)*
_R*B*a+a*(2*A*c-C*b)/(4*a*c-b^2)/c)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^
4*c+_Z^2*b+a))
```

3.29.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.29.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.29.7 Maxima [F]

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(Cx^2+Bx+A)x^4}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((C*b^2 - (2*C*a + A*b)*c)*x^3 + B*a*b + (B*b^2 - 2*B*a*c)*x^2 + (C*a
*b - 2*A*a*c)*x)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c -
 4*a*b*c^2)*x^2) + 1/2*integrate(-(4*B*a*c*x - C*a*b + 2*A*a*c - (C*b^2 -
(6*C*a - A*b)*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)
```

3.29.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5217 vs. $2(361) = 722$.

Time = 1.86 (sec) , antiderivative size = 5217, normalized size of antiderivative = 12.66

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(C*b^2*x^3 - 2*C*a*c*x^3 - A*b*c*x^3 + B*b^2*x^2 - 2*B*a*c*x^2 + C*a*
b*x - 2*A*a*c*x + B*a*b)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2)) + 1/16*((
2*b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^3 -
2*(b^2 - 4*a*c)*b*c^3)*(b^2*c - 4*a*c^2)^2*A + (2*b^4*c^2 - 20*a*b^2*c^3 +
 48*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c +
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 24*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 12*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 6*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2
 + 12*(b^2 - 4*a*c)*a*c^3)*(b^2*c - 4*a*c^2)^2*C - 4*(sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 2*a*b^
4*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^2*c^5 + 16*a^2*b^2*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a...
```

3.29.9 Mupad [B] (verification not implemented)

Time = 8.51 (sec) , antiderivative size = 4754, normalized size of antiderivative = 11.54

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
symsum(log(- root(1572864*a^5*b^2*c^8*z^4 - 983040*a^4*b^4*c^7*z^4 + 32768
0*a^3*b^6*c^6*z^4 - 61440*a^2*b^8*c^5*z^4 + 6144*a*b^10*c^4*z^4 - 256*b^12
*c^3*z^4 - 1048576*a^6*c^9*z^4 + 576*A*C*a*b^8*c^2*z^2 + 24576*A*C*a^4*b^2
*c^5*z^2 - 3072*A*C*a^2*b^6*c^3*z^2 + 2048*A*C*a^3*b^4*c^4*z^2 - 32*A*C*b^
10*c*z^2 + 61440*C^2*a^5*b*c^5*z^2 + 12288*A^2*a^4*b*c^6*z^2 + 432*C^2*a*b
^9*c*z^2 - 49152*A*C*a^5*c^6*z^2 - 61440*C^2*a^4*b^3*c^4*z^2 + 24064*C^2*a
^3*b^5*c^3*z^2 - 4608*C^2*a^2*b^7*c^2*z^2 + 24576*B^2*a^4*b^2*c^5*z^2 - 61
44*B^2*a^3*b^4*c^4*z^2 + 512*B^2*a^2*b^6*c^3*z^2 - 8192*A^2*a^3*b^3*c^5*z^
2 + 1536*A^2*a^2*b^5*c^4*z^2 - 32768*B^2*a^5*c^6*z^2 - 16*A^2*b^9*c^2*z^2
- 16*C^2*b^11*z^2 - 3072*A*B*C*a^3*b^3*c^3*z + 768*A*B*C*a^2*b^5*c^2*z + 4
096*A*B*C*a^4*b*c^4*z - 64*A*B*C*a*b^7*c*z + 672*B*C^2*a^2*b^6*c*z - 32*A^
2*B*a*b^6*c^2*z + 15872*B*C^2*a^4*b^2*c^3*z - 4992*B*C^2*a^3*b^4*c^2*z - 1
536*A^2*B*a^3*b^2*c^4*z + 384*A^2*B*a^2*b^4*c^3*z - 32*B*C^2*a*b^8*z - 184
32*B*C^2*a^5*c^4*z + 2048*A^2*B*a^4*c^5*z + 192*A*B^2*C*a^3*b^2*c^2 - 32*A
*B^2*C*a^2*b^4*c - 960*A^2*C^2*a^3*b^2*c^2 - 16*A^2*B^2*a^2*b^3*c^2 - 18*A
^3*C*a*b^5*c - 960*B^2*C^2*a^4*b*c^2 + 240*B^2*C^2*a^3*b^3*c + 198*A^2*C^2
*a^2*b^4*c + 144*A^3*C*a^2*b^3*c^2 - 192*A^2*B^2*a^3*b*c^3 + 2016*A*C^3*a^
4*b*c^2 - 496*A*C^3*a^3*b^3*c + 224*A^3*C*a^3*b*c^3 + 768*A*B^2*C*a^4*c^3
+ 360*C^4*a^4*b^2*c - 9*A^4*a*b^4*c^2 + 30*A*C^3*a^2*b^5 - 9*A^2*C^2*a*b^6
 - 24*A^4*a^2*b^2*c^3 - 288*A^2*C^2*a^4*c^3 - 16*B^2*C^2*a^2*b^5 - 1296...
```

3.29. $\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

3.30 $\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

3.30.1 Optimal result

Integrand size = 28, antiderivative size = 347

$$\begin{aligned}\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx &= \frac{Bx(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}\\ &+ \frac{a(2Ac-bC)+(Abc-b^2C+2acC)x^2}{2c(b^2-4ac)(a+bx^2+cx^4)}\\ &+ \frac{B\left(b-\frac{b^2+4ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}\\ &+ \frac{B\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}\\ &- \frac{(Ab-2aC)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}\end{aligned}$$

output
```
1/2*B*x*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(a*(2*A*c-C*b)+(A*b*c
+2*C*a*c-C*b^2)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-(A*b-2*C*a)*arctanh((2
*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)+1/4*B*arctan(x*2^(1/2)*c^
(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c-b^2)/(-4*a*c+b^2)^(1/2))/(-
4*a*c+b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*B*arctan(x*2^(
1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2+4*a*c+b*(-4*a*c+b^2)^(1/2)
)/(-4*a*c+b^2)^(3/2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.30.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.03

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{1}{4}\left(-\frac{2(bx^2(Ac-bC+Bcx)+a(2Ac-bC+2cx(B+Cx)))}{c(-b^2+4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}B\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}B\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{2(Ab-2aC)\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}} - \frac{2(Ab-2aC)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

input
```
Integrate[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((-2*(b*x^2*(A*c - b*C + B*c*x) + a*(2*A*c - b*C + 2*c*x*(B + C*x))))/(c*(
-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*B*(-b^2 - 4*a*c + b*Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[
c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*B*(b^2 + 4*
a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*(
A*b - 2*a*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) -
(2*(A*b - 2*a*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2)
)/4
```

3.30.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {2193, 27, 1440, 1480, 218, 1578, 1224, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \text{ 2193}$$

$$\int \frac{x^3\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\int \frac{Bx^4}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x^3\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+B\int \frac{x^4}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{ 1440}$$

$$\int \frac{x^3\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+B\left(\frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int \frac{2a-bx^2}{cx^4+bx^2+a}dx}{2\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{ 1480}$$

$$\int \frac{x^3\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+$$
$$B\left(\frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-\frac{1}{2}\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}\left(\frac{4ac+b^2}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}}{2\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \text{ 218}$$

$$\int \frac{x^3\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+$$
$$B\left(\frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4ac+b^2}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{ 1578}$$

$$\frac{1}{2}\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx^2+$$
$$B\left(\frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4ac+b^2}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{ 1224}$$

3.30. $\int \frac{x^3\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}dx$

$$\frac{1}{2}\left(\frac{(Ab-2aC)\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}+\frac{x^2\left(Abc-C\left(b^2-2ac\right)\right)+a(2Ac-bC)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+B\left(\frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4ac+b^2}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{x^2\left(Abc-C\left(b^2-2ac\right)\right)+a(2Ac-bC)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2(Ab-2aC)\int \frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)}{b^2-4ac}\right)+B\left(\frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4ac+b^2}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{x^2\left(Abc-C\left(b^2-2ac\right)\right)+a(2Ac-bC)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2(Ab-2aC)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)+B\left(\frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4ac+b^2}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2\left(b^2-4ac\right)}\right)$$

input
```
Int[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
B*((x*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (-(((b - (b^2
 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])) - ((b + (b^2 +
4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4
*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*(b^2 - 4*a*c)))
 + ((a*(2*A*c - b*C) + (A*b*c - (b^2 - 2*a*c)*C)*x^2)/(c*(b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) - (2*(A*b - 2*a*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*
c]])/(b^2 - 4*a*c)^(3/2))/2
```

3.30. $\int \frac{x^3\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

3.30.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1224
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c))   Int[(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -
1] &&  !(IntegerQ[p] && NeQ[a, 0] && NiceSqrtQ[b^2 - 4*a*c])
```

rule 1440
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*
(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^4/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

3.30.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.61

method	result
risch	$\frac{-\frac{bB\,x^{3}}{2\left(4ac-b^{2}\right)}-\frac{\left(Abc+2acC-b^{2}C\right)x^{2}}{2c\left(4ac-b^{2}\right)}-\frac{xBa}{4ac-b^{2}}-\frac{a\left(2Ac-Cb\right)}{2\left(4ac-b^{2}\right)c}}{c\,x^{4}+b\,x^{2}+a}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{4}+\_Z^{2}b+a\right)}\frac{\left(-\frac{\_R^{2}Bb}{4ac-b^{2}}-\frac{2\left(Ab-2Ca\right)\_R}{4ac-b^{2}}+\frac{2Ba}{4ac-b^{2}} \right.}{2c\_R^{3}+\_Rb}\right.}{4}$
default	$\frac{-\frac{bB\,x^{3}}{2\left(4ac-b^{2}\right)}-\frac{\left(Abc+2acC-b^{2}C\right)x^{2}}{2c\left(4ac-b^{2}\right)}-\frac{xBa}{4ac-b^{2}}-\frac{a\left(2Ac-Cb\right)}{2\left(4ac-b^{2}\right)c}}{c\,x^{4}+b\,x^{2}+a}+\frac{2c\left(\frac{\frac{\left(-4Abc\sqrt{-4ac+b^{2}}+8C\sqrt{-4ac+b^{2}}\,ac\right)\ln\left(2c\,x^{2}+\sqrt{-4ac+b^{2}}+b\right)}{4c}+\frac{\left(4Ba \right.}{}}{4c} \right.}{}$

input
```
int(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2/(4*a*c-b^2)*b*B*x^3-1/2*(A*b*c+2*C*a*c-C*b^2)/c/(4*a*c-b^2)*x^2-1/(4
*a*c-b^2)*x*B*a-1/2*a*(2*A*c-C*b)/(4*a*c-b^2)/c)/(c*x^4+b*x^2+a)+1/4*sum((
-1/(4*a*c-b^2)*_R^2*B*b-2*(A*b-2*C*a)/(4*a*c-b^2)*_R+2/(4*a*c-b^2)*B*a)/(2
*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.30.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.30.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.30.7 Maxima [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(Cx^2+Bx+A)x^3}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(B*b*c*x^3 + 2*B*a*c*x - C*a*b + 2*A*a*c - (C*b^2 - (2*C*a + A*b)*c)*x
^2)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x
^2) + 1/2*integrate((B*b*x^2 - 2*B*a - 2*(2*C*a - A*b)*x)/(c*x^4 + b*x^2 +
 a), x)/(b^2 - 4*a*c)
```

3.30.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3227 vs. $2(296) = 592$.

Time = 1.22 (sec) , antiderivative size = 3227, normalized size of antiderivative = 9.30

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(B*b*c*x^3 - C*b^2*x^2 + 2*C*a*c*x^2 + A*b*c*x^2 + 2*B*a*c*x - C*a*b +
 2*A*a*c)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2)) + 1/16*((2*b^3*c^2 - 8*a
*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2
)*(b^2 - 4*a*c)^2*B - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c -
 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 16*a^2*b^2*c
^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b
^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*B*abs(b^2 - 4*a*c) - (2*b
^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s...
```

3.30.9 Mupad [B] (verification not implemented)

Time = 8.49 (sec) , antiderivative size = 3278, normalized size of antiderivative = 9.45

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
symsum(log(root(1572864*a^5*b^2*c^6*z^4 - 983040*a^4*b^4*c^5*z^4 + 327680*
a^3*b^6*c^4*z^4 - 61440*a^2*b^8*c^3*z^4 + 6144*a*b^10*c^2*z^4 - 1048576*a^
6*c^7*z^4 - 256*b^12*c*z^4 + 32768*A*C*a^4*b*c^4*z^2 - 512*A*C*a*b^7*c*z^2
 - 24576*A*C*a^3*b^3*c^3*z^2 + 6144*A*C*a^2*b^5*c^2*z^2 + 512*C^2*a^2*b^6*
c*z^2 + 12288*B^2*a^4*b*c^4*z^2 - 1536*A^2*a*b^6*c^2*z^2 + 24576*C^2*a^4*b
^2*c^3*z^2 - 6144*C^2*a^3*b^4*c^2*z^2 - 8192*B^2*a^3*b^3*c^3*z^2 + 1536*B^
2*a^2*b^5*c^2*z^2 - 8192*A^2*a^3*b^2*c^4*z^2 + 6144*A^2*a^2*b^4*c^3*z^2 +
128*A^2*b^8*c*z^2 - 32768*C^2*a^5*c^4*z^2 - 16*B^2*b^9*z^2 + 384*B^2*C*a^2
*b^4*c*z - 1024*A*B^2*a^3*b*c^3*z - 192*A*B^2*a*b^5*c*z - 1536*B^2*C*a^3*b
^2*c^2*z + 768*A*B^2*a^2*b^3*c^2*z - 32*B^2*C*a*b^6*z + 2048*B^2*C*a^4*c^3
*z + 16*A*B^2*b^7*z + 192*A*B^2*C*a^2*b^2*c + 512*A*C^3*a^3*b*c + 128*A^3*
C*a*b^3*c + 16*A*B^2*C*a*b^4 - 384*A^2*C^2*a^2*b^2*c - 192*B^2*C^2*a^3*b*c
 - 48*A^2*B^2*a*b^3*c - 24*B^4*a^2*b^2*c - 16*B^2*C^2*a^2*b^3 - 16*B^4*a^3
*c^2 - 4*A^2*B^2*b^5 - 256*C^4*a^4*c - 16*A^4*b^4*c - 9*B^4*a*b^4, z, k)*(
(256*A*B*a^2*b^2*c^3 + 128*B*C*a^2*b^3*c^2 - 64*A*B*a*b^4*c^2 - 512*B*C*a^
3*b*c^3)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(15728
64*a^5*b^2*c^6*z^4 - 983040*a^4*b^4*c^5*z^4 + 327680*a^3*b^6*c^4*z^4 - 614
40*a^2*b^8*c^3*z^4 + 6144*a*b^10*c^2*z^4 - 1048576*a^6*c^7*z^4 - 256*b^12*
c*z^4 + 32768*A*C*a^4*b*c^4*z^2 - 512*A*C*a*b^7*c*z^2 - 24576*A*C*a^3*b^3*
c^3*z^2 + 6144*A*C*a^2*b^5*c^2*z^2 + 512*C^2*a^2*b^6*c*z^2 + 12288*B^2*...
```

3.30. $\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

3.31 $\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

3.31.1 Optimal result

Integrand size = 28, antiderivative size = 356

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} - \frac{bB\,\mathrm{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
1/2*B*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(A*b-2*C*a+(2*A*c-C*b
)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(3/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2))*(2*A*c-C*b+(-4*A*b*c+(4*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(
1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*A*c-C*b+(4*A*b*c-(4*a*c+b^2)*C)/(-4*
a*c+b^2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.31.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.06

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{1}{4}\left(\frac{4a(B+Cx)+2x(bx(B+Cx)-A(b+2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(-2Ac\left(-2b+\sqrt{b^2-4ac}\right)+\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(-2Ac\left(2b+\sqrt{b^2-4ac}\right)+\left(b^2+4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{2bB\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}} - \frac{2bB\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

input
```
Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 -
 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ (Sqrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 -
 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqr
t[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

3.31.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {2193, 27, 1434, 1159, 1083, 219, 1598, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$\Big\downarrow \text{ 2193}$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\int \frac{Bx^3}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+B\int \frac{x^3}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{ 1434}$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\int \frac{x^2}{\left(cx^4+bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \text{ 1159}$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{b\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}+\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{ 1083}$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\int \frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)}{b^2-4ac}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

$$\Big\downarrow \text{ 1598}$$

$$\frac{\int \frac{-\left(\left(2Ac-bC\right)x^2\right)+Ab-2aC}{cx^4+bx^2+a}dx}{2\left(b^2-4ac\right)}-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+$$
$$\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

$$\Big\downarrow \text{ 1480}$$

3.31. $\int \frac{x^2\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

$$\frac{-\frac{1}{2}\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d}{2(b^2-4ac)}$$

$$\frac{x(-2aC+x^2(2Ac-bC)+Ab)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{1}{2}B\left(\frac{2a+bx^2}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{-\frac{\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2(b^2-4ac)}-$$

$$\frac{x(-2aC+x^2(2Ac-bC)+Ab)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{1}{2}B\left(\frac{2a+bx^2}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}\right)$$

input
```
Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*(x*(A*b - 2*a*C + (2*A*c - b*C)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) + (-(((2*A*c - b*C - (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*S
qrt[b - Sqrt[b^2 - 4*a*c]])) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*(b^2 - 4*a*c)) + (B*(
(2*a + b*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (2*b*ArcTanh[(b + 2*c*
x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/2
```

3.31.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

3.31. $\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

3.31.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.57

method	result
risch	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{x^2Bb}{2(4ac-b^2)}+\frac{(Ab-2Ca)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)}\frac{\left(\frac{(2Ac-Cb)\_R^2}{4ac-b^2}-\frac{2\_RBb}{4ac-b^2}-\frac{Ab-2Ca}{4ac-b^2}\right)\ln(x-\_}{2c\_R^3+\_Rb}\right.}{4}$
default	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{x^2Bb}{2(4ac-b^2)}+\frac{(Ab-2Ca)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{-B\sqrt{-4ac+b^2}\,b\ln\left(2c\,x^2+\sqrt{-4ac+b^2}+b\right)+\frac{\left(-4Abc\sqrt{-4ac+b^2}+8Aa\,c^2-2A\,b^2c+4C\right.}{}}{4c\left(4ac\right.}\right.}{}$

input
```
int(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2/(4*a*c-b^2)*x^2*B*b+1/2*(A*b-2*C*a)/(
4*a*c-b^2)*x-1/(4*a*c-b^2)*B*a)/(c*x^4+b*x^2+a)+1/4*sum(((2*A*c-C*b)/(4*a*
c-b^2)*_R^2-2/(4*a*c-b^2)*_R*B*b-(A*b-2*C*a)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*
ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.31.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.31.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.31.7 Maxima [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(Cx^2+Bx+A)x^2}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*
c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*
x + (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

3.31.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4438 vs. $2(306) = 612$.

Time = 1.59 (sec) , antiderivative size = 4438, normalized size of antiderivative = 12.47

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*
x^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3
*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c
 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*
a*c)*b*c^2)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c
^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*
(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3...
```

3.31.9 Mupad [B] (verification not implemented)

Time = 8.49 (sec) , antiderivative size = 3835, normalized size of antiderivative = 10.77

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*
A*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A
*C^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^
3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^
2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^
8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*
z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*
b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*
c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a
^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 819
2*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A
^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B
*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B
*a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*
c^2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c
+ 48*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*
C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c +
 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6
*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4
*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(25...
```

3.31. $\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

3.32 $\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

3.32.1 Optimal result

Integrand size = 26, antiderivative size = 317

$$\begin{aligned}\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx =& -\frac{Bx(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{Ab-2aC+(2Ac-bC)x^2}{2(b^2-4ac)(a+bx^2+cx^4)} \\ &+ \frac{B\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} \\ &- \frac{B\sqrt{c}\left(2b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} \\ &+ \frac{(2Ac-bC)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}\end{aligned}$$

output
```
-1/2*B*x*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(-A*b+2*C*a-(2*A*c-C
*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+(2*A*c-C*b)*arctanh((2*c*x^2+b)/(-4*
a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)+1/2*B*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a
*c+b^2)^(1/2))^(1/2))*c^(1/2)*(2*b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)*
2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*B*arctan(x*2^(1/2)*c^(1/2)/(b+(-4
*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(2*b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2
)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.32.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.06

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{1}{2}\left(\frac{2aC-A(b+2cx^2)+x(-bB+bCx-2Bcx^2)}{(b^2-4ac)(a+bx^2+cx^4)} - \frac{\sqrt{2}B\sqrt{c}(-2b+\sqrt{b^2-4ac})\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}B\sqrt{c}(2b+\sqrt{b^2-4ac})\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{(-2Ac+bC)\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}} + \frac{(2Ac-bC)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

input
```
Integrate[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((2*a*C - A*(b + 2*c*x^2) + x*(-(b*B) + b*C*x - 2*B*c*x^2))/((b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)) - (Sqrt[2]*B*Sqrt[c]*(-2*b + Sqrt[b^2 - 4*a*c])*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*
Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*B*Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^
(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((-2*A*c + b*C)*Log[-b + Sqrt[b^2 - 4
*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((2*A*c - b*C)*Log[b + Sqrt[b^2 -
4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/2
```

3.32.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {2193, 27, 1439, 1480, 218, 1576, 1159, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \; 2193$$

$$\int \frac{x\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\int \frac{Bx^2}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \; 27$$

$$\int \frac{x\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+B\int \frac{x^2}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \; 1439$$

$$\int \frac{x\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+B\left(\frac{\int \frac{b-2cx^2}{cx^4+bx^2+a}dx}{2\left(b^2-4ac\right)}-\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \; 1480$$

$$\int \frac{x\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+$$
$$B\left(\frac{-c\left(1-\frac{2b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-c\left(\frac{2b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{2\left(b^2-4ac\right)}-\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+c\right.}\right.$$

$$\Big\downarrow \; 218$$

$$\int \frac{x\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+$$
$$B\left(\frac{-\frac{\sqrt{2}\sqrt{c}\left(1-\frac{2b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\left(\frac{2b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}}{2\left(b^2-4ac\right)}-\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \; 1576$$

$$\frac{1}{2}\int \frac{Cx^2+A}{\left(cx^4+bx^2+a\right)^2}dx^2+$$
$$B\left(\frac{-\frac{\sqrt{2}\sqrt{c}\left(1-\frac{2b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\left(\frac{2b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}}{2\left(b^2-4ac\right)}-\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \; 1159$$

3.32. $\int \frac{x\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}dx$

$$\frac{1}{2}\left(-\frac{(2Ac-bC)\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}-\frac{-2aC+x^2(2Ac-bC)+Ab}{(b^2-4ac)(a+bx^2+cx^4)}\right)+B\left(\frac{-\frac{\sqrt{2}\sqrt{c}\left(1-\frac{2b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\left(\frac{2b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}}{2(b^2-4ac)}-\frac{x(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$\Big\downarrow$ 1083

$$\frac{1}{2}\left(\frac{2(2Ac-bC)\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{b^2-4ac}-\frac{-2aC+x^2(2Ac-bC)+Ab}{(b^2-4ac)(a+bx^2+cx^4)}\right)+B\left(\frac{-\frac{\sqrt{2}\sqrt{c}\left(1-\frac{2b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\left(\frac{2b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}}{2(b^2-4ac)}-\frac{x(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$\Big\downarrow$ 219

$$\frac{1}{2}\left(\frac{2(2Ac-bC)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{-2aC+x^2(2Ac-bC)+Ab}{(b^2-4ac)(a+bx^2+cx^4)}\right)+B\left(\frac{-\frac{\sqrt{2}\sqrt{c}\left(1-\frac{2b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\left(\frac{2b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}}{2(b^2-4ac)}-\frac{x(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}\right)$$

input
```
Int[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
B*(-1/2*(x*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (-((Sqrt[2
]*Sqrt[c]*(1 - (2*b)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*(1
+ (2*b)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*(b^2 - 4*a*c))) + (-((A*b - 2*a*
C + (2*A*c - b*C)*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4))) + (2*(2*A*c -
b*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2))/2
```

3.32. $\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

3.32.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1439
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d*(d*x)^(m - 1)*(b + 2*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*(p +
1)*(b^2 - 4*a*c))), x] - Simp[d^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

3.32.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.27 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.62

method	result
risch	$\frac{\frac{Bc\,x^3}{4ac-b^2}+\frac{(2Ac-Cb)x^2}{8ac-2b^2}+\frac{Bbx}{8ac-2b^2}+\frac{Ab-2Ca}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\frac{2c\_R^2B}{4ac-b^2}+\frac{2(2Ac-Cb)\_R}{4ac-b^2}-\frac{bB}{4ac-b^2}\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}\right.}{4}$
default	$16c^2\left(-\frac{\frac{-\frac{B\left(4ac-b^2\right)x}{8c}-\frac{8Aa\,c^2-2A\,b^2c+4C\sqrt{-4ac+b^2}\,ac-C\sqrt{-4ac+b^2}\,b^2-4Cabc+C\,b^3}{16c^2}}{x^2+\frac{b}{2c}-\frac{\sqrt{-4ac+b^2}}{2c}}-\frac{\left(-4Ac\sqrt{-4ac+b^2}+2C\sqrt{-4ac+b^2}\,b\right)\ln\left(-2c\,x^2\right.}{16c}}{4c\left(4ac-b^2\right)^2}\right.$

input
```
int(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(B*c/(4*a*c-b^2)*x^3+1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^2+1/2/(4*a*c-b^2)*x*B*b
+1/2*(A*b-2*C*a)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/4*sum((2*c/(4*a*c-b^2)*_R^
2*B+2*(2*A*c-C*b)/(4*a*c-b^2)*_R-1/(4*a*c-b^2)*b*B)/(2*_R^3*c+_R*b)*ln(x-_
R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.32.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.32.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.32.7 Maxima [F]

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(Cx^2+Bx+A)x}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*B*c*x^3 + B*b*x - (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/((b^2*c - 4*a*c
^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate((2*B*c*x^
2 - B*b - 2*(C*b - 2*A*c)*x)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

3.32.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3014 vs. $2(270) = 540$.

Time = 1.24 (sec) , antiderivative size = 3014, normalized size of antiderivative = 9.51

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(2*B*c*x^3 - C*b*x^2 + 2*A*c*x^2 + B*b*x - 2*C*a + A*b)/((c*x^4 + b*x
^2 + a)*(b^2 - 4*a*c)) - 1/8*((2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*c^2 - 2*(b^2 - 4*a*c)*c^2)*(b^2 - 4*a*c)^2*B - (sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 2*b^5*c + 16*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*
c^2 + 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 3
2*a^2*b*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2 - 4*a*c)*a*b*c^2)*B*abs(b^2 -
 4*a*c) - 2*(2*b^6*c^2 - 16*a*b^4*c^3 + 32*a^2*b^2*c^4 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^4*c^2 + 8*(b^2 - 4*a*c)*a...
```

3.32.9 Mupad [B] (verification not implemented)

Time = 8.31 (sec) , antiderivative size = 3198, normalized size of antiderivative = 10.09

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
symsum(log((4*B^3*a*c^4 + 3*B^3*b^2*c^3 + 8*A^2*B*b*c^4 + 2*B*C^2*b^3*c^2
- 8*A*B*C*b^2*c^3)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) -
root(1572864*a^6*b^2*c^5*z^4 - 983040*a^5*b^4*c^4*z^4 + 327680*a^4*b^6*c^3
*z^4 - 61440*a^3*b^8*c^2*z^4 + 6144*a^2*b^10*c*z^4 - 1048576*a^7*c^6*z^4 -
 256*a*b^12*z^4 + 32768*A*C*a^4*b*c^4*z^2 - 512*A*C*a*b^7*c*z^2 - 24576*A*
C*a^3*b^3*c^3*z^2 + 6144*A*C*a^2*b^5*c^2*z^2 - 1536*C^2*a^2*b^6*c*z^2 + 12
288*B^2*a^4*b*c^4*z^2 + 512*A^2*a*b^6*c^2*z^2 - 8192*C^2*a^4*b^2*c^3*z^2 +
 6144*C^2*a^3*b^4*c^2*z^2 - 8192*B^2*a^3*b^3*c^3*z^2 + 1536*B^2*a^2*b^5*c^
2*z^2 + 24576*A^2*a^3*b^2*c^4*z^2 - 6144*A^2*a^2*b^4*c^3*z^2 + 128*C^2*a*b
^8*z^2 - 32768*A^2*a^4*c^5*z^2 - 16*B^2*b^9*z^2 + 1024*B^2*C*a^3*b*c^3*z -
 384*A*B^2*a*b^4*c^2*z + 192*B^2*C*a*b^5*c*z - 768*B^2*C*a^2*b^3*c^2*z + 1
536*A*B^2*a^2*b^2*c^3*z + 32*A*B^2*b^6*c*z - 2048*A*B^2*a^3*c^4*z - 16*B^2
*C*b^7*z + 192*A*B^2*C*a*b^2*c^2 + 512*A^3*C*a*b*c^3 + 128*A*C^3*a*b^3*c +
 16*A*B^2*C*b^4*c - 384*A^2*C^2*a*b^2*c^2 - 48*B^2*C^2*a*b^3*c - 192*A^2*B
^2*a*b*c^3 - 24*B^4*a*b^2*c^2 - 16*A^2*B^2*b^3*c^2 - 16*B^4*a^2*c^3 - 4*B^
2*C^2*b^5 - 9*B^4*b^4*c - 16*C^4*a*b^4 - 256*A^4*a*c^4, z, k)*(root(157286
4*a^6*b^2*c^5*z^4 - 983040*a^5*b^4*c^4*z^4 + 327680*a^4*b^6*c^3*z^4 - 6144
0*a^3*b^8*c^2*z^4 + 6144*a^2*b^10*c*z^4 - 1048576*a^7*c^6*z^4 - 256*a*b^12
*z^4 + 32768*A*C*a^4*b*c^4*z^2 - 512*A*C*a*b^7*c*z^2 - 24576*A*C*a^3*b^3*c
^3*z^2 + 6144*A*C*a^2*b^5*c^2*z^2 - 1536*C^2*a^2*b^6*c*z^2 + 12288*B^2*...
```

3.32. $\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

3.33 $\int \frac{A+Bx+Cx^2}{(a+bx^2+cx^4)^2}\,dx$

3.33.1 Optimal result

Integrand size = 25, antiderivative size = 368

$$\int \frac{A+Bx+Cx^2}{(a+bx^2+cx^4)^2}\,dx = -\frac{B(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{x(Ab^2-2aAc-abC+c(Ab-2aC)x^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{c}\left(Ab-2aC+\frac{A(b^2-12ac)+4abC}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c}\left(Ab-2aC-\frac{Ab^2-12aAc+4abC}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} + \frac{2Bc\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
-1/2*B*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*x*(A*b^2-2*a*A*c-a*b*C
+c*(A*b-2*C*a)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+2*B*c*arctanh((2*c*x^2+
b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-
(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(A*b-2*C*a+(A*(-12*a*c+b^2)+4*a*b*C)/(-
4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*
arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(A*b-2*C*a+
(12*A*a*c-A*b^2-4*C*a*b)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/(b+(-4
*a*c+b^2)^(1/2))^(1/2)
```

3.33.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx+Cx^2}{(a+bx^2+cx^4)^2}\,dx$$
$$= \frac{1}{4}\left(\frac{2ab(B+Cx)-2Abx(b+cx^2)+4acx(A+x(B+Cx))}{a(-b^2+4ac)(a+bx^2+cx^4)}\right.$$
$$+\frac{\sqrt{2}\sqrt{c}\left(A\left(b^2-12ac+b\sqrt{b^2-4ac}\right)-2a\left(-2b+\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\sqrt{2}\sqrt{c}\left(A\left(b^2-12ac-b\sqrt{b^2-4ac}\right)+2a\left(2b+\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$
$$\left.-\frac{4Bc\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}}+\frac{4Bc\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

input
```
Integrate[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((2*a*b*(B + C*x) - 2*A*b*x*(b + c*x^2) + 4*a*c*x*(A + x*(B + C*x)))/(a*(-
b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(A*(b^2 - 12*a*c + b*
Sqrt[b^2 - 4*a*c]) - 2*a*(-2*b + Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt
[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*(A*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])
 + 2*a*(2*b + Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sq
rt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (
4*B*c*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + (4*B*c*
Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

3.33.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2202, 27, 1432, 1086, 1083, 219, 1492, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{Cx^2+A}{(cx^4+bx^2+a)^2}dx+\int \frac{Bx}{(cx^4+bx^2+a)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{Cx^2+A}{(cx^4+bx^2+a)^2}dx+B\int \frac{x}{(cx^4+bx^2+a)^2}dx$$

$$\Big\downarrow \text{1432}$$

$$\int \frac{Cx^2+A}{(cx^4+bx^2+a)^2}dx+\frac{1}{2}B\int \frac{1}{(cx^4+bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{1086}$$

$$\int \frac{Cx^2+A}{(cx^4+bx^2+a)^2}dx+\frac{1}{2}B\left(-\frac{2c\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \text{1083}$$

$$\int \frac{Cx^2+A}{(cx^4+bx^2+a)^2}dx+\frac{1}{2}B\left(\frac{4c\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{b^2-4ac}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \text{219}$$

$$\int \frac{Cx^2+A}{(cx^4+bx^2+a)^2}dx+\frac{1}{2}B\left(\frac{4c\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \text{1492}$$

$$-\frac{\int -\frac{Ab^2+aCb+c(Ab-2aC)x^2-6aAc}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}+\frac{x\left(cx^2(Ab-2aC)-2aAc-abC+Ab^2\right)}{2a(b^2-4ac)(a+bx^2+cx^4)}+\frac{1}{2}B\left(\frac{4c\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{c(Ab-2aC)x^2+A(b^2-6ac)+abC}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}+\frac{x\left(cx^2(Ab-2aC)-2aAc-abC+Ab^2\right)}{2a(b^2-4ac)(a+bx^2+cx^4)}+\frac{1}{2}B\left(\frac{4c\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)$$

3.33. $\int \frac{A+Bx+Cx^2}{(a+bx^2+cx^4)^2}\,dx$

$\Big\downarrow$ 1480

$$\frac{\frac{1}{2}c\left(\frac{A(b^2-12ac)+4abC}{\sqrt{b^2-4ac}}-2aC+Ab\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(-\frac{-12aAc+4abC+Ab^2}{\sqrt{b^2-4ac}}-2aC+Ab\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}}{2a\left(b^2-4ac\right)}$$

$$\frac{x\left(cx^2(Ab-2aC)-2aAc-abC+Ab^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+$$

$$\frac{1}{2}B\left(\frac{4c\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$\Big\downarrow$ 218

$$\frac{\frac{\sqrt{c}\left(\frac{A(b^2-12ac)+4abC}{\sqrt{b^2-4ac}}-2aC+Ab\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{-12aAc+4abC+Ab^2}{\sqrt{b^2-4ac}}-2aC+Ab\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a\left(b^2-4ac\right)}+$$

$$\frac{x\left(cx^2(Ab-2aC)-2aAc-abC+Ab^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+$$

$$\frac{1}{2}B\left(\frac{4c\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

input
```
Int[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(x*(A*b^2 - 2*a*A*c - a*b*C + c*(A*b - 2*a*C)*x^2))/(2*a*(b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + ((Sqrt[c]*(A*b - 2*a*C + (A*(b^2 - 12*a*c) + 4*a*b*C)/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(A*b - 2*a*C - (A*b^2 -
 12*a*A*c + 4*a*b*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*(b^2 -
4*a*c)) + (B*(-((b + 2*c*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4))) + (4*c*
ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/2
```

3.33. $\int \frac{A+Bx+Cx^2}{\left(a+bx^2+cx^4\right)^2}\,dx$

3.33.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.33. $\int \frac{A+Bx+Cx^2}{(a+bx^2+cx^4)^2}\,dx$

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

3.33.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.25 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.63

method	result
risch	$\frac{-\frac{c(Ab-2Ca)x^3}{2a\left(4ac-b^2\right)}+\frac{c\,x^2B}{4ac-b^2}+\frac{\left(2Aac-A\,b^2+abC\right)x}{2a\left(4ac-b^2\right)}+\frac{Bb}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(-\frac{c(Ab-2Ca)\_R^2}{a\left(4ac-b^2\right)}+\frac{4\_RBc}{4ac-b^2}+\frac{6Aac-}{a(4a}\right.}{2c\_R^3+\_Rb}\right.}{4}$
default	$16c^2\left(-\frac{-\frac{\left(-4A\sqrt{-4ac+b^2}\,ac+A\sqrt{-4ac+b^2}\,b^2-4Aabc+A\,b^3+8a^2cC-2Ca\,b^2\right)x}{16ac}-\frac{B\left(4ac-b^2\right)}{8c}}{x^2+\frac{b}{2c}-\frac{\sqrt{-4ac+b^2}}{2c}}+\frac{2Ba\sqrt{-4ac+b^2}\,\ln\left(-2c\,x^2+\sqrt{-4ac+b^2}-b\right)}{4c(4ac-}\right.$

input
```
int((C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*c*(A*b-2*C*a)/a/(4*a*c-b^2)*x^3+c/(4*a*c-b^2)*x^2*B+1/2*(2*A*a*c-A*b
^2+C*a*b)/a/(4*a*c-b^2)*x+1/2/(4*a*c-b^2)*b*B)/(c*x^4+b*x^2+a)+1/4*sum((-c
*(A*b-2*C*a)/a/(4*a*c-b^2)*_R^2+4/(4*a*c-b^2)*_R*B*c+(6*A*a*c-A*b^2-C*a*b)
/a/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.33.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.33.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.33.7 Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx^2+cx^4)^2}\,dx = \int \frac{Cx^2+Bx+A}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*B*a*c*x^2 + (2*C*a - A*b)*c*x^3 + B*a*b + (C*a*b - A*b^2 + 2*A*a*c
)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*
x^2) + 1/2*integrate(-(4*B*a*c*x + (2*C*a - A*b)*c*x^2 - C*a*b - A*b^2 + 6
*A*a*c)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

3.33.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5156 vs. $2(323) = 646$.

Time = 1.59 (sec) , antiderivative size = 5156, normalized size of antiderivative = 14.01

$$\int \frac{A + Bx + Cx^2}{(a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(2*C*a*c*x^3 - A*b*c*x^3 + 2*B*a*c*x^2 + C*a*b*x - A*b^2*x + 2*A*a*c*
x + B*a*b)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) + 1/16*((2*b^3*c^2 - 8*
a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 +
4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^
2)*(a*b^2 - 4*a^2*c)^2*A - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*C +
 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b^5*c - 2*a*b^6*c + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*
c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 -
 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 19
2*a^4*c^4 + 2*(b^2 - 4*a*c)*a*b^4*c - 20*(b^2 - 4*a*c)*a^2*b^2*c^2 + 48...
```

3.33.9 Mupad [B] (verification not implemented)

Time = 8.40 (sec) , antiderivative size = 4707, normalized size of antiderivative = 12.79

$$\int \frac{A + Bx + Cx^2}{(a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(a + b*x^2 + c*x^4)^2,x)
```

output
```
((B*b)/(2*(4*a*c - b^2)) + (x*(2*A*a*c - A*b^2 + C*a*b))/(2*a*(4*a*c - b^2
)) + (B*c*x^2)/(4*a*c - b^2) - (c*x^3*(A*b - 2*C*a))/(2*a*(4*a*c - b^2)))/
(a + b*x^2 + c*x^4) + symsum(log((5*A^3*b^3*c^4 + 8*C^3*a^3*c^4 + 6*C^3*a^
2*b^2*c^3 - 36*A^3*a*b*c^5 - 96*A*B^2*a^2*c^5 + 72*A^2*C*a^2*c^5 - 3*A^2*C
*b^4*c^3 + 16*A*B^2*a*b^2*c^4 + 3*A*C^2*a*b^3*c^3 - 60*A*C^2*a^2*b*c^4 + 1
8*A^2*C*a*b^2*c^4 + 16*B^2*C*a^2*b*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*
b^4*c + 48*a^4*b^2*c^2)) - root(1572864*a^8*b^2*c^5*z^4 - 983040*a^7*b^4*c
^4*z^4 + 327680*a^6*b^6*c^3*z^4 - 61440*a^5*b^8*c^2*z^4 + 6144*a^4*b^10*c*
z^4 - 1048576*a^9*c^6*z^4 - 256*a^3*b^12*z^4 + 576*A*C*a^2*b^8*c*z^2 + 245
76*A*C*a^5*b^2*c^4*z^2 - 3072*A*C*a^3*b^6*c^2*z^2 + 2048*A*C*a^4*b^4*c^3*z
^2 - 32*A*C*a*b^10*z^2 + 12288*C^2*a^6*b*c^4*z^2 + 61440*A^2*a^5*b*c^5*z^2
 + 432*A^2*a*b^9*c*z^2 - 49152*A*C*a^6*c^5*z^2 - 8192*C^2*a^5*b^3*c^3*z^2
+ 1536*C^2*a^4*b^5*c^2*z^2 + 24576*B^2*a^5*b^2*c^4*z^2 - 6144*B^2*a^4*b^4*
c^3*z^2 + 512*B^2*a^3*b^6*c^2*z^2 - 61440*A^2*a^4*b^3*c^4*z^2 + 24064*A^2*
a^3*b^5*c^3*z^2 - 4608*A^2*a^2*b^7*c^2*z^2 - 32768*B^2*a^6*c^5*z^2 - 16*C^
2*a^2*b^9*z^2 - 16*A^2*b^11*z^2 + 3072*A*B*C*a^3*b^3*c^3*z - 768*A*B*C*a^2
*b^5*c^2*z - 4096*A*B*C*a^4*b*c^4*z + 64*A*B*C*a*b^7*c*z + 32*B*C^2*a^2*b^
6*c*z - 672*A^2*B*a*b^6*c^2*z + 1536*B*C^2*a^4*b^2*c^3*z - 384*B*C^2*a^3*b
^4*c^2*z - 15872*A^2*B*a^3*b^2*c^4*z + 4992*A^2*B*a^2*b^4*c^3*z + 32*A^2*B
*b^8*c*z - 2048*B*C^2*a^5*c^4*z + 18432*A^2*B*a^4*c^5*z + 192*A*B^2*C*a...
```

3.33. $\int \frac{A+Bx+Cx^2}{(a+bx^2+cx^4)^2}\,dx$

3.34 $\int \frac{A+Bx+Cx^2}{x(a+bx^2+cx^4)^2}\,dx$

3.34.1 Optimal result

Integrand size = 28, antiderivative size = 403

$$\int \frac{A+Bx+Cx^2}{x(a+bx^2+cx^4)^2}\,dx = \frac{Bx(b^2-2ac+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{B\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{B\sqrt{c}\left(b^2-12ac-b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{(A(b^3-6abc)+4a^2cC)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2(b^2-4ac)^{3/2}} + \frac{A\log(x)}{a^2} - \frac{A\log(a+bx^2+cx^4)}{4a^2}$$

output
```
1/2*B*x*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(A*(-2*a*c+
b^2)-a*b*C+c*(A*b-2*C*a)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(A*(-6*a*
b*c+b^3)+4*a^2*c*C)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^
2)^(3/2)+A*ln(x)/a^2-1/4*A*ln(c*x^4+b*x^2+a)/a^2+1/4*B*arctan(x*2^(1/2)*c^
(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c+b*(-4*a*c+b^2)^(1/
2))/a/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*B*arctan
(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c-b*(-4
*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.34.2 Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx$$
$$= \frac{-\frac{2a(abC+2acx(B+Cx)-bBx(b+cx^2)-A(b^2-2ac+bcx^2))}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}aB\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}aB\sqrt{c}\left(-b^2+1\right.}{(}}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)^2),x]
```

output
```
((-2*a*(a*b*C + 2*a*c*x*(B + C*x) - b*B*x*(b + c*x^2) - A*(b^2 - 2*a*c + b
*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*a*B*Sqrt[c]*(b^2
- 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b
^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2
]*a*B*Sqrt[c]*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2
- 4*a*c]]) + 4*A*Log[x] - ((A*(b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a
*c*Sqrt[b^2 - 4*a*c]) + 4*a^2*c*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/
(b^2 - 4*a*c)^(3/2) - ((A*(-b^3 + 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*
Sqrt[b^2 - 4*a*c]) - 4*a^2*c*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2
 - 4*a*c)^(3/2))/(4*a^2)
```

3.34.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {2193, 27, 1405, 25, 1480, 218, 1578, 1235, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)^2}dx$$
$$\Big\downarrow \text{2193}$$
$$\int \frac{Cx^2+A}{x\left(cx^4+bx^2+a\right)^2}dx+\int \frac{B}{\left(cx^4+bx^2+a\right)^2}dx$$
$$\Big\downarrow \text{27}$$

$$\int \frac{Cx^2+A}{x\left(cx^4+bx^2+a\right)^2}dx+B\int \frac{1}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \ 1405$$

$$\int \frac{Cx^2+A}{x\left(cx^4+bx^2+a\right)^2}dx+B\left(\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{b^2+cx^2b-6ac}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \ 25$$

$$\int \frac{Cx^2+A}{x\left(cx^4+bx^2+a\right)^2}dx+B\left(\frac{\int \frac{b^2+cx^2b-6ac}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \ 1480$$

$$\int \frac{Cx^2+A}{x\left(cx^4+bx^2+a\right)^2}dx+$$
$$B\left(\frac{\frac{1}{2}c\left(\frac{b^2-12ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(b-\frac{b^2-12ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{2a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^2\right.}{2a\left(b^2-4ac\right)\left(a+bx^2+\right.}\right.$$

$$\Big\downarrow \ 218$$

$$\int \frac{Cx^2+A}{x\left(cx^4+bx^2+a\right)^2}dx+$$
$$B\left(\frac{\frac{\sqrt{c}\left(\frac{b^2-12ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-12ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \ 1578$$

$$\frac{1}{2}\int \frac{Cx^2+A}{x^2\left(cx^4+bx^2+a\right)^2}dx^2+$$
$$B\left(\frac{\frac{\sqrt{c}\left(\frac{b^2-12ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-12ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \ 1235$$

3.34. $\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx$

$$\frac{1}{2}\left(\frac{cx^2(Ab-2aC)-2aAc-abC+Ab^2}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{c(Ab-2aC)x^2+A\left(b^2-4ac\right)}{x^2\left(cx^4+bx^2+a\right)}dx^2}{a\left(b^2-4ac\right)}\right)+$$
$$B\left(\frac{\frac{\sqrt{c}\left(\frac{b^2-12ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-12ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(\frac{\int \frac{c(Ab-2aC)x^2+A\left(b^2-4ac\right)}{x^2\left(cx^4+bx^2+a\right)}dx^2}{a\left(b^2-4ac\right)}+\frac{cx^2(Ab-2aC)-2aAc-abC+Ab^2}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+$$
$$B\left(\frac{\frac{\sqrt{c}\left(\frac{b^2-12ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-12ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \; 1200$$

$$\frac{1}{2}\left(\frac{\int\left(\frac{-Ab^3+5aAcb-Ac\left(b^2-4ac\right)x^2-2a^2cC}{a\left(cx^4+bx^2+a\right)}-\frac{A\left(4ac-b^2\right)}{ax^2}\right)dx^2}{a\left(b^2-4ac\right)}+\frac{cx^2(Ab-2aC)-2aAc-abC+Ab^2}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+$$
$$B\left(\frac{\frac{\sqrt{c}\left(\frac{b^2-12ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-12ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{\frac{\left(4a^2cC+A\left(b^3-6abc\right)\right)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}+\frac{A\log\left(x^2\right)\left(b^2-4ac\right)}{a}-\frac{A\left(b^2-4ac\right)\log\left(a+bx^2+cx^4\right)}{2a}}{a\left(b^2-4ac\right)}+\frac{cx^2(Ab-2aC)-2aAc-}{a\left(b^2-4ac\right)\left(a+bx^2+\right.}\right.$$
$$B\left(\frac{\frac{\sqrt{c}\left(\frac{b^2-12ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-12ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

3.34. $\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx$

input
```
Int[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)^2),x]
```

output
```
B*((x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (
(Sqrt[c]*(b + (b^2 - 12*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sq
rt[c]*(b - (b^2 - 12*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*(b
^2 - 4*a*c))) + ((A*b^2 - 2*a*A*c - a*b*C + c*(A*b - 2*a*C)*x^2)/(a*(b^2 -
 4*a*c)*(a + b*x^2 + c*x^4)) + (((A*(b^3 - 6*a*b*c) + 4*a^2*c*C)*ArcTanh[(
b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + (A*(b^2 - 4*a*c)*
Log[x^2])/a - (A*(b^2 - 4*a*c)*Log[a + b*x^2 + c*x^4])/(2*a))/(a*(b^2 - 4*
a*c)))/2
```

3.34.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

3.34. $\int \frac{A+Bx+Cx^2}{x(a+bx^2+cx^4)^2}\,dx$

3.34.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.40

method	result
default	$\frac{A\ln(x)}{a^2} - \frac{\frac{Babc\,x^3}{8ac-2b^2} + \frac{ac(Ab-2Ca)x^2}{8ac-2b^2} - \frac{aB\left(2ac-b^2\right)x}{2\left(4ac-b^2\right)} - \frac{a\left(2Aac-A\,b^2+abC\right)}{2\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a} + \frac{2c\left(\frac{\left(12Aabc\sqrt{-4ac+b^2}-2A\,b^3\sqrt{-4ac+b^2}+32A\,a^2c^2-16Aa\,b^2 \cdots\right.}{4c \cdots} \cdots\right.}{\cdots}$
risch	Expression too large to display

input
```
int((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
A*ln(x)/a^2-1/a^2*((1/2*B*a*b*c/(4*a*c-b^2)*x^3+1/2*a*c*(A*b-2*C*a)/(4*a*c
-b^2)*x^2-1/2*a*B*(2*a*c-b^2)/(4*a*c-b^2)*x-1/2*a*(2*A*a*c-A*b^2+C*a*b)/(4
*a*c-b^2))/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(1/(16*a*c-4*b^2)*(1/4*(12*A*a*
b*c*(-4*a*c+b^2)^(1/2)-2*A*b^3*(-4*a*c+b^2)^(1/2)+32*A*a^2*c^2-16*A*a*b^2*
c+2*A*b^4-8*C*(-4*a*c+b^2)^(1/2)*a^2*c)/c*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)
+1/2*(-12*a^2*B*c*(-4*a*c+b^2)^(1/2)+B*a*b^2*(-4*a*c+b^2)^(1/2)+4*a^2*b*B*
c-B*a*b^3)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+1/(16*a*c-4*b^2)*(-1/4*(12*A*a*b*c*(-4*a*c
+b^2)^(1/2)-2*A*b^3*(-4*a*c+b^2)^(1/2)-32*A*a^2*c^2+16*A*a*b^2*c-2*A*b^4-8
*C*(-4*a*c+b^2)^(1/2)*a^2*c)/c*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)+1/2*(-12*
a^2*B*c*(-4*a*c+b^2)^(1/2)+B*a*b^2*(-4*a*c+b^2)^(1/2)-4*a^2*b*B*c+B*a*b^3)
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2))))
```

3.34.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.34.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/x/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.34.7 Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{Cx^2+Bx+A}{\left(cx^4+bx^2+a\right)^2x}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(B*b*c*x^3 - (2*C*a - A*b)*c*x^2 - C*a*b + A*b^2 - 2*A*a*c + (B*b^2 -
2*B*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^
2*b*c)*x^2) + 1/2*integrate((B*a*b*c*x^2 + B*a*b^2 - 6*B*a^2*c - 2*(A*b^2*
c - 4*A*a*c^2)*x^3 - 2*(A*b^3 + (2*C*a^2 - 5*A*a*b)*c)*x)/(c*x^4 + b*x^2 +
 a), x)/(a^2*b^2 - 4*a^3*c) + A*log(x)/a^2
```

3.34.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6021 vs. $2(348) = 696$.

Time = 1.65 (sec) , antiderivative size = 6021, normalized size of antiderivative = 14.94

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/4*A*log(abs(c*x^4 + b*x^2 + a))/a^2 + A*log(abs(x))/a^2 - 1/16*((a^4*b^
4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)^2*(2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*B - 2*(sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^8*c - 18*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^5*b^6*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^
2 - 2*a^4*b^8*c^2 + 120*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^
3 + 28*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^3 + sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^3 + 36*a^5*b^6*c^3 - 352*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^4 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^6*b^3*c^4 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b
^4*c^4 - 240*a^6*b^4*c^4 + 384*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8
*c^5 + 192*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^5 + 64*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^5 + 704*a^7*b^2*c^5 - 96*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*c^6 - 768*a^8*c^6 + 2*(b^2 - 4*a*c)*a
^4*b^6*c^2 - 28*(b^2 - 4*a*c)*a^5*b^4*c^3 + 128*(b^2 - 4*a*c)*a^6*b^2*c^4
- 192*(b^2 - 4*a*c)*a^7*c^5)*B*abs(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)
 + (2*a^8*b^11*c^4 - 56*a^9*b^9*c^5 + 576*a^10*b^7*c^6 - 2816*a^11*b^5*...
```

3.34.9 Mupad [B] (verification not implemented)

Time = 8.53 (sec) , antiderivative size = 8129, normalized size of antiderivative = 20.17

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)^2),x)
```

output
```
((2*A*a*c - A*b^2 + C*a*b)/(2*a*(4*a*c - b^2)) + (B*x*(2*a*c - b^2))/(2*a*
(4*a*c - b^2)) - (c*x^2*(A*b - 2*C*a))/(2*a*(4*a*c - b^2)) - (B*b*c*x^3)/(
2*a*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) + symsum(log(root(1572864*a^9*b^2*
c^5*z^4 - 983040*a^8*b^4*c^4*z^4 + 327680*a^7*b^6*c^3*z^4 - 61440*a^6*b^8*
c^2*z^4 + 6144*a^5*b^10*c*z^4 - 1048576*a^10*c^6*z^4 - 256*a^4*b^12*z^4 +
1572864*A*a^7*b^2*c^5*z^3 - 983040*A*a^6*b^4*c^4*z^3 + 327680*A*a^5*b^6*c^
3*z^3 - 61440*A*a^4*b^8*c^2*z^3 + 6144*A*a^3*b^10*c*z^3 - 1048576*A*a^8*c^
6*z^3 - 256*A*a^2*b^12*z^3 + 98304*A*C*a^6*b*c^5*z^2 + 256*A*C*a^2*b^9*c*z
^2 - 90112*A*C*a^5*b^3*c^4*z^2 + 30720*A*C*a^4*b^5*c^3*z^2 - 4608*A*C*a^3*
b^7*c^2*z^2 + 61440*B^2*a^6*b*c^5*z^2 + 432*B^2*a^2*b^9*c*z^2 + 1536*A^2*a
*b^10*c*z^2 + 24576*C^2*a^6*b^2*c^4*z^2 - 6144*C^2*a^5*b^4*c^3*z^2 + 512*C
^2*a^4*b^6*c^2*z^2 - 61440*B^2*a^5*b^3*c^4*z^2 + 24064*B^2*a^4*b^5*c^3*z^2
 - 4608*B^2*a^3*b^7*c^2*z^2 + 516096*A^2*a^5*b^2*c^5*z^2 - 288768*A^2*a^4*
b^4*c^4*z^2 + 88576*A^2*a^3*b^6*c^3*z^2 - 15744*A^2*a^2*b^8*c^2*z^2 - 16*B
^2*a*b^11*z^2 - 32768*C^2*a^7*c^5*z^2 - 393216*A^2*a^6*c^6*z^2 - 64*A^2*b^
12*z^2 + 49152*A^2*C*a^4*b*c^5*z - 2304*A^2*C*a*b^7*c^2*z + 3072*A*B^2*a^4
*b*c^5*z - 48*A*B^2*a*b^7*c^2*z + 32*B^2*C*a*b^8*c*z - 15872*B^2*C*a^4*b^2
*c^4*z + 4992*B^2*C*a^3*b^4*c^3*z - 672*B^2*C*a^2*b^6*c^2*z - 45056*A^2*C*
a^3*b^3*c^4*z + 15360*A^2*C*a^2*b^5*c^3*z + 12288*A*C^2*a^4*b^2*c^4*z - 30
72*A*C^2*a^3*b^4*c^3*z + 256*A*C^2*a^2*b^6*c^2*z - 2304*A*B^2*a^3*b^3*c...
```

3.34. $\int \frac{A+Bx+Cx^2}{x(a+bx^2+cx^4)^2}\,dx$

3.35 $\int \frac{A+Bx+Cx^2}{x^2(a+bx^2+cx^4)^2}\,dx$

3.35.1 Optimal result

Integrand size = 28, antiderivative size = 514

$$\int \frac{A+Bx+Cx^2}{x^2(a+bx^2+cx^4)^2}\,dx = -\frac{3Ab^2-10aAc-abC}{2a^2(b^2-4ac)\,x}$$
$$+\frac{B(b^2-2ac+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}+\frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a(b^2-4ac)\,x\,(a+bx^2+cx^4)}$$
$$-\frac{\sqrt{c}\left(A\left(3b^3-16abc+3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}\right)-a\left(b^2-12ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{}}{\sqrt{b-\sqrt{b}}}\right.}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\sqrt{c}\left(3Ab^2-10aAc-abC-\frac{A(3b^3-16abc)-a(b^2-12ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$
$$+\frac{bB(b^2-6ac)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2(b^2-4ac)^{3/2}}+\frac{B\log(x)}{a^2}-\frac{B\log(a+bx^2+cx^4)}{4a^2}$$

output
```
1/2*(10*A*a*c-3*A*b^2+C*a*b)/a^2/(-4*a*c+b^2)/x+1/2*B*(b*c*x^2-2*a*c+b^2)/
a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(A*(-2*a*c+b^2)-a*b*C+c*(A*b-2*C*a)*x^2
)/a/(-4*a*c+b^2)/x/(c*x^4+b*x^2+a)+1/2*b*B*(-6*a*c+b^2)*arctanh((2*c*x^2+b
)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)+B*ln(x)/a^2-1/4*B*ln(c*x^4+b*
x^2+a)/a^2-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1
/2)*(-a*C*(b^2-12*a*c+b*(-4*a*c+b^2)^(1/2))+A*(3*b^3-16*a*b*c+3*b^2*(-4*a*
c+b^2)^(1/2)-10*a*c*(-4*a*c+b^2)^(1/2)))/a^2/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b
-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1
/2))^(1/2))*c^(1/2)*(3*A*b^2-10*a*A*c-a*b*C+(-A*(-16*a*b*c+3*b^3)+a*(-12*a
*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1
/2))^(1/2)
```

3.35.2 Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx+Cx^2}{x^2(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{-\frac{4A}{x}+\frac{-4a^2c(B+Cx)-2Ab^2x(b+cx^2)+2a(2Ac^2x^3+b^2(B+Cx)+bcx(3A+x(B+Cx)))}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\sqrt{c}\left(A\left(-3b^3+16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-\ldots}\right.\right.}{(b^2-4ac\ldots}}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

output
```
((-4*A)/x + (-4*a^2*c*(B + C*x) - 2*A*b^2*x*(b + c*x^2) + 2*a*(2*A*c^2*x^3
 + b^2*(B + C*x) + b*c*x*(3*A + x*(B + C*x))))/((b^2 - 4*a*c)*(a + b*x^2 +
 c*x^4)) + (Sqrt[2]*Sqrt[c]*(A*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c
] + 10*a*c*Sqrt[b^2 - 4*a*c]) + a*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*C)*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3
/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(A*(3*b^3 - 16*a*b*c -
 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]) + a*(-b^2 + 12*a*c +
b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + 4*B*Log[x] - (B
*(b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log[-b
+ Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (B*(-b^3 + 6*a*b*c +
 b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c
] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(4*a^2)
```

3.35.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2193, 27, 1434, 1165, 25, 1200, 1600, 25, 1604, 1480, 218, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^2 \left(a+bx^2+cx^4\right)^2} \, dx$$

$$\Big\downarrow \text{2193}$$

$$\int \frac{Cx^2+A}{x^2 \left(cx^4+bx^2+a\right)^2}dx+\int \frac{B}{x \left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{Cx^2+A}{x^2 \left(cx^4+bx^2+a\right)^2}dx+B\int \frac{1}{x \left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{1434}$$

$$\int \frac{Cx^2+A}{x^2 \left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\int \frac{1}{x^2 \left(cx^4+bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \text{1165}$$

$$\int \frac{Cx^2+A}{x^2 \left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{-2ac+b^2+bcx^2}{a \left(b^2-4ac\right) \left(a+bx^2+cx^4\right)}-\frac{\int -\frac{b^2+cx^2b-4ac}{x^2 \left(cx^4+bx^2+a\right)}dx^2}{a \left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$\int \frac{Cx^2+A}{x^2 \left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{\int \frac{b^2+cx^2b-4ac}{x^2 \left(cx^4+bx^2+a\right)}dx^2}{a \left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{a \left(b^2-4ac\right) \left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{1200}$$

$$\int \frac{Cx^2+A}{x^2 \left(cx^4+bx^2+a\right)^2}dx+$$
$$\frac{1}{2}B\left(\frac{\int \left(\frac{b^2-4ac}{ax^2}+\frac{-c \left(b^2-4ac\right) x^2-b \left(b^2-5ac\right)}{a \left(cx^4+bx^2+a\right)}\right)dx^2}{a \left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{a \left(b^2-4ac\right) \left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{1600}$$

$$-\frac{\int -\frac{3Ab^2-aCb+3c(Ab-2aC)x^2-10aAc}{x^2(cx^4+bx^2+a)}dx}{2a\left(b^2-4ac\right)}+\frac{1}{2}B\left(\frac{\int \left(\frac{b^2-4ac}{ax^2}+\frac{-c(b^2-4ac)x^2-b(b^2-5ac)}{a(cx^4+bx^2+a)}\right)dx^2}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+\frac{A\left(b^2-2ac\right)+cx^2(Ab-2aC)-abC}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{3Ab^2-aCb+3c(Ab-2aC)x^2-10aAc}{x^2(cx^4+bx^2+a)}dx}{2a\left(b^2-4ac\right)}+\frac{1}{2}B\left(\frac{\int \left(\frac{b^2-4ac}{ax^2}+\frac{-c(b^2-4ac)x^2-b(b^2-5ac)}{a(cx^4+bx^2+a)}\right)dx^2}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+\frac{A\left(b^2-2ac\right)+cx^2(Ab-2aC)-abC}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 1604$$

$$\frac{-\frac{\int \frac{c\left(3Ab^2-aCb-10aAc\right)x^2+A\left(3b^3-13abc\right)-a\left(b^2-6ac\right)C}{cx^4+bx^2+a}dx}{a}-\frac{-10aAc-abC+3Ab^2}{ax}}{2a\left(b^2-4ac\right)}+\frac{1}{2}B\left(\frac{\int \left(\frac{b^2-4ac}{ax^2}+\frac{-c(b^2-4ac)x^2-b(b^2-5ac)}{a(cx^4+bx^2+a)}\right)dx^2}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+\frac{A\left(b^2-2ac\right)+cx^2(Ab-2aC)-abC}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 1480$$

$$-\frac{\frac{\frac{1}{2}c\left(\frac{A\left(3b^3-16abc\right)-aC\left(b^2-12ac\right)}{\sqrt{b^2-4ac}}-10aAc-abC+3Ab^2\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{c\left(A\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}-16abc+3b^3\right)-aC\left(-b\sqrt{b^2-4ac}\right.\cdots\right.}{2\sqrt{b^2-4ac}}\cdots}{a}\cdots}{2a\left(b^2-4ac\right)}+\frac{1}{2}B\left(\frac{\int \left(\frac{b^2-4ac}{ax^2}+\frac{-c(b^2-4ac)x^2-b(b^2-5ac)}{a(cx^4+bx^2+a)}\right)dx^2}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+\frac{A\left(b^2-2ac\right)+cx^2(Ab-2aC)-abC}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 218$$

3.35. $\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$

$$\frac{1}{2}B\left(\frac{\int\left(\frac{b^2-4ac}{ax^2}+\frac{-c(b^2-4ac)x^2-b(b^2-5ac)}{a(cx^4+bx^2+a)}\right)dx^2}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx^2}{a(b^2-4ac)(a+bx^2+cx^4)}\right)+$$

$$-\frac{\frac{\sqrt{c}\left(\frac{A(3b^3-16abc)-aC(b^2-12ac)}{\sqrt{b^2-4ac}}-10aAc-abC+3Ab^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}-16abc+3b^3\right)-aC\left(-b\sqrt{b^2-4ac}-\right.\right.}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{a}}{2a(b^2-4ac)}$$

$$\frac{A(b^2-2ac)+cx^2(Ab-2aC)-abC}{2ax(b^2-4ac)(a+bx^2+cx^4)}$$

$\downarrow$ 2009

$$-\frac{\frac{\sqrt{c}\left(\frac{A(3b^3-16abc)-aC(b^2-12ac)}{\sqrt{b^2-4ac}}-10aAc-abC+3Ab^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}-16abc+3b^3\right)-aC\left(-b\sqrt{b^2-4ac}-\right.\right.}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{a}}{2a(b^2-4ac)}$$

$$\frac{A(b^2-2ac)+cx^2(Ab-2aC)-abC}{2ax(b^2-4ac)(a+bx^2+cx^4)}+$$

$$\frac{1}{2}B\left(\frac{\frac{b(b^2-6ac)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}+\frac{\log(x^2)(b^2-4ac)}{a}-\frac{(b^2-4ac)\log(a+bx^2+cx^4)}{2a}}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx^2}{a(b^2-4ac)(a+bx^2+cx^4)}\right)$$

input
```
Int[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

output
```
(A*(b^2 - 2*a*c) - a*b*C + c*(A*b - 2*a*C)*x^2)/(2*a*(b^2 - 4*a*c)*x*(a +
b*x^2 + c*x^4)) + (-((3*A*b^2 - 10*a*A*c - a*b*C)/(a*x)) - ((Sqrt[c]*(3*A*
b^2 - 10*a*A*c - a*b*C + (A*(3*b^3 - 16*a*b*c) - a*(b^2 - 12*a*c)*C)/Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sq
rt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(A*(3*b^3 - 16*a*b*c - 3*b^2
*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]) - a*(b^2 - 12*a*c - b*Sqrt[
b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/
(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a)/(2*a*(b^2 - 4*
a*c)) + (B*((b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4))
+ ((b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2
- 4*a*c]) + ((b^2 - 4*a*c)*Log[x^2])/a - ((b^2 - 4*a*c)*Log[a + b*x^2 + c*
x^4])/(2*a))/(a*(b^2 - 4*a*c))))/2
```

3.35. $\int \frac{A+Bx+Cx^2}{x^2(a+bx^2+cx^4)^2}\,dx$

3.35.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1600
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^m*(a + b*x^2 + c
*x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1))
- a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Int
egerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

3.35.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 667, normalized size of antiderivative = 1.30

method	result
default	$-\frac{A}{a^{2} x}+\frac{\ln \left(x \right) B}{a^{2}}-\frac{\frac{c \left(2 A a c -A \,b^{2}+a b C \right) x^{3}}{8 a c -2 b^{2}}+\frac{x^{2} B a b c}{8 a c -2 b^{2}}+\frac{\left(3 A a b c -A \,b^{3}-2 a^{2} c C +C a \,b^{2}\right) x}{8 a c -2 b^{2}}-\frac{a B \left(2 a c -b^{2}\right)}{2 \left(4 a c -b^{2}\right)}}{c \,x^{4}+b \,x^{2}+a}+\frac{2 c \left(\frac{\left(12 B a b c \sqrt{-4 a c +b^{2}}-2 B \,b^{3} \sqrt{\cdots}\right.}{\cdots}\cdots\right.}{\cdots}$
risch	Expression too large to display

input
```
int((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-A/a^2/x+ln(x)/a^2*B-1/a^2*((1/2*c*(2*A*a*c-A*b^2+C*a*b)/(4*a*c-b^2)*x^3+1
/2/(4*a*c-b^2)*x^2*B*a*b*c+1/2*(3*A*a*b*c-A*b^3-2*C*a^2*c+C*a*b^2)/(4*a*c-
b^2)*x-1/2*a*B*(2*a*c-b^2)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(1
/(16*a*c-4*b^2)*(1/4*(12*B*a*b*c*(-4*a*c+b^2)^(1/2)-2*B*b^3*(-4*a*c+b^2)^(
1/2)+32*B*a^2*c^2-16*B*a*b^2*c+2*B*b^4)/c*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)
+1/2*(16*A*a*b*c*(-4*a*c+b^2)^(1/2)-3*A*b^3*(-4*a*c+b^2)^(1/2)+40*A*a^2*c^
2-22*A*a*b^2*c+3*A*b^4-12*C*(-4*a*c+b^2)^(1/2)*a^2*c+C*(-4*a*c+b^2)^(1/2)*
a*b^2+4*C*a^2*b*c-C*a*b^3)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+1/(16*a*c-4*b^2)*(-1/4*(12
*B*a*b*c*(-4*a*c+b^2)^(1/2)-2*B*b^3*(-4*a*c+b^2)^(1/2)-32*B*a^2*c^2+16*B*a
*b^2*c-2*B*b^4)/c*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)+1/2*(16*A*a*b*c*(-4*a*
c+b^2)^(1/2)-3*A*b^3*(-4*a*c+b^2)^(1/2)-40*A*a^2*c^2+22*A*a*b^2*c-3*A*b^4-
12*C*(-4*a*c+b^2)^(1/2)*a^2*c+C*(-4*a*c+b^2)^(1/2)*a*b^2-4*C*a^2*b*c+C*a*b
^3)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2))))
```

3.35.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.35.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/x**2/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.35.7 Maxima [F]

$$\int \frac{A + Bx + Cx^2}{x^2 (a + bx^2 + cx^4)^2} \, dx = \int \frac{Cx^2 + Bx + A}{(cx^4 + bx^2 + a)^2 x^2} \, dx$$

input
```
integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(B*a*b*c*x^3 + (10*A*a*c^2 + (C*a*b - 3*A*b^2)*c)*x^4 - 2*A*a*b^2 + 8*
A*a^2*c + (C*a*b^2 - 3*A*b^3 - (2*C*a^2 - 11*A*a*b)*c)*x^2 + (B*a*b^2 - 2*
B*a^2*c)*x)/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^
3*b^2 - 4*a^4*c)*x) + 1/2*integrate((C*a*b^2 - 3*A*b^3 - 2*(B*b^2*c - 4*B*
a*c^2)*x^3 + (10*A*a*c^2 + (C*a*b - 3*A*b^2)*c)*x^2 - (6*C*a^2 - 13*A*a*b)
*c - 2*(B*b^3 - 5*B*a*b*c)*x)/(c*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c)
+ B*log(x)/a^2
```

3.35.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9013 vs. 2(453) = 906.

Time = 2.08 (sec) , antiderivative size = 9013, normalized size of antiderivative = 17.54

$$\int \frac{A + Bx + Cx^2}{x^2 (a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/4*B*log(abs(c*x^4 + b*x^2 + a))/a^2 + B*log(abs(x))/a^2 + 1/2*(C*a*b*c*
x^4 - 3*A*b^2*c*x^4 + 10*A*a*c^2*x^4 + B*a*b*c*x^3 + C*a*b^2*x^2 - 3*A*b^3
*x^2 - 2*C*a^2*c*x^2 + 11*A*a*b*c*x^2 + B*a*b^2*x - 2*B*a^2*c*x - 2*A*a*b^
2 + 8*A*a^2*c)/((c*x^5 + b*x^3 + a*x)*(a^2*b^2 - 4*a^3*c)) + 1/16*((a^4*b^
4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)^2*(6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2*c^4
 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 22*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 6*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 40*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2 + 20*
(b^2 - 4*a*c)*a*c^3)*A - (a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)^2*(2*a*b
^3*c^2 - 8*a^2*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 -
 2*(b^2 - 4*a*c)*a*b*c^2)*C - 2*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^4*b^9*c - 49*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^7*c^2 - 6*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^8*c^2 - 6*a^4*b^9*c^2 + 300...
```

3.35.9 Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 8684, normalized size of antiderivative = 16.89

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x)
```

output
```
symsum(log(root(1572864*a^10*b^2*c^5*z^4 - 983040*a^9*b^4*c^4*z^4 + 327680
*a^8*b^6*c^3*z^4 - 61440*a^7*b^8*c^2*z^4 + 6144*a^6*b^10*c*z^4 - 1048576*a
^11*c^6*z^4 - 256*a^5*b^12*z^4 + 1572864*B*a^8*b^2*c^5*z^3 - 983040*B*a^7*
b^4*c^4*z^3 + 327680*B*a^6*b^6*c^3*z^3 - 61440*B*a^5*b^8*c^2*z^3 + 6144*B*
a^4*b^10*c*z^3 - 1048576*B*a^9*c^6*z^3 - 256*B*a^3*b^12*z^3 - 2432*A*C*a^2
*b^10*c*z^2 - 491520*A*C*a^6*b^2*c^5*z^2 + 358400*A*C*a^5*b^4*c^4*z^2 - 12
9024*A*C*a^4*b^6*c^3*z^2 + 24768*A*C*a^3*b^8*c^2*z^2 + 96*A*C*a*b^12*z^2 +
 61440*C^2*a^7*b*c^5*z^2 + 432*C^2*a^3*b^9*c*z^2 + 1536*B^2*a^2*b^10*c*z^2
 - 430080*A^2*a^6*b*c^6*z^2 + 3408*A^2*a*b^11*c*z^2 + 245760*A*C*a^7*c^6*z
^2 - 61440*C^2*a^6*b^3*c^4*z^2 + 24064*C^2*a^5*b^5*c^3*z^2 - 4608*C^2*a^4*
b^7*c^2*z^2 + 516096*B^2*a^6*b^2*c^5*z^2 - 288768*B^2*a^5*b^4*c^4*z^2 + 88
576*B^2*a^4*b^6*c^3*z^2 - 15744*B^2*a^3*b^8*c^2*z^2 + 716800*A^2*a^5*b^3*c
^5*z^2 - 483840*A^2*a^4*b^5*c^4*z^2 + 170496*A^2*a^3*b^7*c^3*z^2 - 33232*A
^2*a^2*b^9*c^2*z^2 - 64*B^2*a*b^12*z^2 - 393216*B^2*a^7*c^6*z^2 - 16*C^2*a
^2*b^11*z^2 - 144*A^2*b^13*z^2 - 110592*A*B*C*a^4*b^2*c^5*z + 36864*A*B*C*
a^3*b^4*c^4*z - 5376*A*B*C*a^2*b^6*c^3*z + 288*A*B*C*a*b^8*c^2*z + 3072*B*
C^2*a^5*b*c^5*z - 138240*A^2*B*a^4*b*c^6*z + 7344*A^2*B*a*b^7*c^3*z + 1228
80*A*B*C*a^5*c^6*z - 2304*B*C^2*a^4*b^3*c^4*z + 576*B*C^2*a^3*b^5*c^3*z -
48*B*C^2*a^2*b^7*c^2*z + 131328*A^2*B*a^3*b^3*c^5*z - 46656*A^2*B*a^2*b^5*
c^4*z + 61440*B^3*a^4*b^2*c^5*z - 21504*B^3*a^3*b^4*c^4*z + 3328*B^3*a^...
```

3.35. $\int \frac{A+Bx+Cx^2}{x^2(a+bx^2+cx^4)^2} \, dx$

3.36 $\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$

3.36.1 Optimal result

Integrand size = 28, antiderivative size = 534

$$\begin{aligned}
&\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx \\
&= -\frac{2Ab^2-6aAc-abC}{2a^2\left(b^2-4ac\right)x^2}-\frac{B(3b^2-10ac)}{2a^2\left(b^2-4ac\right)x} \\
&\quad+\frac{B(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}+\frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} \\
&\quad-\frac{B\sqrt{c}\left(3b^3-16abc+(3b^2-10ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad+\frac{B\sqrt{c}\left(3b^3-16abc-(3b^2-10ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} \\
&\quad-\frac{\left(2A(b^4-6ab^2c+6a^2c^2)-ab(b^2-6ac)C\right)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{3/2}} \\
&\quad-\frac{(2Ab-aC)\log(x)}{a^3}+\frac{(2Ab-aC)\log\left(a+bx^2+cx^4\right)}{4a^3}
\end{aligned}$$

output
```
1/2*(6*A*a*c-2*A*b^2+C*a*b)/a^2/(-4*a*c+b^2)/x^2-1/2*B*(-10*a*c+3*b^2)/a^2
/(-4*a*c+b^2)/x+1/2*B*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x/(c*x^4+b*x^2+a)
+1/2*(A*(-2*a*c+b^2)-a*b*C+c*(A*b-2*C*a)*x^2)/a/(-4*a*c+b^2)/x^2/(c*x^4+b*
x^2+a)-1/2*(2*A*(6*a^2*c^2-6*a*b^2*c+b^4)-a*b*(-6*a*c+b^2)*C)*arctanh((2*c
*x^2+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)-(2*A*b-C*a)*ln(x)/a^3+1
/4*(2*A*b-C*a)*ln(c*x^4+b*x^2+a)/a^3-1/4*B*arctan(x*2^(1/2)*c^(1/2)/(b-(-4
*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^3-16*a*b*c+(-10*a*c+3*b^2)*(-4*a*c+b^
2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*
B*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^3-16
*a*b*c-(-10*a*c+3*b^2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)*2^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.36.2 Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 655, normalized size of antiderivative = 1.23

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{-\frac{2aA}{x^2}-\frac{4aB}{x}-\frac{2a\left(2a^2cC+b^2Bx\left(b+cx^2\right)+A\left(b^3-3abc+b^2cx^2-2ac^2x^2\right)-a\left(b^2C+2Bc^2x^3+bcx(3B+Cx)\right)\right)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}aB\sqrt{c}\left(-3b^3+16abc-3b^2\right.}{(b}}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

output
```
((-2*a*A)/x^2 - (4*a*B)/x - (2*a*(2*a^2*c*C + b^2*B*x*(b + c*x^2) + A*(b^3
 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2) - a*(b^2*C + 2*B*c^2*x^3 + b*c*x*(3*
B + C*x))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*a*B*Sqrt[c]*(-3
*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*
Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*a*B*Sqrt[c]*(3*b^3 - 16*a*b*c - 3*
b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 -
4*a*c]]) + 4*(-2*A*b + a*C)*Log[x] + ((2*A*(b^4 - 6*a*b^2*c + 6*a^2*c^2 +
b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c]) + a*(-b^3 + 6*a*b*c - b
^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*C)*Log[-b + Sqrt[b^2 - 4*a
*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((2*A*(-b^4 + 6*a*b^2*c - 6*a^2*c^2
+ b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c]) + a*(b^3 - 6*a*b*c -
b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*C)*Log[b + Sqrt[b^2 - 4*a
*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(4*a^3)
```

3.36.3 Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2193, 27, 1441, 25, 1578, 1235, 25, 1200, 1604, 1480, 218, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^3 \left(a+bx^2+cx^4\right)^2} \, dx$$

$$\Big\downarrow \text{2193}$$

$$\int \frac{Cx^2+A}{x^3 \left(cx^4+bx^2+a\right)^2}dx+\int \frac{B}{x^2 \left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{Cx^2+A}{x^3 \left(cx^4+bx^2+a\right)^2}dx+B\int \frac{1}{x^2 \left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{1441}$$

$$\int \frac{Cx^2+A}{x^3 \left(cx^4+bx^2+a\right)^2}dx+B\left(\frac{-2ac+b^2+bcx^2}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{3b^2+3cx^2b-10ac}{x^2\left(cx^4+bx^2+a\right)}dx}{2a\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$\int \frac{Cx^2+A}{x^3 \left(cx^4+bx^2+a\right)^2}dx+B\left(\frac{\int \frac{3b^2+3cx^2b-10ac}{x^2\left(cx^4+bx^2+a\right)}dx}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{Cx^2+A}{x^4 \left(cx^4+bx^2+a\right)^2}dx^2+B\left(\frac{\int \frac{3b^2+3cx^2b-10ac}{x^2\left(cx^4+bx^2+a\right)}dx}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{1235}$$

$$\frac{1}{2}\left(\frac{cx^2(Ab-2aC)-2aAc-abC+Ab^2}{ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{2Ab^2-aCb+2c(Ab-2aC)x^2-6aAc}{x^4\left(cx^4+bx^2+a\right)}dx^2}{a\left(b^2-4ac\right)}\right)+ B\left(\frac{\int \frac{3b^2+3cx^2b-10ac}{x^2\left(cx^4+bx^2+a\right)}dx}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{2c(Ab-2aC)x^2+2A(b^2-3ac)-abC}{x^4(cx^4+bx^2+a)}dx^2}{a\left(b^2-4ac\right)}+\frac{cx^2(Ab-2aC)-2aAc-abC+Ab^2}{ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+B\left(\frac{\int \frac{3b^2+3cx^2b-10ac}{x^2(cx^4+bx^2+a)}dx}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$\downarrow$ 1200

$$\frac{1}{2}\left(\frac{\int \left(-\frac{(4ac-b^2)(aC-2Ab)}{a^2x^2}+\frac{c(b^2-4ac)(2Ab-aC)x^2+2A(b^4-5acb^2+3a^2c^2)-ab(b^2-5ac)C}{a^2(cx^4+bx^2+a)}+\frac{2A(b^2-3ac)-abC}{ax^4}\right)dx^2}{a\left(b^2-4ac\right)}+\frac{cx^2(Ab-}{ax^2(b}\right.$$
$$B\left(\frac{\int \frac{3b^2+3cx^2b-10ac}{x^2(cx^4+bx^2+a)}dx}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$\downarrow$ 1604

$$\frac{1}{2}\left(\frac{\int \left(-\frac{(4ac-b^2)(aC-2Ab)}{a^2x^2}+\frac{c(b^2-4ac)(2Ab-aC)x^2+2A(b^4-5acb^2+3a^2c^2)-ab(b^2-5ac)C}{a^2(cx^4+bx^2+a)}+\frac{2A(b^2-3ac)-abC}{ax^4}\right)dx^2}{a\left(b^2-4ac\right)}+\frac{cx^2(Ab-}{ax^2(b}\right.$$
$$B\left(\frac{-\frac{\int \frac{c\left(3b^2-10ac\right)x^2+b\left(3b^2-13ac\right)}{cx^4+bx^2+a}dx}{a}-\frac{3b^2-10ac}{ax}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$\downarrow$ 1480

$$\frac{1}{2}\left(\frac{\int \left(-\frac{(4ac-b^2)(aC-2Ab)}{a^2x^2}+\frac{c(b^2-4ac)(2Ab-aC)x^2+2A(b^4-5acb^2+3a^2c^2)-ab(b^2-5ac)C}{a^2(cx^4+bx^2+a)}+\frac{2A(b^2-3ac)-abC}{ax^4}\right)dx^2}{a\left(b^2-4ac\right)}+\frac{cx^2(Ab-}{ax^2(b}\right.$$
$$B\left(\frac{-\frac{\frac{1}{2}c\left(-\frac{16abc}{\sqrt{b^2-4ac}}+\frac{3b^3}{\sqrt{b^2-4ac}}-10ac+3b^2\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{c\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{2\sqrt{b^2-4ac}}}{a}-\frac{3b^2-}{ax}}{2a\left(b^2-4ac\right)}\right.$$

$\downarrow$ 218

3.36. $\int \frac{A+Bx+Cx^2}{x^3(a+bx^2+cx^4)^2}\,dx$

$$\frac{1}{2}\left(\frac{\int \left(-\frac{(4ac-b^2)(aC-2Ab)}{a^2x^2}+\frac{c(b^2-4ac)(2Ab-aC)x^2+2A(b^4-5acb^2+3a^2c^2)-ab(b^2-5ac)C}{a^2(cx^4+bx^2+a)}+\frac{2A(b^2-3ac)-abC}{ax^4}\right)dx^2}{a(b^2-4ac)}+\frac{cx^2(Ab-}{ax^2(b}\right.$$

$$B\left(\frac{-\frac{\frac{\sqrt{c}\left(-\frac{16abc}{\sqrt{b^2-4ac}}+\frac{3b^3}{\sqrt{b^2-4ac}}-10ac+3b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{3b^2-10ac}{ax}}{2a(b^2-4ac)}\right.$$

$$\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{-\frac{(2A(6a^2c^2-6ab^2c+b^4)-abC(b^2-6ac))\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}-\frac{\log(x^2)(b^2-4ac)(2Ab-aC)}{a^2}+\frac{(b^2-4ac)(2Ab-aC)\log(a+bx^2+cx^4)}{2a^2}}{a(b^2-4ac)}\right.$$

$$B\left(\frac{-\frac{\frac{\sqrt{c}\left(-\frac{16abc}{\sqrt{b^2-4ac}}+\frac{3b^3}{\sqrt{b^2-4ac}}-10ac+3b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{3b^2-10ac}{ax}}{2a(b^2-4ac)}\right.$$

input
```
Int[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

output
```
B*((b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) + (-(
(3*b^2 - 10*a*c)/(a*x)) - ((Sqrt[c]*(3*b^2 - 10*a*c + (3*b^3)/Sqrt[b^2 - 4
*a*c] - (16*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(3*b
^3 - 16*a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqr
t[b^2 - 4*a*c]]))/a)/(2*a*(b^2 - 4*a*c))) + ((A*b^2 - 2*a*A*c - a*b*C + c*
(A*b - 2*a*C)*x^2)/(a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)) + (-((2*A*b^2
 - 6*a*A*c - a*b*C)/(a*x^2)) - ((2*A*(b^4 - 6*a*b^2*c + 6*a^2*c^2) - a*b*(
b^2 - 6*a*c)*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 -
4*a*c]) - ((b^2 - 4*a*c)*(2*A*b - a*C)*Log[x^2])/a^2 + ((b^2 - 4*a*c)*(2*A
*b - a*C)*Log[a + b*x^2 + c*x^4])/(2*a^2))/(a*(b^2 - 4*a*c)))/2
```

3.36. $\int \frac{A+Bx+Cx^2}{x^3(a+bx^2+cx^4)^2}\,dx$

3.36.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1441
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))
  Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

3.36. $\int \frac{A+Bx+Cx^2}{x^3(a+bx^2+cx^4)^2}\,dx$

3.36.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 778, normalized size of antiderivative = 1.46

method	result
default	$-\frac{A}{2a^{2}x^{2}}-\frac{B}{a^{2}x}+\frac{\left(-2Ab+Ca\right)\ln\left(x\right)}{a^{3}}-\frac{\frac{\frac{Bac\left(2ac-b^{2}\right)x^{3}}{8ac-2b^{2}}+\frac{ac\left(2Aac-A\,b^{2}+abC\right)x^{2}}{8ac-2b^{2}}+\frac{Bab\left(3ac-b^{2}\right)x}{8ac-2b^{2}}+\frac{a\left(3Aabc-A\,b^{3}-2a^{2}cC+Ca\,b^{2}\right)}{8ac-2b^{2}}}{c\,x^{4}+b\,x^{2}+a}}{}$
risch	Expression too large to display

input
```
int((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*A/a^2/x^2-1/a^2*B/x+(-2*A*b+C*a)/a^3*ln(x)-1/a^3*((1/2*B*a*c*(2*a*c-b
^2)/(4*a*c-b^2)*x^3+1/2*a*c*(2*A*a*c-A*b^2+C*a*b)/(4*a*c-b^2)*x^2+1/2*B*a*
b*(3*a*c-b^2)/(4*a*c-b^2)*x+1/2*a*(3*A*a*b*c-A*b^3-2*C*a^2*c+C*a*b^2)/(4*a
*c-b^2))/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(1/(16*a*c-4*b^2)*(1/4*(24*A*(-4*
a*c+b^2)^(1/2)*a^2*c^2-24*A*(-4*a*c+b^2)^(1/2)*a*b^2*c+4*A*(-4*a*c+b^2)^(1
/2)*b^4-64*A*a^2*b*c^2+32*A*a*b^3*c-4*A*b^5+12*C*(-4*a*c+b^2)^(1/2)*a^2*b*
c-2*C*(-4*a*c+b^2)^(1/2)*a*b^3+32*C*a^3*c^2-16*C*a^2*b^2*c+2*C*a*b^4)/c*ln
(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)+1/2*(16*B*(-4*a*c+b^2)^(1/2)*a^2*b*c-3*B*(-
4*a*c+b^2)^(1/2)*a*b^3+40*a^3*B*c^2-22*B*a^2*b^2*c+3*B*a*b^4)*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)))+1/(16*a*c-4*b^2)*(-1/4*(24*A*(-4*a*c+b^2)^(1/2)*a^2*c^2-24*A*(-4*
a*c+b^2)^(1/2)*a*b^2*c+4*A*(-4*a*c+b^2)^(1/2)*b^4+64*A*a^2*b*c^2-32*A*a*b^
3*c+4*A*b^5+12*C*(-4*a*c+b^2)^(1/2)*a^2*b*c-2*C*(-4*a*c+b^2)^(1/2)*a*b^3-3
2*C*a^3*c^2+16*C*a^2*b^2*c-2*C*a*b^4)/c*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)+
1/2*(16*B*(-4*a*c+b^2)^(1/2)*a^2*b*c-3*B*(-4*a*c+b^2)^(1/2)*a*b^3-40*a^3*B
*c^2+22*B*a^2*b^2*c-3*B*a*b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

3.36.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.36.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/x**3/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.36.7 Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{Cx^2+Bx+A}{\left(cx^4+bx^2+a\right)^2x^3}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((3*B*b^2*c - 10*B*a*c^2)*x^5 - (6*A*a*c^2 + (C*a*b - 2*A*b^2)*c)*x^4
 + A*a*b^2 - 4*A*a^2*c + (3*B*b^3 - 11*B*a*b*c)*x^3 - (C*a*b^2 - 2*A*b^3 -
 (2*C*a^2 - 7*A*a*b)*c)*x^2 + 2*(B*a*b^2 - 4*B*a^2*c)*x)/((a^2*b^2*c - 4*a
^3*c^2)*x^6 + (a^2*b^3 - 4*a^3*b*c)*x^4 + (a^3*b^2 - 4*a^4*c)*x^2) - 1/2*i
ntegrate((3*B*a*b^3 - 13*B*a^2*b*c - 2*(4*(C*a^2 - 2*A*a*b)*c^2 - (C*a*b^2
 - 2*A*b^3)*c)*x^3 + (3*B*a*b^2*c - 10*B*a^2*c^2)*x^2 + 2*(C*a*b^3 - 2*A*b
^4 - 6*A*a^2*c^2 - 5*(C*a^2*b - 2*A*a*b^2)*c)*x)/(c*x^4 + b*x^2 + a), x)/(
a^3*b^2 - 4*a^4*c) + (C*a - 2*A*b)*log(x)/a^3
```

3.36.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6939 vs. 2(470) = 940.

Time = 1.72 (sec) , antiderivative size = 6939, normalized size of antiderivative = 12.99

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/16*((a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)^2*(6*b^4*c^2 - 44*a*b^2*c^
3 + 80*a^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^
2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c -
40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)
*b^2*c^2 + 20*(b^2 - 4*a*c)*a*c^3)*B + 2*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^6*b^9*c - 49*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c
^2 - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c^2 - 6*a^6*b^9*c^2
 + 300*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^3 + 74*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^3 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^6*b^7*c^3 + 98*a^7*b^7*c^3 - 816*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^9*b^3*c^4 - 304*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8
*b^4*c^4 - 37*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^4 - 600*a^
8*b^5*c^4 + 832*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b*c^5 + 416*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^5 + 152*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^5 + 1632*a^9*b^3*c^5 - 208*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^9*b*c^6 - 1664*a^10*b*c^6 + 6*(b^2 - 4*a*c)...
```

3.36.9 Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 10595, normalized size of antiderivative = 19.84

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x)
```

output
```
symsum(log(root(1572864*a^11*b^2*c^5*z^4 - 983040*a^10*b^4*c^4*z^4 + 32768
0*a^9*b^6*c^3*z^4 - 61440*a^8*b^8*c^2*z^4 + 6144*a^7*b^10*c*z^4 - 1048576*
a^12*c^6*z^4 - 256*a^6*b^12*z^4 + 1572864*C*a^9*b^2*c^5*z^3 - 983040*C*a^8
*b^4*c^4*z^3 + 327680*C*a^7*b^6*c^3*z^3 - 61440*C*a^6*b^8*c^2*z^3 - 314572
8*A*a^8*b^3*c^5*z^3 + 1966080*A*a^7*b^5*c^4*z^3 - 655360*A*a^6*b^7*c^3*z^3
 + 122880*A*a^5*b^9*c^2*z^3 + 6144*C*a^5*b^10*c*z^3 + 2097152*A*a^9*b*c^6*
z^3 - 12288*A*a^4*b^11*c*z^3 - 1048576*C*a^10*c^6*z^3 - 256*C*a^4*b^12*z^3
 + 512*A*a^3*b^13*z^3 + 1277952*A*C*a^7*b*c^6*z^2 - 6144*A*C*a^2*b^11*c*z^
2 - 1794048*A*C*a^6*b^3*c^5*z^2 + 1062912*A*C*a^5*b^5*c^4*z^2 - 340480*A*C
*a^4*b^7*c^3*z^2 + 62208*A*C*a^3*b^9*c^2*z^2 + 256*A*C*a*b^13*z^2 + 1536*C
^2*a^3*b^10*c*z^2 - 430080*B^2*a^7*b*c^6*z^2 + 3408*B^2*a^2*b^11*c*z^2 + 6
144*A^2*a*b^12*c*z^2 + 516096*C^2*a^7*b^2*c^5*z^2 - 288768*C^2*a^6*b^4*c^4
*z^2 + 88576*C^2*a^5*b^6*c^3*z^2 - 15744*C^2*a^4*b^8*c^2*z^2 + 716800*B^2*
a^6*b^3*c^5*z^2 - 483840*B^2*a^5*b^5*c^4*z^2 + 170496*B^2*a^4*b^7*c^3*z^2
- 33232*B^2*a^3*b^9*c^2*z^2 + 1468416*A^2*a^5*b^4*c^5*z^2 - 966144*A^2*a^4
*b^6*c^4*z^2 - 761856*A^2*a^6*b^2*c^6*z^2 + 326656*A^2*a^3*b^8*c^3*z^2 - 6
1440*A^2*a^2*b^10*c^2*z^2 - 144*B^2*a*b^13*z^2 - 393216*C^2*a^8*c^6*z^2 -
64*C^2*a^2*b^12*z^2 - 294912*A^2*a^7*c^7*z^2 - 256*A^2*b^14*z^2 - 138240*B
^2*C*a^5*b*c^6*z - 432*B^2*C*a*b^9*c^2*z + 245760*A*C^2*a^5*b*c^6*z + 1228
8*A^2*C*a*b^8*c^3*z + 768*A*C^2*a*b^9*c^2*z + 576*A*B^2*a*b^8*c^3*z + 1...
```

3.36. $\int \frac{A+Bx+Cx^2}{x^3(a+bx^2+cx^4)^2}\,dx$

3.37 $\int (dx)^m (A+Bx+Cx^2)(a+bx^2+cx^4)^3\,dx$

3.37.1 Optimal result

Integrand size = 30, antiderivative size = 399

$$\begin{aligned}\int (dx)^m (A+Bx+Cx^2)(a+bx^2+cx^4)^3\,dx &= \frac{a^3A(dx)^{1+m}}{d(1+m)}+\frac{a^3B(dx)^{2+m}}{d^2(2+m)}+\frac{a^2(3Ab+aC)(dx)^{3+m}}{d^3(3+m)}+\frac{3a^2bB(dx)^{4+m}}{d^4(4+m)}\\ &+\frac{3a(A(b^2+ac)+abC)(dx)^{5+m}}{d^5(5+m)}+\frac{3aB(b^2+ac)(dx)^{6+m}}{d^6(6+m)}\\ &+\frac{(A(b^3+6abc)+3a(b^2+ac)C)(dx)^{7+m}}{d^7(7+m)}+\frac{bB(b^2+6ac)(dx)^{8+m}}{d^8(8+m)}\\ &+\frac{(3Ac(b^2+ac)+b(b^2+6ac)C)(dx)^{9+m}}{d^9(9+m)}+\frac{3Bc(b^2+ac)(dx)^{10+m}}{d^{10}(10+m)}\\ &+\frac{3c(Abc+(b^2+ac)C)(dx)^{11+m}}{d^{11}(11+m)}+\frac{3bBc^2(dx)^{12+m}}{d^{12}(12+m)}\\ &+\frac{c^2(Ac+3bC)(dx)^{13+m}}{d^{13}(13+m)}+\frac{Bc^3(dx)^{14+m}}{d^{14}(14+m)}+\frac{c^3C(dx)^{15+m}}{d^{15}(15+m)}\end{aligned}$$

output
```
a^3*A*(d*x)^(1+m)/d/(1+m)+a^3*B*(d*x)^(2+m)/d^2/(2+m)+a^2*(3*A*b+C*a)*(d*x
)^(3+m)/d^3/(3+m)+3*a^2*b*B*(d*x)^(4+m)/d^4/(4+m)+3*a*(A*(a*c+b^2)+a*b*C)*
(d*x)^(5+m)/d^5/(5+m)+3*a*B*(a*c+b^2)*(d*x)^(6+m)/d^6/(6+m)+(A*(6*a*b*c+b^
3)+3*a*(a*c+b^2)*C)*(d*x)^(7+m)/d^7/(7+m)+b*B*(6*a*c+b^2)*(d*x)^(8+m)/d^8/
(8+m)+(3*A*c*(a*c+b^2)+b*(6*a*c+b^2)*C)*(d*x)^(9+m)/d^9/(9+m)+3*B*c*(a*c+b
^2)*(d*x)^(10+m)/d^10/(10+m)+3*c*(A*b*c+(a*c+b^2)*C)*(d*x)^(11+m)/d^11/(11
+m)+3*b*B*c^2*(d*x)^(12+m)/d^12/(12+m)+c^2*(A*c+3*C*b)*(d*x)^(13+m)/d^13/(
13+m)+B*c^3*(d*x)^(14+m)/d^14/(14+m)+c^3*C*(d*x)^(15+m)/d^15/(15+m)
```

3.37.2 Mathematica [A] (verified)

Time = 2.10 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.74

$$\int (dx)^m (A+Bx+Cx^2) (a+bx^2+cx^4)^3 \, dx = x(dx)^m \left(\frac{a^3A}{1+m}+\frac{a^3Bx}{2+m}+\frac{a^2(3Ab+aC)x^2}{3+m}+\frac{3a^2bBx^3}{4+m}+\frac{3a(A(b^2+ac)+abC)x^4}{5+m} +\frac{3aB(b^2+ac)x^5}{6+m}+\frac{(A(b^3+6abc)+3a(b^2+ac)C)x^6}{7+m}+\frac{bB(b^2+6ac)x^7}{8+m} +\frac{(3Ac(b^2+ac)+b(b^2+6ac)C)x^8}{9+m}+\frac{3Bc(b^2+ac)x^9}{10+m}+\frac{3c(Abc+(b^2+ac)C)x^{10}}{11+m} +\frac{3bBc^2x^{11}}{12+m}+\frac{c^2(Ac+3bC)x^{12}}{13+m}+\frac{Bc^3x^{13}}{14+m}+\frac{c^3Cx^{14}}{15+m}\right)$$

input
```
Integrate[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

output
```
x*(d*x)^m*((a^3*A)/(1 + m) + (a^3*B*x)/(2 + m) + (a^2*(3*A*b + a*C)*x^2)/(
3 + m) + (3*a^2*b*B*x^3)/(4 + m) + (3*a*(A*(b^2 + a*c) + a*b*C)*x^4)/(5 +
m) + (3*a*B*(b^2 + a*c)*x^5)/(6 + m) + ((A*(b^3 + 6*a*b*c) + 3*a*(b^2 + a*
c)*C)*x^6)/(7 + m) + (b*B*(b^2 + 6*a*c)*x^7)/(8 + m) + ((3*A*c*(b^2 + a*c)
 + b*(b^2 + 6*a*c)*C)*x^8)/(9 + m) + (3*B*c*(b^2 + a*c)*x^9)/(10 + m) + (3
*c*(A*b*c + (b^2 + a*c)*C)*x^10)/(11 + m) + (3*b*B*c^2*x^11)/(12 + m) + (c
^2*(A*c + 3*b*C)*x^12)/(13 + m) + (B*c^3*x^13)/(14 + m) + (c^3*C*x^14)/(15
 + m))
```

3.37.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m (a+bx^2+cx^4)^3 (A+Bx+Cx^2) \, dx$$

$$\Big\downarrow \; 2159$$

$$\int \left(a^3A(dx)^m + \frac{a^3B(dx)^{m+1}}{d} + \frac{a^2(dx)^{m+2}(aC+3Ab)}{d^2} + \frac{3a^2bB(dx)^{m+3}}{d^3} + \frac{3c(dx)^{m+10}\left(C\left(ac+b^2\right)+Abc\right)}{d^{10}} + \right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3A(dx)^{m+1}}{d(m+1)} + \frac{a^3B(dx)^{m+2}}{d^2(m+2)} + \frac{a^2(dx)^{m+3}(aC+3Ab)}{d^3(m+3)} + \frac{3a^2bB(dx)^{m+4}}{d^4(m+4)} + \frac{3c(dx)^{m+11}\left(C\left(ac+b^2\right)+Abc\right)}{d^{11}(m+11)} + \frac{(dx)^{m+9}\left(3Ac\left(ac+b^2\right)+bC\left(6ac+b^2\right)\right)}{d^9(m+9)} + \frac{3a(dx)^{m+5}\left(A\left(ac+b^2\right)+abC\right)}{d^5(m+5)} + \frac{(dx)^{m+7}\left(A\left(6abc+b^3\right)+3aC\left(ac+b^2\right)\right)}{d^7(m+7)} + \frac{3Bc\left(ac+b^2\right)(dx)^{m+10}}{d^{10}(m+10)} + \frac{bB\left(6ac+b^2\right)(dx)^{m+8}}{d^8(m+8)} + \frac{3aB\left(ac+b^2\right)(dx)^{m+6}}{d^6(m+6)} + \frac{c^2(dx)^{m+13}(Ac+3bC)}{d^{13}(m+13)} + \frac{3bBc^2(dx)^{m+12}}{d^{12}(m+12)} + \frac{Bc^3(dx)^{m+14}}{d^{14}(m+14)} + \frac{c^3C(dx)^{m+15}}{d^{15}(m+15)}$$

input
```
Int[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

output
```
(a^3*A*(d*x)^(1 + m))/(d*(1 + m)) + (a^3*B*(d*x)^(2 + m))/(d^2*(2 + m)) +
(a^2*(3*A*b + a*C)*(d*x)^(3 + m))/(d^3*(3 + m)) + (3*a^2*b*B*(d*x)^(4 + m)
)/(d^4*(4 + m)) + (3*a*(A*(b^2 + a*c) + a*b*C)*(d*x)^(5 + m))/(d^5*(5 + m)
) + (3*a*B*(b^2 + a*c)*(d*x)^(6 + m))/(d^6*(6 + m)) + ((A*(b^3 + 6*a*b*c)
+ 3*a*(b^2 + a*c)*C)*(d*x)^(7 + m))/(d^7*(7 + m)) + (b*B*(b^2 + 6*a*c)*(d*
x)^(8 + m))/(d^8*(8 + m)) + ((3*A*c*(b^2 + a*c) + b*(b^2 + 6*a*c)*C)*(d*x)
^(9 + m))/(d^9*(9 + m)) + (3*B*c*(b^2 + a*c)*(d*x)^(10 + m))/(d^10*(10 + m
)) + (3*c*(A*b*c + (b^2 + a*c)*C)*(d*x)^(11 + m))/(d^11*(11 + m)) + (3*b*B
*c^2*(d*x)^(12 + m))/(d^12*(12 + m)) + (c^2*(A*c + 3*b*C)*(d*x)^(13 + m))/
(d^13*(13 + m)) + (B*c^3*(d*x)^(14 + m))/(d^14*(14 + m)) + (c^3*C*(d*x)^(1
5 + m))/(d^15*(15 + m))
```

3.37.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.37.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5519 vs. $2(399) = 798$.

Time = 0.41 (sec) , antiderivative size = 5520, normalized size of antiderivative = 13.83

method	result	size
gosper	Expression too large to display	5520
risch	Expression too large to display	5520
parallelrisch	Expression too large to display	7809

input
```
int((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.37.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3898 vs. $2(399) = 798$.

Time = 0.38 (sec) , antiderivative size = 3898, normalized size of antiderivative = 9.77

$$\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
((C*c^3*m^14 + 105*C*c^3*m^13 + 5005*C*c^3*m^12 + 143325*C*c^3*m^11 + 2749
747*C*c^3*m^10 + 37312275*C*c^3*m^9 + 368411615*C*c^3*m^8 + 2681453775*C*c
^3*m^7 + 14409322928*C*c^3*m^6 + 56663366760*C*c^3*m^5 + 159721605680*C*c^
3*m^4 + 310989260400*C*c^3*m^3 + 392156797824*C*c^3*m^2 + 283465647360*C*c
^3*m + 87178291200*C*c^3)*x^15 + (B*c^3*m^14 + 106*B*c^3*m^13 + 5096*B*c^3
*m^12 + 147056*B*c^3*m^11 + 2840838*B*c^3*m^10 + 38786748*B*c^3*m^9 + 3850
81268*B*c^3*m^8 + 2816490248*B*c^3*m^7 + 15200266081*B*c^3*m^6 + 599994855
46*B*c^3*m^5 + 169679309436*B*c^3*m^4 + 331303013496*B*c^3*m^3 + 418753514
880*B*c^3*m^2 + 303268406400*B*c^3*m + 93405312000*B*c^3)*x^14 + ((3*C*b*c
^2 + A*c^3)*m^14 + 107*(3*C*b*c^2 + A*c^3)*m^13 + 5189*(3*C*b*c^2 + A*c^3)
*m^12 + 150943*(3*C*b*c^2 + A*c^3)*m^11 + 2937363*(3*C*b*c^2 + A*c^3)*m^10
 + 40372761*(3*C*b*c^2 + A*c^3)*m^9 + 403249847*(3*C*b*c^2 + A*c^3)*m^8 +
2965379989*(3*C*b*c^2 + A*c^3)*m^7 + 16081189696*(3*C*b*c^2 + A*c^3)*m^6 +
 63747744632*(3*C*b*c^2 + A*c^3)*m^5 + 180951426864*(3*C*b*c^2 + A*c^3)*m^
4 + 301771008000*C*b*c^2 + 100590336000*A*c^3 + 354444796368*(3*C*b*c^2 +
A*c^3)*m^3 + 449213351040*(3*C*b*c^2 + A*c^3)*m^2 + 326044051200*(3*C*b*c^
2 + A*c^3)*m)*x^13 + 3*(B*b*c^2*m^14 + 108*B*b*c^2*m^13 + 5284*B*b*c^2*m^1
2 + 154992*B*b*c^2*m^11 + 3039718*B*b*c^2*m^10 + 42081864*B*b*c^2*m^9 + 42
3113372*B*b*c^2*m^8 + 3130267536*B*b*c^2*m^7 + 17067919121*B*b*c^2*m^6 + 6
7988181228*B*b*c^2*m^5 + 193813932344*B*b*c^2*m^4 + 381046157472*B*b*c^...
```

3.37.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47658 vs. $2(379) = 758$.

Time = 3.03 (sec) , antiderivative size = 47658, normalized size of antiderivative = 119.44

$$\int (dx)^m \left(A + Bx + Cx^2\right) \left(a + bx^2 + cx^4\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((d*x)**m*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**3,x)
```

output
```
Piecewise(((-A*a**3/(14*x**14) - A*a**2*b/(4*x**12) - 3*A*a**2*c/(10*x**10
) - 3*A*a*b**2/(10*x**10) - 3*A*a*b*c/(4*x**8) - A*a*c**2/(2*x**6) - A*b**
3/(8*x**8) - A*b**2*c/(2*x**6) - 3*A*b*c**2/(4*x**4) - A*c**3/(2*x**2) - B
*a**3/(13*x**13) - 3*B*a**2*b/(11*x**11) - B*a**2*c/(3*x**9) - B*a*b**2/(3
*x**9) - 6*B*a*b*c/(7*x**7) - 3*B*a*c**2/(5*x**5) - B*b**3/(7*x**7) - 3*B*
b**2*c/(5*x**5) - B*b*c**2/x**3 - B*c**3/x - C*a**3/(12*x**12) - 3*C*a**2*
b/(10*x**10) - 3*C*a**2*c/(8*x**8) - 3*C*a*b**2/(8*x**8) - C*a*b*c/x**6 -
3*C*a*c**2/(4*x**4) - C*b**3/(6*x**6) - 3*C*b**2*c/(4*x**4) - 3*C*b*c**2/(
2*x**2) + C*c**3*log(x))/d**15, Eq(m, -15)), ((-A*a**3/(13*x**13) - 3*A*a*
*2*b/(11*x**11) - A*a**2*c/(3*x**9) - A*a*b**2/(3*x**9) - 6*A*a*b*c/(7*x**
7) - 3*A*a*c**2/(5*x**5) - A*b**3/(7*x**7) - 3*A*b**2*c/(5*x**5) - A*b*c**
2/x**3 - A*c**3/x - B*a**3/(12*x**12) - 3*B*a**2*b/(10*x**10) - 3*B*a**2*c
/(8*x**8) - 3*B*a*b**2/(8*x**8) - B*a*b*c/x**6 - 3*B*a*c**2/(4*x**4) - B*b
**3/(6*x**6) - 3*B*b**2*c/(4*x**4) - 3*B*b*c**2/(2*x**2) + B*c**3*log(x) -
 C*a**3/(11*x**11) - C*a**2*b/(3*x**9) - 3*C*a**2*c/(7*x**7) - 3*C*a*b**2/
(7*x**7) - 6*C*a*b*c/(5*x**5) - C*a*c**2/x**3 - C*b**3/(5*x**5) - C*b**2*c
/x**3 - 3*C*b*c**2/x + C*c**3*x)/d**14, Eq(m, -14)), ((-A*a**3/(12*x**12)
- 3*A*a**2*b/(10*x**10) - 3*A*a**2*c/(8*x**8) - 3*A*a*b**2/(8*x**8) - A*a*
b*c/x**6 - 3*A*a*c**2/(4*x**4) - A*b**3/(6*x**6) - 3*A*b**2*c/(4*x**4) - 3
*A*b*c**2/(2*x**2) + A*c**3*log(x) - B*a**3/(11*x**11) - B*a**2*b/(3*x*...
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 611, normalized size of antiderivative = 1.53

$$\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^3 \, dx = \frac{Cc^3d^mx^{15}x^m}{m+15} + \frac{Bc^3d^mx^{14}x^m}{m+14} + \frac{3\,Cbc^2d^mx^{13}x^m}{m+13} + \frac{Ac^3d^mx^{13}x^m}{m+13} + \frac{3\,Bbc^2d^mx^{12}x^m}{m+12} + \frac{3\,Cb^2cd^mx^{11}x^m}{m+11} + \frac{3\,Cac^2d^mx^{11}x^m}{m+11} + \frac{3\,Abc^2d^mx^{11}x^m}{m+11} + \frac{3\,Bb^2cd^mx^{10}x^m}{m+10} + \frac{3\,Bac^2d^mx^{10}x^m}{m+10} + \frac{Cb^3d^mx^9x^m}{m+9} + \frac{6\,Cabcd^mx^9x^m}{m+9} + \frac{3\,Ab^2cd^mx^9x^m}{m+9} + \frac{3\,Aac^2d^mx^9x^m}{m+9} + \frac{Bb^3d^mx^8x^m}{m+8} + \frac{6\,Babcd^mx^8x^m}{m+8} + \frac{3\,Cab^2d^mx^7x^m}{m+7} + \frac{Ab^3d^mx^7x^m}{m+7} + \frac{3\,Ca^2cd^mx^7x^m}{m+7} + \frac{6\,Aabcd^mx^7x^m}{m+7} + \frac{3\,Bab^2d^mx^6x^m}{m+6} + \frac{3\,Ba^2cd^mx^6x^m}{m+6} + \frac{3\,Ca^2bd^mx^5x^m}{m+5} + \frac{3\,Aab^2d^mx^5x^m}{m+5} + \frac{3\,Aa^2cd^mx^5x^m}{m+5} + \frac{3\,Ba^2bd^mx^4x^m}{m+4} + \frac{Ca^3d^mx^3x^m}{m+3} + \frac{3\,Aa^2bd^mx^3x^m}{m+3} + \frac{Ba^3d^mx^2x^m}{m+2} + \frac{(dx)^{m+1}\,Aa^3}{d(m+1)}$$

input
```
integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
C*c^3*d^m*x^15*x^m/(m + 15) + B*c^3*d^m*x^14*x^m/(m + 14) + 3*C*b*c^2*d^m*
x^13*x^m/(m + 13) + A*c^3*d^m*x^13*x^m/(m + 13) + 3*B*b*c^2*d^m*x^12*x^m/(
m + 12) + 3*C*b^2*c*d^m*x^11*x^m/(m + 11) + 3*C*a*c^2*d^m*x^11*x^m/(m + 11
) + 3*A*b*c^2*d^m*x^11*x^m/(m + 11) + 3*B*b^2*c*d^m*x^10*x^m/(m + 10) + 3*
B*a*c^2*d^m*x^10*x^m/(m + 10) + C*b^3*d^m*x^9*x^m/(m + 9) + 6*C*a*b*c*d^m*
x^9*x^m/(m + 9) + 3*A*b^2*c*d^m*x^9*x^m/(m + 9) + 3*A*a*c^2*d^m*x^9*x^m/(m
 + 9) + B*b^3*d^m*x^8*x^m/(m + 8) + 6*B*a*b*c*d^m*x^8*x^m/(m + 8) + 3*C*a*
b^2*d^m*x^7*x^m/(m + 7) + A*b^3*d^m*x^7*x^m/(m + 7) + 3*C*a^2*c*d^m*x^7*x^
m/(m + 7) + 6*A*a*b*c*d^m*x^7*x^m/(m + 7) + 3*B*a*b^2*d^m*x^6*x^m/(m + 6)
+ 3*B*a^2*c*d^m*x^6*x^m/(m + 6) + 3*C*a^2*b*d^m*x^5*x^m/(m + 5) + 3*A*a*b^
2*d^m*x^5*x^m/(m + 5) + 3*A*a^2*c*d^m*x^5*x^m/(m + 5) + 3*B*a^2*b*d^m*x^4*
x^m/(m + 4) + C*a^3*d^m*x^3*x^m/(m + 3) + 3*A*a^2*b*d^m*x^3*x^m/(m + 3) +
B*a^3*d^m*x^2*x^m/(m + 2) + (d*x)^(m + 1)*A*a^3/(d*(m + 1))
```

3.37.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7808 vs. $2(399) = 798$.

Time = 0.40 (sec) , antiderivative size = 7808, normalized size of antiderivative = 19.57

$$\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
((d*x)^m*C*c^3*m^14*x^15 + (d*x)^m*B*c^3*m^14*x^14 + 105*(d*x)^m*C*c^3*m^1
3*x^15 + 3*(d*x)^m*C*b*c^2*m^14*x^13 + (d*x)^m*A*c^3*m^14*x^13 + 106*(d*x)
^m*B*c^3*m^13*x^14 + 5005*(d*x)^m*C*c^3*m^12*x^15 + 3*(d*x)^m*B*b*c^2*m^14
*x^12 + 321*(d*x)^m*C*b*c^2*m^13*x^13 + 107*(d*x)^m*A*c^3*m^13*x^13 + 5096
*(d*x)^m*B*c^3*m^12*x^14 + 143325*(d*x)^m*C*c^3*m^11*x^15 + 3*(d*x)^m*C*b^
2*c*m^14*x^11 + 3*(d*x)^m*C*a*c^2*m^14*x^11 + 3*(d*x)^m*A*b*c^2*m^14*x^11
+ 324*(d*x)^m*B*b*c^2*m^13*x^12 + 15567*(d*x)^m*C*b*c^2*m^12*x^13 + 5189*(
d*x)^m*A*c^3*m^12*x^13 + 147056*(d*x)^m*B*c^3*m^11*x^14 + 2749747*(d*x)^m*
C*c^3*m^10*x^15 + 3*(d*x)^m*B*b^2*c*m^14*x^10 + 3*(d*x)^m*B*a*c^2*m^14*x^1
0 + 327*(d*x)^m*C*b^2*c*m^13*x^11 + 327*(d*x)^m*C*a*c^2*m^13*x^11 + 327*(d
*x)^m*A*b*c^2*m^13*x^11 + 15852*(d*x)^m*B*b*c^2*m^12*x^12 + 452829*(d*x)^m
*C*b*c^2*m^11*x^13 + 150943*(d*x)^m*A*c^3*m^11*x^13 + 2840838*(d*x)^m*B*c^
3*m^10*x^14 + 37312275*(d*x)^m*C*c^3*m^9*x^15 + (d*x)^m*C*b^3*m^14*x^9 + 6
*(d*x)^m*C*a*b*c*m^14*x^9 + 3*(d*x)^m*A*b^2*c*m^14*x^9 + 3*(d*x)^m*A*a*c^2
*m^14*x^9 + 330*(d*x)^m*B*b^2*c*m^13*x^10 + 330*(d*x)^m*B*a*c^2*m^13*x^10
+ 16143*(d*x)^m*C*b^2*c*m^12*x^11 + 16143*(d*x)^m*C*a*c^2*m^12*x^11 + 1614
3*(d*x)^m*A*b*c^2*m^12*x^11 + 464976*(d*x)^m*B*b*c^2*m^11*x^12 + 8812089*(
d*x)^m*C*b*c^2*m^10*x^13 + 2937363*(d*x)^m*A*c^3*m^10*x^13 + 38786748*(d*x
)^m*B*c^3*m^9*x^14 + 368411615*(d*x)^m*C*c^3*m^8*x^15 + (d*x)^m*B*b^3*m^14
*x^8 + 6*(d*x)^m*B*a*b*c*m^14*x^8 + 111*(d*x)^m*C*b^3*m^13*x^9 + 666*(d...
```

3.37.9 Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 2443, normalized size of antiderivative = 6.12

$$\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^3 \, dx = \text{Too large to display}$$

input
```
int((d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^3,x)
```

output
```
(x^7*(d*x)^m*(A*b^3 + 3*C*a*b^2 + 3*C*a^2*c + 6*A*a*b*c)*(593193196800*m +
 796089202560*m^2 + 608700928752*m^3 + 299730345264*m^4 + 101420251688*m^5
 + 24483279856*m^6 + 4306835671*m^7 + 557256047*m^8 + 52977099*m^9 + 36544
83*m^10 + 177877*m^11 + 5789*m^12 + 113*m^13 + m^14 + 186810624000))/(4339
163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 2706813345600*m^4 +
1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 8207628000*m^8 +
928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*m^13 + 1
20*m^14 + m^15 + 1307674368000) + (x^9*(d*x)^m*(C*b^3 + 3*A*a*c^2 + 3*A*b^
2*c + 6*C*a*b*c)*(465985094400*m + 633314724480*m^2 + 491520108816*m^3 + 2
46143692976*m^4 + 84836490456*m^5 + 20885191136*m^6 + 3749548713*m^7 + 495
342143*m^8 + 48083733*m^9 + 3386083*m^10 + 168171*m^11 + 5581*m^12 + 111*m
^13 + m^14 + 145297152000))/(4339163001600*m + 6165817614720*m^2 + 5056995
703824*m^3 + 2706813345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54
631129553*m^7 + 8207628000*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m
^11 + 218400*m^12 + 6580*m^13 + 120*m^14 + m^15 + 1307674368000) + (B*c^3*
x^14*(d*x)^m*(303268406400*m + 418753514880*m^2 + 331303013496*m^3 + 16967
9309436*m^4 + 59999485546*m^5 + 15200266081*m^6 + 2816490248*m^7 + 3850812
68*m^8 + 38786748*m^9 + 2840838*m^10 + 147056*m^11 + 5096*m^12 + 106*m^13
+ m^14 + 93405312000))/(4339163001600*m + 6165817614720*m^2 + 505699570382
4*m^3 + 2706813345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 5463...
```

3.37. $\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^3 \, dx$

3.38 $\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^2 \, dx$

3.38.1 Optimal result

Integrand size = 30, antiderivative size = 260

$$\begin{aligned}&\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^2 \, dx\\
&=\frac{a^2A(dx)^{1+m}}{d(1+m)}+\frac{a^2B(dx)^{2+m}}{d^2(2+m)}+\frac{a(2Ab+aC)(dx)^{3+m}}{d^3(3+m)}+\frac{2abB(dx)^{4+m}}{d^4(4+m)}\\
&+\frac{\left(A(b^2+2ac)+2abC\right)(dx)^{5+m}}{d^5(5+m)}+\frac{B\left(b^2+2ac\right)(dx)^{6+m}}{d^6(6+m)}\\
&+\frac{\left(2Abc+\left(b^2+2ac\right)C\right)(dx)^{7+m}}{d^7(7+m)}+\frac{2bBc(dx)^{8+m}}{d^8(8+m)}\\
&+\frac{c(Ac+2bC)(dx)^{9+m}}{d^9(9+m)}+\frac{Bc^2(dx)^{10+m}}{d^{10}(10+m)}+\frac{c^2C(dx)^{11+m}}{d^{11}(11+m)}\end{aligned}$$

output
```
a^2*A*(d*x)^(1+m)/d/(1+m)+a^2*B*(d*x)^(2+m)/d^2/(2+m)+a*(2*A*b+C*a)*(d*x)^
(3+m)/d^3/(3+m)+2*a*b*B*(d*x)^(4+m)/d^4/(4+m)+(A*(2*a*c+b^2)+2*a*b*C)*(d*x
)^(5+m)/d^5/(5+m)+B*(2*a*c+b^2)*(d*x)^(6+m)/d^6/(6+m)+(2*A*b*c+(2*a*c+b^2)
*C)*(d*x)^(7+m)/d^7/(7+m)+2*b*B*c*(d*x)^(8+m)/d^8/(8+m)+c*(A*c+2*C*b)*(d*x
)^(9+m)/d^9/(9+m)+B*c^2*(d*x)^(10+m)/d^10/(10+m)+c^2*C*(d*x)^(11+m)/d^11/(
11+m)
```

3.38.2 Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.71

$$\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^2 \, dx$$
$$= x(dx)^m \left(\frac{a^2A}{1+m}+\frac{a^2Bx}{2+m}+\frac{a(2Ab+aC)x^2}{3+m}+\frac{2abBx^3}{4+m}+\frac{(A(b^2+2ac)+2abC)\,x^4}{5+m}\right.$$
$$+\frac{B(b^2+2ac)\,x^5}{6+m}+\frac{(2Abc+(b^2+2ac)\,C)\,x^6}{7+m}+\frac{2bBcx^7}{8+m}+\frac{c(Ac+2bC)x^8}{9+m}+\frac{Bc^2x^9}{10+m}$$
$$\left.+\frac{c^2Cx^{10}}{11+m}\right)$$

input
```
Integrate[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

output
```
x*(d*x)^m*((a^2*A)/(1 + m) + (a^2*B*x)/(2 + m) + (a*(2*A*b + a*C)*x^2)/(3
+ m) + (2*a*b*B*x^3)/(4 + m) + ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^4)/(5 + m) +
 (B*(b^2 + 2*a*c)*x^5)/(6 + m) + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^6)/(7 + m)
 + (2*b*B*c*x^7)/(8 + m) + (c*(A*c + 2*b*C)*x^8)/(9 + m) + (B*c^2*x^9)/(10
 + m) + (c^2*C*x^10)/(11 + m))
```

3.38.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+bx^2+cx^4\right)^2 \left(A+Bx+Cx^2\right) \, dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(a^2A(dx)^m+\frac{a^2B(dx)^{m+1}}{d}+\frac{(dx)^{m+6}\left(C\left(2ac+b^2\right)+2Abc\right)}{d^6}+\frac{(dx)^{m+4}\left(A\left(2ac+b^2\right)+2abC\right)}{d^4}+\frac{a(dx)^{m+2}(\cdots}{\cdots}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2A(dx)^{m+1}}{d(m+1)} + \frac{a^2B(dx)^{m+2}}{d^2(m+2)} + \frac{(dx)^{m+7}\left(C\left(2ac+b^2\right)+2Abc\right)}{d^7(m+7)} + \frac{(dx)^{m+5}\left(A\left(2ac+b^2\right)+2abC\right)}{d^5(m+5)} + \frac{a(dx)^{m+3}(aC+2Ab)}{d^3(m+3)} + \frac{B\left(2ac+b^2\right)(dx)^{m+6}}{d^6(m+6)} + \frac{2abB(dx)^{m+4}}{d^4(m+4)} + \frac{c(dx)^{m+9}(Ac+2bC)}{d^9(m+9)} + \frac{2bBc(dx)^{m+8}}{d^8(m+8)} + \frac{Bc^2(dx)^{m+10}}{d^{10}(m+10)} + \frac{c^2C(dx)^{m+11}}{d^{11}(m+11)}$$

input
```
Int[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

output
```
(a^2*A*(d*x)^(1 + m))/(d*(1 + m)) + (a^2*B*(d*x)^(2 + m))/(d^2*(2 + m)) +
(a*(2*A*b + a*C)*(d*x)^(3 + m))/(d^3*(3 + m)) + (2*a*b*B*(d*x)^(4 + m))/(d
^4*(4 + m)) + ((A*(b^2 + 2*a*c) + 2*a*b*C)*(d*x)^(5 + m))/(d^5*(5 + m)) +
(B*(b^2 + 2*a*c)*(d*x)^(6 + m))/(d^6*(6 + m)) + ((2*A*b*c + (b^2 + 2*a*c)*
C)*(d*x)^(7 + m))/(d^7*(7 + m)) + (2*b*B*c*(d*x)^(8 + m))/(d^8*(8 + m)) +
(c*(A*c + 2*b*C)*(d*x)^(9 + m))/(d^9*(9 + m)) + (B*c^2*(d*x)^(10 + m))/(d^
10*(10 + m)) + (c^2*C*(d*x)^(11 + m))/(d^11*(11 + m))
```

3.38.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.38.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2186 vs. $2(260) = 520$.

Time = 0.18 (sec) , antiderivative size = 2187, normalized size of antiderivative = 8.41

method	result	size
gosper	Expression too large to display	2187
risch	Expression too large to display	2187
parallelrisch	Expression too large to display	3204

input
```
int((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x*(C*c^2*m^10*x^10+B*c^2*m^10*x^9+55*C*c^2*m^9*x^10+A*c^2*m^10*x^8+56*B*c^
2*m^9*x^9+2*C*b*c*m^10*x^8+1320*C*c^2*m^8*x^10+57*A*c^2*m^9*x^8+2*B*b*c*m^
10*x^7+1365*B*c^2*m^8*x^9+114*C*b*c*m^9*x^8+18150*C*c^2*m^7*x^10+2*A*b*c*m
^10*x^6+1412*A*c^2*m^8*x^8+116*B*b*c*m^9*x^7+19020*B*c^2*m^7*x^9+2*C*a*c*m
^10*x^6+C*b^2*m^10*x^6+2824*C*b*c*m^8*x^8+157773*C*c^2*m^6*x^10+118*A*b*c*
m^9*x^6+19962*A*c^2*m^7*x^8+2*B*a*c*m^10*x^5+B*b^2*m^10*x^5+2922*B*b*c*m^8
*x^7+167223*B*c^2*m^6*x^9+118*C*a*c*m^9*x^6+59*C*b^2*m^9*x^6+39924*C*b*c*m
^7*x^8+902055*C*c^2*m^5*x^10+2*A*a*c*m^10*x^4+A*b^2*m^10*x^4+3024*A*b*c*m^
8*x^6+177765*A*c^2*m^6*x^8+120*B*a*c*m^9*x^5+60*B*b^2*m^9*x^5+41964*B*b*c*
m^7*x^7+965328*B*c^2*m^5*x^9+2*C*a*b*m^10*x^4+3024*C*a*c*m^8*x^6+1512*C*b^
2*m^8*x^6+355530*C*b*c*m^6*x^8+3416930*C*c^2*m^4*x^10+122*A*a*c*m^9*x^4+61
*A*b^2*m^9*x^4+44172*A*b*c*m^7*x^6+1037673*A*c^2*m^5*x^8+2*B*a*b*m^10*x^3+
3130*B*a*c*m^8*x^5+1565*B*b^2*m^8*x^5+379134*B*b*c*m^6*x^7+3686255*B*c^2*m
^4*x^9+122*C*a*b*m^9*x^4+44172*C*a*c*m^7*x^6+22086*C*b^2*m^7*x^6+2075346*C
*b*c*m^5*x^8+8409500*C*c^2*m^3*x^10+2*A*a*b*m^10*x^2+3240*A*a*c*m^8*x^4+16
20*A*b^2*m^8*x^4+405642*A*b*c*m^6*x^6+4000478*A*c^2*m^4*x^8+124*B*a*b*m^9*
x^3+46560*B*a*c*m^7*x^5+23280*B*b^2*m^7*x^5+2242044*B*b*c*m^5*x^7+9133180*
B*c^2*m^3*x^9+C*a^2*m^10*x^2+3240*C*a*b*m^8*x^4+405642*C*a*c*m^6*x^6+20282
1*C*b^2*m^6*x^6+8000956*C*b*c*m^4*x^8+12753576*C*c^2*m^2*x^10+126*A*a*b*m^
9*x^2+49140*A*a*c*m^7*x^4+24570*A*b^2*m^7*x^4+2435622*A*b*c*m^5*x^6+999...
```

3.38.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1603 vs. $2(260) = 520$.

Time = 0.33 (sec) , antiderivative size = 1603, normalized size of antiderivative = 6.17

$$\int (dx)^m \left(A + Bx + Cx^2\right) \left(a + bx^2 + cx^4\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
((C*c^2*m^10 + 55*C*c^2*m^9 + 1320*C*c^2*m^8 + 18150*C*c^2*m^7 + 157773*C*
c^2*m^6 + 902055*C*c^2*m^5 + 3416930*C*c^2*m^4 + 8409500*C*c^2*m^3 + 12753
576*C*c^2*m^2 + 10628640*C*c^2*m + 3628800*C*c^2)*x^11 + (B*c^2*m^10 + 56*
B*c^2*m^9 + 1365*B*c^2*m^8 + 19020*B*c^2*m^7 + 167223*B*c^2*m^6 + 965328*B
*c^2*m^5 + 3686255*B*c^2*m^4 + 9133180*B*c^2*m^3 + 13926276*B*c^2*m^2 + 11
655216*B*c^2*m + 3991680*B*c^2)*x^10 + ((2*C*b*c + A*c^2)*m^10 + 57*(2*C*b
*c + A*c^2)*m^9 + 1412*(2*C*b*c + A*c^2)*m^8 + 19962*(2*C*b*c + A*c^2)*m^7
 + 177765*(2*C*b*c + A*c^2)*m^6 + 1037673*(2*C*b*c + A*c^2)*m^5 + 4000478*
(2*C*b*c + A*c^2)*m^4 + 9991428*(2*C*b*c + A*c^2)*m^3 + 8870400*C*b*c + 44
35200*A*c^2 + 15335224*(2*C*b*c + A*c^2)*m^2 + 12900960*(2*C*b*c + A*c^2)*
m)*x^9 + 2*(B*b*c*m^10 + 58*B*b*c*m^9 + 1461*B*b*c*m^8 + 20982*B*b*c*m^7 +
 189567*B*b*c*m^6 + 1121022*B*b*c*m^5 + 4371359*B*b*c*m^4 + 11024858*B*b*c
*m^3 + 17059212*B*b*c*m^2 + 14444280*B*b*c*m + 4989600*B*b*c)*x^8 + ((C*b^
2 + 2*(C*a + A*b)*c)*m^10 + 59*(C*b^2 + 2*(C*a + A*b)*c)*m^9 + 1512*(C*b^2
 + 2*(C*a + A*b)*c)*m^8 + 22086*(C*b^2 + 2*(C*a + A*b)*c)*m^7 + 202821*(C*
b^2 + 2*(C*a + A*b)*c)*m^6 + 1217811*(C*b^2 + 2*(C*a + A*b)*c)*m^5 + 48148
58*(C*b^2 + 2*(C*a + A*b)*c)*m^4 + 12291724*(C*b^2 + 2*(C*a + A*b)*c)*m^3
+ 5702400*C*b^2 + 19216008*(C*b^2 + 2*(C*a + A*b)*c)*m^2 + 11404800*(C*a +
 A*b)*c + 16405920*(C*b^2 + 2*(C*a + A*b)*c)*m)*x^7 + ((B*b^2 + 2*B*a*c)*m
^10 + 60*(B*b^2 + 2*B*a*c)*m^9 + 1565*(B*b^2 + 2*B*a*c)*m^8 + 23280*(B*...
```

3.38.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16323 vs. $2(245) = 490$.

Time = 1.42 (sec) , antiderivative size = 16323, normalized size of antiderivative = 62.78

$$\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((d*x)**m*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)
```

output
```
Piecewise(((-A*a**2/(10*x**10) - A*a*b/(4*x**8) - A*a*c/(3*x**6) - A*b**2/
(6*x**6) - A*b*c/(2*x**4) - A*c**2/(2*x**2) - B*a**2/(9*x**9) - 2*B*a*b/(7
*x**7) - 2*B*a*c/(5*x**5) - B*b**2/(5*x**5) - 2*B*b*c/(3*x**3) - B*c**2/x
- C*a**2/(8*x**8) - C*a*b/(3*x**6) - C*a*c/(2*x**4) - C*b**2/(4*x**4) - C*
b*c/x**2 + C*c**2*log(x))/d**11, Eq(m, -11)), ((-A*a**2/(9*x**9) - 2*A*a*b
/(7*x**7) - 2*A*a*c/(5*x**5) - A*b**2/(5*x**5) - 2*A*b*c/(3*x**3) - A*c**2
/x - B*a**2/(8*x**8) - B*a*b/(3*x**6) - B*a*c/(2*x**4) - B*b**2/(4*x**4) -
 B*b*c/x**2 + B*c**2*log(x) - C*a**2/(7*x**7) - 2*C*a*b/(5*x**5) - 2*C*a*c
/(3*x**3) - C*b**2/(3*x**3) - 2*C*b*c/x + C*c**2*x)/d**10, Eq(m, -10)), ((
-A*a**2/(8*x**8) - A*a*b/(3*x**6) - A*a*c/(2*x**4) - A*b**2/(4*x**4) - A*b
*c/x**2 + A*c**2*log(x) - B*a**2/(7*x**7) - 2*B*a*b/(5*x**5) - 2*B*a*c/(3*
x**3) - B*b**2/(3*x**3) - 2*B*b*c/x + B*c**2*x - C*a**2/(6*x**6) - C*a*b/(
2*x**4) - C*a*c/x**2 - C*b**2/(2*x**2) + 2*C*b*c*log(x) + C*c**2*x**2/2)/d
**9, Eq(m, -9)), ((-A*a**2/(7*x**7) - 2*A*a*b/(5*x**5) - 2*A*a*c/(3*x**3)
- A*b**2/(3*x**3) - 2*A*b*c/x + A*c**2*x - B*a**2/(6*x**6) - B*a*b/(2*x**4
) - B*a*c/x**2 - B*b**2/(2*x**2) + 2*B*b*c*log(x) + B*c**2*x**2/2 - C*a**2
/(5*x**5) - 2*C*a*b/(3*x**3) - 2*C*a*c/x - C*b**2/x + 2*C*b*c*x + C*c**2*x
**3/3)/d**8, Eq(m, -8)), ((-A*a**2/(6*x**6) - A*a*b/(2*x**4) - A*a*c/x**2
- A*b**2/(2*x**2) + 2*A*b*c*log(x) + A*c**2*x**2/2 - B*a**2/(5*x**5) - 2*B
*a*b/(3*x**3) - 2*B*a*c/x - B*b**2/x + 2*B*b*c*x + B*c**2*x**3/3 - C*a*...
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.32

$$\int (dx)^m \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx = \frac{Cc^2d^mx^{11}x^m}{m+11}+\frac{Bc^2d^mx^{10}x^m}{m+10}+\frac{2\,Cbcd^mx^9x^m}{m+9}+\frac{Ac^2d^mx^9x^m}{m+9}+\frac{2\,Bbcd^mx^8x^m}{m+8} + \frac{Cb^2d^mx^7x^m}{m+7}+\frac{2\,Cacd^mx^7x^m}{m+7}+\frac{2\,Abcd^mx^7x^m}{m+7}+\frac{Bb^2d^mx^6x^m}{m+6} + \frac{2\,Bacd^mx^6x^m}{m+6}+\frac{2\,Cabd^mx^5x^m}{m+5}+\frac{Ab^2d^mx^5x^m}{m+5}+\frac{2\,Aacd^mx^5x^m}{m+5} + \frac{2\,Babd^mx^4x^m}{m+4}+\frac{Ca^2d^mx^3x^m}{m+3}+\frac{2\,Aabd^mx^3x^m}{m+3}+\frac{Ba^2d^mx^2x^m}{m+2}+\frac{(dx)^{m+1}Aa^2}{d(m+1)}$$

input
```
integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
C*c^2*d^m*x^11*x^m/(m + 11) + B*c^2*d^m*x^10*x^m/(m + 10) + 2*C*b*c*d^m*x^
9*x^m/(m + 9) + A*c^2*d^m*x^9*x^m/(m + 9) + 2*B*b*c*d^m*x^8*x^m/(m + 8) +
C*b^2*d^m*x^7*x^m/(m + 7) + 2*C*a*c*d^m*x^7*x^m/(m + 7) + 2*A*b*c*d^m*x^7*
x^m/(m + 7) + B*b^2*d^m*x^6*x^m/(m + 6) + 2*B*a*c*d^m*x^6*x^m/(m + 6) + 2*
C*a*b*d^m*x^5*x^m/(m + 5) + A*b^2*d^m*x^5*x^m/(m + 5) + 2*A*a*c*d^m*x^5*x^
m/(m + 5) + 2*B*a*b*d^m*x^4*x^m/(m + 4) + C*a^2*d^m*x^3*x^m/(m + 3) + 2*A*
a*b*d^m*x^3*x^m/(m + 3) + B*a^2*d^m*x^2*x^m/(m + 2) + (d*x)^(m + 1)*A*a^2/
(d*(m + 1))
```

3.38.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3203 vs. $2(260) = 520$.

Time = 0.37 (sec) , antiderivative size = 3203, normalized size of antiderivative = 12.32

$$\int (dx)^m (A + Bx + Cx^2) (a + bx^2 + cx^4)^2 \, dx = \text{Too large to display}$$

input
```
integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
((d*x)^m*C*c^2*m^10*x^11 + (d*x)^m*B*c^2*m^10*x^10 + 55*(d*x)^m*C*c^2*m^9*
x^11 + 2*(d*x)^m*C*b*c*m^10*x^9 + (d*x)^m*A*c^2*m^10*x^9 + 56*(d*x)^m*B*c^
2*m^9*x^10 + 1320*(d*x)^m*C*c^2*m^8*x^11 + 2*(d*x)^m*B*b*c*m^10*x^8 + 114*
(d*x)^m*C*b*c*m^9*x^9 + 57*(d*x)^m*A*c^2*m^9*x^9 + 1365*(d*x)^m*B*c^2*m^8*
x^10 + 18150*(d*x)^m*C*c^2*m^7*x^11 + (d*x)^m*C*b^2*m^10*x^7 + 2*(d*x)^m*C
*a*c*m^10*x^7 + 2*(d*x)^m*A*b*c*m^10*x^7 + 116*(d*x)^m*B*b*c*m^9*x^8 + 282
4*(d*x)^m*C*b*c*m^8*x^9 + 1412*(d*x)^m*A*c^2*m^8*x^9 + 19020*(d*x)^m*B*c^2
*m^7*x^10 + 157773*(d*x)^m*C*c^2*m^6*x^11 + (d*x)^m*B*b^2*m^10*x^6 + 2*(d*
x)^m*B*a*c*m^10*x^6 + 59*(d*x)^m*C*b^2*m^9*x^7 + 118*(d*x)^m*C*a*c*m^9*x^7
 + 118*(d*x)^m*A*b*c*m^9*x^7 + 2922*(d*x)^m*B*b*c*m^8*x^8 + 39924*(d*x)^m*
C*b*c*m^7*x^9 + 19962*(d*x)^m*A*c^2*m^7*x^9 + 167223*(d*x)^m*B*c^2*m^6*x^1
0 + 902055*(d*x)^m*C*c^2*m^5*x^11 + 2*(d*x)^m*C*a*b*m^10*x^5 + (d*x)^m*A*b
^2*m^10*x^5 + 2*(d*x)^m*A*a*c*m^10*x^5 + 60*(d*x)^m*B*b^2*m^9*x^6 + 120*(d
*x)^m*B*a*c*m^9*x^6 + 1512*(d*x)^m*C*b^2*m^8*x^7 + 3024*(d*x)^m*C*a*c*m^8*
x^7 + 3024*(d*x)^m*A*b*c*m^8*x^7 + 41964*(d*x)^m*B*b*c*m^7*x^8 + 355530*(d
*x)^m*C*b*c*m^6*x^9 + 177765*(d*x)^m*A*c^2*m^6*x^9 + 965328*(d*x)^m*B*c^2*
m^5*x^10 + 3416930*(d*x)^m*C*c^2*m^4*x^11 + 2*(d*x)^m*B*a*b*m^10*x^4 + 122
*(d*x)^m*C*a*b*m^9*x^5 + 61*(d*x)^m*A*b^2*m^9*x^5 + 122*(d*x)^m*A*a*c*m^9*
x^5 + 1565*(d*x)^m*B*b^2*m^8*x^6 + 3130*(d*x)^m*B*a*c*m^8*x^6 + 22086*(d*x
)^m*C*b^2*m^7*x^7 + 44172*(d*x)^m*C*a*c*m^7*x^7 + 44172*(d*x)^m*A*b*c*m...
```

3.38.9 Mupad [B] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 1314, normalized size of antiderivative = 5.05

$$\int (dx)^m (A + Bx + Cx^2) (a + bx^2 + cx^4)^2 \, dx = \text{Too large to display}$$

input
```
int((d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x)
```

output
```
(x^5*(d*x)^m*(A*b^2 + 2*A*a*c + 2*C*a*b)*(22512096*m + 25681176*m^2 + 1591
5380*m^3 + 6016070*m^4 + 1464693*m^5 + 234573*m^6 + 24570*m^7 + 1620*m^8 +
 61*m^9 + m^10 + 7983360))/(120543840*m + 150917976*m^2 + 105258076*m^3 +
45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*
m^9 + 66*m^10 + m^11 + 39916800) + (x^7*(d*x)^m*(C*b^2 + 2*A*b*c + 2*C*a*c
)*(16405920*m + 19216008*m^2 + 12291724*m^3 + 4814858*m^4 + 1217811*m^5 +
202821*m^6 + 22086*m^7 + 1512*m^8 + 59*m^9 + m^10 + 5702400))/(120543840*m
 + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m
^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800) + (B*x
^6*(d*x)^m*(2*a*c + b^2)*(18981840*m + 21989356*m^2 + 13878120*m^3 + 53529
35*m^4 + 1331100*m^5 + 217743*m^6 + 23280*m^7 + 1565*m^8 + 60*m^9 + m^10 +
 6652800))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 1
3339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 +
m^11 + 39916800) + (A*a^2*x*(d*x)^m*(80627040*m + 70290936*m^2 + 34967140*
m^3 + 11028590*m^4 + 2310945*m^5 + 326613*m^6 + 30810*m^7 + 1860*m^8 + 65*
m^9 + m^10 + 39916800))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 459
95730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9
 + 66*m^10 + m^11 + 39916800) + (c*x^9*(d*x)^m*(A*c + 2*C*b)*(12900960*m +
 15335224*m^2 + 9991428*m^3 + 4000478*m^4 + 1037673*m^5 + 177765*m^6 + 199
62*m^7 + 1412*m^8 + 57*m^9 + m^10 + 4435200))/(120543840*m + 150917976*...
```

3.39 $\int (dx)^m (A+Bx+Cx^2)(a+bx^2+cx^4)\,dx$

3.39.1 Optimal result

Integrand size = 28, antiderivative size = 137

$$\int (dx)^m (A+Bx+Cx^2)(a+bx^2+cx^4)\,dx = \frac{aA(dx)^{1+m}}{d(1+m)}+\frac{aB(dx)^{2+m}}{d^2(2+m)}+\frac{(Ab+aC)(dx)^{3+m}}{d^3(3+m)} + \frac{bB(dx)^{4+m}}{d^4(4+m)}+\frac{(Ac+bC)(dx)^{5+m}}{d^5(5+m)} + \frac{Bc(dx)^{6+m}}{d^6(6+m)}+\frac{cC(dx)^{7+m}}{d^7(7+m)}$$

output
```
a*A*(d*x)^(1+m)/d/(1+m)+a*B*(d*x)^(2+m)/d^2/(2+m)+(A*b+C*a)*(d*x)^(3+m)/d^
3/(3+m)+b*B*(d*x)^(4+m)/d^4/(4+m)+(A*c+C*b)*(d*x)^(5+m)/d^5/(5+m)+B*c*(d*x
)^(6+m)/d^6/(6+m)+c*C*(d*x)^(7+m)/d^7/(7+m)
```

3.39.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.66

$$\int (dx)^m (A+Bx+Cx^2)(a+bx^2+cx^4)\,dx = x(dx)^m\left(\frac{aA}{1+m}+\frac{aBx}{2+m}+\frac{(Ab+aC)x^2}{3+m} + \frac{bBx^3}{4+m}+\frac{(Ac+bC)x^4}{5+m}+\frac{Bcx^5}{6+m}+\frac{cCx^6}{7+m}\right)$$

input
```
Integrate[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

output
```
x*(d*x)^m*((a*A)/(1 + m) + (a*B*x)/(2 + m) + ((A*b + a*C)*x^2)/(3 + m) + (
b*B*x^3)/(4 + m) + ((A*c + b*C)*x^4)/(5 + m) + (B*c*x^5)/(6 + m) + (c*C*x^
6)/(7 + m))
```

3.39.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+bx^2+cx^4\right) \left(A+Bx+Cx^2\right) \, dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{(dx)^{m+2}(aC+Ab)}{d^2}+aA(dx)^m+\frac{aB(dx)^{m+1}}{d}+\frac{(dx)^{m+4}(Ac+bC)}{d^4}+\frac{bB(dx)^{m+3}}{d^3}+\frac{Bc(dx)^{m+5}}{d^5}+\frac{cC(dx)^{m+6}}{d^6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(dx)^{m+3}(aC+Ab)}{d^3(m+3)}+\frac{aA(dx)^{m+1}}{d(m+1)}+\frac{aB(dx)^{m+2}}{d^2(m+2)}+\frac{(dx)^{m+5}(Ac+bC)}{d^5(m+5)}+\frac{bB(dx)^{m+4}}{d^4(m+4)}+\frac{Bc(dx)^{m+6}}{d^6(m+6)}+\frac{cC(dx)^{m+7}}{d^7(m+7)}$$

input
```
Int[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

output
```
(a*A*(d*x)^(1 + m))/(d*(1 + m)) + (a*B*(d*x)^(2 + m))/(d^2*(2 + m)) + ((A*
b + a*C)*(d*x)^(3 + m))/(d^3*(3 + m)) + (b*B*(d*x)^(4 + m))/(d^4*(4 + m))
+ ((A*c + b*C)*(d*x)^(5 + m))/(d^5*(5 + m)) + (B*c*(d*x)^(6 + m))/(d^6*(6
+ m)) + (c*C*(d*x)^(7 + m))/(d^7*(7 + m))
```

3.39.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.39.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99

method	result
norman	$\frac{(Ab+Ca)x^3 e^{m\ln(dx)}}{3+m}+\frac{(Ac+Cb)x^5 e^{m\ln(dx)}}{5+m}+\frac{Aax\, e^{m\ln(dx)}}{1+m}+\frac{Ba\, x^2 e^{m\ln(dx)}}{2+m}+\frac{Bb\, x^4 e^{m\ln(dx)}}{4+m}+\frac{Bc\, x^6 e^{m\ln(dx)}}{6+m}+$
gosper	$\frac{x(Cc\, m^6x^6+Bc\, m^6x^5+21Cc\, m^5x^6+Ac\, m^6x^4+22Bc\, m^5x^5+Cb\, m^6x^4+175Cc\, m^4x^6+23Ac\, m^5x^4+Bb\, m^6x^3+190Bc\, m^4x^5+2}{}$
risch	$\frac{x(Cc\, m^6x^6+Bc\, m^6x^5+21Cc\, m^5x^6+Ac\, m^6x^4+22Bc\, m^5x^5+Cb\, m^6x^4+175Cc\, m^4x^6+23Ac\, m^5x^4+Bb\, m^6x^3+190Bc\, m^4x^5+2}{}$
parallelrisch	$\frac{720C\, x^7(dx)^m c+840B\, x^6(dx)^m c+1008A\, x^5(dx)^m c+1008C\, x^5(dx)^m b+1260B\, x^4(dx)^m b+1680A\, x^3(dx)^m b+1680C\, x^3(dx)^m}{}$

input
```
int((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
(A*b+C*a)/(3+m)*x^3*exp(m*ln(d*x))+(A*c+C*b)/(5+m)*x^5*exp(m*ln(d*x))+A*a/
(1+m)*x*exp(m*ln(d*x))+B*a/(2+m)*x^2*exp(m*ln(d*x))+B*b/(4+m)*x^4*exp(m*ln
(d*x))+B*c/(6+m)*x^6*exp(m*ln(d*x))+C*c/(7+m)*x^7*exp(m*ln(d*x))
```

3.39.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(137) = 274.

Time = 0.30 (sec) , antiderivative size = 444, normalized size of antiderivative = 3.24

$$\int (dx)^m \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)\,dx = \frac{((Ccm^6+21\,Ccm^5+175\,Ccm^4+735\,Ccm^3+1624\,Ccm^2+1764\,Ccm+720\,Cc)x^7+(Bcm^6+22\,Bcm}{}$$

input
```
integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
((C*c*m^6 + 21*C*c*m^5 + 175*C*c*m^4 + 735*C*c*m^3 + 1624*C*c*m^2 + 1764*C
*c*m + 720*C*c)*x^7 + (B*c*m^6 + 22*B*c*m^5 + 190*B*c*m^4 + 820*B*c*m^3 +
1849*B*c*m^2 + 2038*B*c*m + 840*B*c)*x^6 + ((C*b + A*c)*m^6 + 23*(C*b + A*
c)*m^5 + 207*(C*b + A*c)*m^4 + 925*(C*b + A*c)*m^3 + 2144*(C*b + A*c)*m^2
+ 1008*C*b + 1008*A*c + 2412*(C*b + A*c)*m)*x^5 + (B*b*m^6 + 24*B*b*m^5 +
226*B*b*m^4 + 1056*B*b*m^3 + 2545*B*b*m^2 + 2952*B*b*m + 1260*B*b)*x^4 + (
(C*a + A*b)*m^6 + 25*(C*a + A*b)*m^5 + 247*(C*a + A*b)*m^4 + 1219*(C*a + A
*b)*m^3 + 3112*(C*a + A*b)*m^2 + 1680*C*a + 1680*A*b + 3796*(C*a + A*b)*m)
*x^3 + (B*a*m^6 + 26*B*a*m^5 + 270*B*a*m^4 + 1420*B*a*m^3 + 3929*B*a*m^2 +
 5274*B*a*m + 2520*B*a)*x^2 + (A*a*m^6 + 27*A*a*m^5 + 295*A*a*m^4 + 1665*A
*a*m^3 + 5104*A*a*m^2 + 8028*A*a*m + 5040*A*a)*x)*(d*x)^m/(m^7 + 28*m^6 +
322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)
```

3.39.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3628 vs. $2(122) = 244$.

Time = 0.63 (sec) , antiderivative size = 3628, normalized size of antiderivative = 26.48

$$\int (dx)^m \left(A + Bx + Cx^2\right) \left(a + bx^2 + cx^4\right) \, dx = \text{Too large to display}$$

input
```
integrate((d*x)**m*(C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
```

output
```
Piecewise(((-A*a/(6*x**6) - A*b/(4*x**4) - A*c/(2*x**2) - B*a/(5*x**5) - B
*b/(3*x**3) - B*c/x - C*a/(4*x**4) - C*b/(2*x**2) + C*c*log(x))/d**7, Eq(m
, -7)), ((-A*a/(5*x**5) - A*b/(3*x**3) - A*c/x - B*a/(4*x**4) - B*b/(2*x**
2) + B*c*log(x) - C*a/(3*x**3) - C*b/x + C*c*x)/d**6, Eq(m, -6)), ((-A*a/(
4*x**4) - A*b/(2*x**2) + A*c*log(x) - B*a/(3*x**3) - B*b/x + B*c*x - C*a/(
2*x**2) + C*b*log(x) + C*c*x**2/2)/d**5, Eq(m, -5)), ((-A*a/(3*x**3) - A*b
/x + A*c*x - B*a/(2*x**2) + B*b*log(x) + B*c*x**2/2 - C*a/x + C*b*x + C*c*
x**3/3)/d**4, Eq(m, -4)), ((-A*a/(2*x**2) + A*b*log(x) + A*c*x**2/2 - B*a/
x + B*b*x + B*c*x**3/3 + C*a*log(x) + C*b*x**2/2 + C*c*x**4/4)/d**3, Eq(m,
 -3)), ((-A*a/x + A*b*x + A*c*x**3/3 + B*a*log(x) + B*b*x**2/2 + B*c*x**4/
4 + C*a*x + C*b*x**3/3 + C*c*x**5/5)/d**2, Eq(m, -2)), ((A*a*log(x) + A*b*
x**2/2 + A*c*x**4/4 + B*a*x + B*b*x**3/3 + B*c*x**5/5 + C*a*x**2/2 + C*b*x
**4/4 + C*c*x**6/6)/d, Eq(m, -1)), (A*a*m**6*x*(d*x)**m/(m**7 + 28*m**6 +
322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 27*A*a*m
**5*x*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*
m**2 + 13068*m + 5040) + 295*A*a*m**4*x*(d*x)**m/(m**7 + 28*m**6 + 322*m**
5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1665*A*a*m**3*x
*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2
+ 13068*m + 5040) + 5104*A*a*m**2*x*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 +
1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 8028*A*a*m*x*(d*...
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.13

$$\int (dx)^m (A+Bx+Cx^2)(a+bx^2+cx^4)\,dx = \frac{Ccd^mx^7x^m}{m+7} + \frac{Bcd^mx^6x^m}{m+6} + \frac{Cbd^mx^5x^m}{m+5} + \frac{Acd^mx^5x^m}{m+5} + \frac{Bbd^mx^4x^m}{m+4} + \frac{Cad^mx^3x^m}{m+3} + \frac{Abd^mx^3x^m}{m+3} + \frac{Bad^mx^2x^m}{m+2} + \frac{(dx)^{m+1}Aa}{d(m+1)}$$

input
```
integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
C*c*d^m*x^7*x^m/(m + 7) + B*c*d^m*x^6*x^m/(m + 6) + C*b*d^m*x^5*x^m/(m + 5
) + A*c*d^m*x^5*x^m/(m + 5) + B*b*d^m*x^4*x^m/(m + 4) + C*a*d^m*x^3*x^m/(m
 + 3) + A*b*d^m*x^3*x^m/(m + 3) + B*a*d^m*x^2*x^m/(m + 2) + (d*x)^(m + 1)*
A*a/(d*(m + 1))
```

3.39.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 914 vs. $2(137) = 274$.

Time = 0.38 (sec) , antiderivative size = 914, normalized size of antiderivative = 6.67

$$\int (dx)^m (A + Bx + Cx^2) (a + bx^2 + cx^4) \, dx$$
$$= \frac{(dx)^m Ccm^6x^7 + (dx)^m Bcm^6x^6 + 21 (dx)^m Ccm^5x^7 + (dx)^m Cbm^6x^5 + (dx)^m Acm^6x^5 + 22 (dx)^m Bcm^5}{}$$

input
```
integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
((d*x)^m*C*c*m^6*x^7 + (d*x)^m*B*c*m^6*x^6 + 21*(d*x)^m*C*c*m^5*x^7 + (d*x
)^m*C*b*m^6*x^5 + (d*x)^m*A*c*m^6*x^5 + 22*(d*x)^m*B*c*m^5*x^6 + 175*(d*x)
^m*C*c*m^4*x^7 + (d*x)^m*B*b*m^6*x^4 + 23*(d*x)^m*C*b*m^5*x^5 + 23*(d*x)^m
*A*c*m^5*x^5 + 190*(d*x)^m*B*c*m^4*x^6 + 735*(d*x)^m*C*c*m^3*x^7 + (d*x)^m
*C*a*m^6*x^3 + (d*x)^m*A*b*m^6*x^3 + 24*(d*x)^m*B*b*m^5*x^4 + 207*(d*x)^m*
C*b*m^4*x^5 + 207*(d*x)^m*A*c*m^4*x^5 + 820*(d*x)^m*B*c*m^3*x^6 + 1624*(d*
x)^m*C*c*m^2*x^7 + (d*x)^m*B*a*m^6*x^2 + 25*(d*x)^m*C*a*m^5*x^3 + 25*(d*x)
^m*A*b*m^5*x^3 + 226*(d*x)^m*B*b*m^4*x^4 + 925*(d*x)^m*C*b*m^3*x^5 + 925*(
d*x)^m*A*c*m^3*x^5 + 1849*(d*x)^m*B*c*m^2*x^6 + 1764*(d*x)^m*C*c*m*x^7 + (
d*x)^m*A*a*m^6*x + 26*(d*x)^m*B*a*m^5*x^2 + 247*(d*x)^m*C*a*m^4*x^3 + 247*
(d*x)^m*A*b*m^4*x^3 + 1056*(d*x)^m*B*b*m^3*x^4 + 2144*(d*x)^m*C*b*m^2*x^5
+ 2144*(d*x)^m*A*c*m^2*x^5 + 2038*(d*x)^m*B*c*m*x^6 + 720*(d*x)^m*C*c*x^7
+ 27*(d*x)^m*A*a*m^5*x + 270*(d*x)^m*B*a*m^4*x^2 + 1219*(d*x)^m*C*a*m^3*x^
3 + 1219*(d*x)^m*A*b*m^3*x^3 + 2545*(d*x)^m*B*b*m^2*x^4 + 2412*(d*x)^m*C*b
*m*x^5 + 2412*(d*x)^m*A*c*m*x^5 + 840*(d*x)^m*B*c*x^6 + 295*(d*x)^m*A*a*m^
4*x + 1420*(d*x)^m*B*a*m^3*x^2 + 3112*(d*x)^m*C*a*m^2*x^3 + 3112*(d*x)^m*A
*b*m^2*x^3 + 2952*(d*x)^m*B*b*m*x^4 + 1008*(d*x)^m*C*b*x^5 + 1008*(d*x)^m*
A*c*x^5 + 1665*(d*x)^m*A*a*m^3*x + 3929*(d*x)^m*B*a*m^2*x^2 + 3796*(d*x)^m
*C*a*m*x^3 + 3796*(d*x)^m*A*b*m*x^3 + 1260*(d*x)^m*B*b*x^4 + 5104*(d*x)^m*
A*a*m^2*x + 5274*(d*x)^m*B*a*m*x^2 + 1680*(d*x)^m*C*a*x^3 + 1680*(d*x)^...
```

3.39.9 Mupad [B] (verification not implemented)

Time = 7.95 (sec) , antiderivative size = 527, normalized size of antiderivative = 3.85

$$\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right) dx = \frac{x^3 (d x)^m (A b+C a) (m^6+25 m^5+247 m^4+1219 m^3+3112 m^2+3796 m+1680)}{m^7+28 m^6+322 m^5+1960 m^4+6769 m^3+13132 m^2+13068 m+5040} + \frac{x^5 (d x)^m (A c+C b) (m^6+23 m^5+207 m^4+925 m^3+2144 m^2+2412 m+1008)}{m^7+28 m^6+322 m^5+1960 m^4+6769 m^3+13132 m^2+13068 m+5040} + \frac{A a x (d x)^m (m^6+27 m^5+295 m^4+1665 m^3+5104 m^2+8028 m+5040)}{m^7+28 m^6+322 m^5+1960 m^4+6769 m^3+13132 m^2+13068 m+5040} + \frac{B a x^2 (d x)^m (m^6+26 m^5+270 m^4+1420 m^3+3929 m^2+5274 m+2520)}{m^7+28 m^6+322 m^5+1960 m^4+6769 m^3+13132 m^2+13068 m+5040} + \frac{B b x^4 (d x)^m (m^6+24 m^5+226 m^4+1056 m^3+2545 m^2+2952 m+1260)}{m^7+28 m^6+322 m^5+1960 m^4+6769 m^3+13132 m^2+13068 m+5040} + \frac{B c x^6 (d x)^m (m^6+22 m^5+190 m^4+820 m^3+1849 m^2+2038 m+840)}{m^7+28 m^6+322 m^5+1960 m^4+6769 m^3+13132 m^2+13068 m+5040} + \frac{C c x^7 (d x)^m (m^6+21 m^5+175 m^4+735 m^3+1624 m^2+1764 m+720)}{m^7+28 m^6+322 m^5+1960 m^4+6769 m^3+13132 m^2+13068 m+5040}$$

input
```
int((d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x)
```

output
```
(x^3*(d*x)^m*(A*b + C*a)*(3796*m + 3112*m^2 + 1219*m^3 + 247*m^4 + 25*m^5
+ m^6 + 1680))/(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m
^6 + m^7 + 5040) + (x^5*(d*x)^m*(A*c + C*b)*(2412*m + 2144*m^2 + 925*m^3 +
 207*m^4 + 23*m^5 + m^6 + 1008))/(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^
4 + 322*m^5 + 28*m^6 + m^7 + 5040) + (A*a*x*(d*x)^m*(8028*m + 5104*m^2 + 1
665*m^3 + 295*m^4 + 27*m^5 + m^6 + 5040))/(13068*m + 13132*m^2 + 6769*m^3
+ 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) + (B*a*x^2*(d*x)^m*(5274*m + 3
929*m^2 + 1420*m^3 + 270*m^4 + 26*m^5 + m^6 + 2520))/(13068*m + 13132*m^2
+ 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) + (B*b*x^4*(d*x)^m*
(2952*m + 2545*m^2 + 1056*m^3 + 226*m^4 + 24*m^5 + m^6 + 1260))/(13068*m +
 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) + (B*c*x
^6*(d*x)^m*(2038*m + 1849*m^2 + 820*m^3 + 190*m^4 + 22*m^5 + m^6 + 840))/(
13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)
 + (C*c*x^7*(d*x)^m*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6
+ 720))/(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^
7 + 5040)
```

3.40 $\int \frac{(dx)^m(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$

3.40.1 Optimal result

Integrand size = 30, antiderivative size = 368

$$\begin{aligned}\int \frac{(dx)^m(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx &= \frac{\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)(dx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)d(1+m)}\\
&\quad+\frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)(dx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{\left(b+\sqrt{b^2-4ac}\right)d(1+m)}\\
&\quad+\frac{2Bc(dx)^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)d^2(2+m)}\\
&\quad-\frac{2Bc(dx)^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)d^2(2+m)}\end{aligned}$$

output
```
(d*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b-(-4*a*c+b^2)^
(1/2)))*(C+(2*A*c-C*b)/(-4*a*c+b^2)^(1/2))/d/(1+m)/(b-(-4*a*c+b^2)^(1/2))+
2*B*c*(d*x)^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-2*c*x^2/(b-(-4*a*c+b^2
)^(1/2)))/d^2/(2+m)/(b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)+(d*x)^(1+m)*
hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(C+(
-2*A*c+C*b)/(-4*a*c+b^2)^(1/2))/d/(1+m)/(b+(-4*a*c+b^2)^(1/2))-2*B*c*(d*x)
^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/d
^2/(2+m)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

3.40.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 2.26 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.19

$$\int \frac{(dx)^m (A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$$
$$= \frac{(dx)^m \left(A(2+3m+m^2)\,\text{RootSum}\left[a+b\#1^2+c\#1^4\&, \frac{\text{Hypergeometric2F1}\left(-m,-m,1-m,-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}}{b\#1+2c\#1^3}\& \right.\right.}{}$$

input
```
Integrate[((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
((d*x)^m*(A*(2 + 3*m + m^2)*RootSum[a + b*#1^2 + c*#1^4 & , Hypergeometric
2F1[-m, -m, 1 - m, -(#1/(x - #1))]/((x/(x - #1))^m*(b*#1 + 2*c*#1^3)) & ]
+ B*(2 + m)*RootSum[a + b*#1^2 + c*#1^4 & , (m*x + (Hypergeometric2F1[-m,
-m, 1 - m, -(#1/(x - #1))]*#1)/(x/(x - #1))^m + (m*Hypergeometric2F1[-m, -
m, 1 - m, -(#1/(x - #1))]*#1)/(x/(x - #1))^m)/(b*#1 + 2*c*#1^3) & ] + C*Ro
otSum[a + b*#1^2 + c*#1^4 & , (m*x^2 + m^2*x^2 + 2*m*x*#1 + m^2*x*#1 + (2*
Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (3
*m*Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m +
 (m^2*Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^
m + (m*#1^2)/(x/#1)^m)/(b*#1 + 2*c*#1^3) & ]))/(2*m*(1 + m)*(2 + m))
```

3.40.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2193, 27, 1451, 27, 278, 1608, 27, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m (A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$$
$$\Big\downarrow \; 2193$$
$$\int \frac{(dx)^m (Cx^2+A)}{cx^4+bx^2+a}dx+\frac{\int \frac{B(dx)^{m+1}}{cx^4+bx^2+a}dx}{d}$$

$$\Big\downarrow \ 27$$

$$\int \frac{(dx)^m\left(Cx^2+A\right)}{cx^4+bx^2+a}dx+\frac{B\int \frac{(dx)^{m+1}}{cx^4+bx^2+a}dx}{d}$$

$$\Big\downarrow \ 1451$$

$$\int \frac{(dx)^m\left(Cx^2+A\right)}{cx^4+bx^2+a}dx+\frac{B\left(\frac{c\int \frac{2(dx)^{m+1}}{2cx^2+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{2(dx)^{m+1}}{2cx^2+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}\right)}{d}$$

$$\Big\downarrow \ 27$$

$$\int \frac{(dx)^m\left(Cx^2+A\right)}{cx^4+bx^2+a}dx+\frac{B\left(\frac{2c\int \frac{(dx)^{m+1}}{2cx^2+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{(dx)^{m+1}}{2cx^2+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}\right)}{d}$$

$$\Big\downarrow \ 278$$

$$\int \frac{(dx)^m\left(Cx^2+A\right)}{cx^4+bx^2+a}dx+\frac{B\left(\frac{2c(dx)^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d(m+2)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+2)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}\right)}{d}$$

$$\Big\downarrow \ 1608$$

$$\frac{1}{2}\left(\frac{2Ac-bC}{\sqrt{b^2-4ac}}+C\right)\int \frac{2(dx)^m}{2cx^2+b-\sqrt{b^2-4ac}}dx+\frac{1}{2}\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\int \frac{2(dx)^m}{2cx^2+b+\sqrt{b^2-4ac}}dx+\frac{B\left(\frac{2c(dx)^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d(m+2)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+2)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}\right)}{d}$$

$$\Big\downarrow \ 27$$

$$\left(\frac{2Ac-bC}{\sqrt{b^2-4ac}}+C\right)\int \frac{(dx)^m}{2cx^2+b-\sqrt{b^2-4ac}}dx+\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\int \frac{(dx)^m}{2cx^2+b+\sqrt{b^2-4ac}}dx+\frac{B\left(\frac{2c(dx)^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d(m+2)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+2)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}\right)}{d}$$

$$\Big\downarrow \ 278$$

3.40. $\int \frac{(dx)^m\left(A+Bx+Cx^2\right)}{a+bx^2+cx^4}\,dx$

$$\frac{(dx)^{m+1}\left(\frac{2Ac-bC}{\sqrt{b^2-4ac}}+C\right)\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\left(b-\sqrt{b^2-4ac}\right)}+\frac{(dx)^{m+1}\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\left(\sqrt{b^2-4ac}+b\right)}+\frac{B\left(\frac{2c(dx)^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d(m+2)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+2)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}\right)}{d}$$

input
```
Int[((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
((C + (2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*(d*x)^(1 + m)*Hypergeometric2F1[1,
(1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^2
- 4*a*c])*d*(1 + m)) + ((C - (2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*(d*x)^(1 + m
)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c])])/((b + Sqrt[b^2 - 4*a*c])*d*(1 + m)) + (B*((2*c*(d*x)^(2 + m)*Hyper
geometric2F1[1, (2 + m)/2, (4 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])
/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*(2 + m)) - (2*c*(d*x)^(2 + m
)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d*(2 + m))))/d
```

3.40.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 1451
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[(d*x)^m/(b/2 - q/2 + c*x^2), x]
, x] - Simp[c/q   Int[(d*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b,
c, d, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

rule 1608
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (b_.)*(x_)^2 + (c_.)
*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d -
b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d
- b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

3.40.4 Maple [F]

$$\int \frac{(dx)^m \left(C\,x^2+Bx+A\right)}{c\,x^4+b\,x^2+a}dx$$

input
```
int((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

output
```
int((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

3.40.5 Fricas [F]

$$\int \frac{(dx)^m \left(A+Bx+Cx^2\right)}{a+bx^2+cx^4}\,dx=\int \frac{(Cx^2+Bx+A)(dx)^m}{cx^4+bx^2+a}\,dx$$

input
```
integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
integral((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a), x)
```

3.40.6 Sympy [F]

$$\int \frac{(dx)^m (A + Bx + Cx^2)}{a + bx^2 + cx^4} dx = \int \frac{(dx)^m (A + Bx + Cx^2)}{a + bx^2 + cx^4} dx$$

input
```
integrate((d*x)**m*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

output
```
Integral((d*x)**m*(A + B*x + C*x**2)/(a + b*x**2 + c*x**4), x)
```

3.40.7 Maxima [F]

$$\int \frac{(dx)^m (A + Bx + Cx^2)}{a + bx^2 + cx^4} dx = \int \frac{(Cx^2 + Bx + A)(dx)^m}{cx^4 + bx^2 + a} dx$$

input
```
integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a), x)
```

3.40.8 Giac [F]

$$\int \frac{(dx)^m (A + Bx + Cx^2)}{a + bx^2 + cx^4} dx = \int \frac{(Cx^2 + Bx + A)(dx)^m}{cx^4 + bx^2 + a} dx$$

input
```
integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a), x)
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m (A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \int \frac{(d\,x)^m\,(C\,x^2+B\,x+A)}{c\,x^4+b\,x^2+a}\,dx$$

input
```
int(((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x)
```

output
```
int(((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4), x)
```

3.41 $\int \frac{(dx)^m(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

3.41.1 Optimal result

Integrand size = 30, antiderivative size = 685

$$\int \frac{(dx)^m\,(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$$
$$= \frac{B(dx)^{2+m}\,(b^2-2ac+bcx^2)}{2a\,(b^2-4ac)\,d^2\,(a+bx^2+cx^4)} + \frac{(dx)^{1+m}\,(A(b^2-2ac)-abC+c(Ab-2aC)x^2)}{2a\,(b^2-4ac)\,d\,(a+bx^2+cx^4)}$$
$$+ \frac{c\big(2aC\big(2b-\sqrt{b^2-4ac}(1-m)\big)+A\big(b^2(1-m)+b\sqrt{b^2-4ac}(1-m)-4ac(3-m)\big)\big)\,(dx)^{1+m}\,\text{Hyperg}}{2a\,(b^2-4ac)^{3/2}\,\big(b-\sqrt{b^2-4ac}\big)\,d(1+m)}$$
$$- \frac{c\big(2aC\big(2b+\sqrt{b^2-4ac}(1-m)\big)+A\big(b^2(1-m)-b\sqrt{b^2-4ac}(1-m)-4ac(3-m)\big)\big)\,(dx)^{1+m}\,\text{Hyperg}}{2a\,(b^2-4ac)^{3/2}\,\big(b+\sqrt{b^2-4ac}\big)\,d(1+m)}$$
$$- \frac{Bc\big(4ac(2-m)+b\big(b+\sqrt{b^2-4ac}\big)\,m\big)\,(dx)^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{2a\,(b^2-4ac)^{3/2}\,\big(b-\sqrt{b^2-4ac}\big)\,d^2(2+m)}$$
$$+ \frac{Bc\big(4ac(2-m)+b\big(b-\sqrt{b^2-4ac}\big)\,m\big)\,(dx)^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{2a\,(b^2-4ac)^{3/2}\,\big(b+\sqrt{b^2-4ac}\big)\,d^2(2+m)}$$

output
```
1/2*B*(d*x)^(2+m)*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/d^2/(c*x^4+b*x^2+a)+1
/2*(d*x)^(1+m)*(A*(-2*a*c+b^2)-a*b*C+c*(A*b-2*C*a)*x^2)/a/(-4*a*c+b^2)/d/(
c*x^4+b*x^2+a)+1/2*B*c*(d*x)^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-2*c*x
^2/(b+(-4*a*c+b^2)^(1/2)))*(4*a*c*(2-m)+b*m*(b-(-4*a*c+b^2)^(1/2)))/a/(-4*
a*c+b^2)^(3/2)/d^2/(2+m)/(b+(-4*a*c+b^2)^(1/2))-1/2*B*c*(d*x)^(2+m)*hyperg
eom([1, 1+1/2*m],[2+1/2*m],-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))*(4*a*c*(2-m)+b
*m*(b+(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)^(3/2)/d^2/(2+m)/(b-(-4*a*c+b^2)^
(1/2))-1/2*c*(d*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b+
(-4*a*c+b^2)^(1/2)))*(2*a*C*(2*b+(1-m)*(-4*a*c+b^2)^(1/2))+A*(b^2*(1-m)-4*
a*c*(3-m)-b*(1-m)*(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)^(3/2)/d/(1+m)/(b+(-4
*a*c+b^2)^(1/2))+1/2*c*(d*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2
*c*x^2/(b-(-4*a*c+b^2)^(1/2)))*(2*a*C*(2*b-(1-m)*(-4*a*c+b^2)^(1/2))+A*(b^
2*(1-m)-4*a*c*(3-m)+b*(1-m)*(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)^(3/2)/d/(1
+m)/(b-(-4*a*c+b^2)^(1/2))
```

3.41.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 2.28 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.35

$$\int \frac{(dx)^m (A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$$
$$= \frac{x(dx)^m \Big(A(6+5m+m^2)\,\text{AppellF1}\left(\frac{1+m}{2},2,2,\frac{3+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+(1+m)x\Big(B(3+m)\,\text{App}}{a^2(1+}$$

input
```
Integrate[((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(x*(d*x)^m*(A*(6 + 5*m + m^2)*AppellF1[(1 + m)/2, 2, 2, (3 + m)/2, (-2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (1 + m)
*x*(B*(3 + m)*AppellF1[(2 + m)/2, 2, 2, (4 + m)/2, (-2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + C*(2 + m)*x*AppellF1[(3
 + m)/2, 2, 2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-
b + Sqrt[b^2 - 4*a*c])])))/(a^2*(1 + m)*(2 + m)*(3 + m))
```

3.41.3 Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 687, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2193, 27, 1441, 1600, 25, 1608, 27, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m (A+Bx+Cx^2)}{(a+bx^2+cx^4)^2} \, dx$$

$$\Big\downarrow \text{2193}$$

$$\int \frac{(dx)^m (Cx^2+A)}{(cx^4+bx^2+a)^2}dx+\frac{\int \frac{B(dx)^{m+1}}{(cx^4+bx^2+a)^2}dx}{d}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(dx)^m (Cx^2+A)}{(cx^4+bx^2+a)^2}dx+\frac{B\int \frac{(dx)^{m+1}}{(cx^4+bx^2+a)^2}dx}{d}$$

$$\Big\downarrow \text{1441}$$

$$\int \frac{(dx)^m (Cx^2+A)}{(cx^4+bx^2+a)^2}dx+\frac{B\left(\frac{(dx)^{m+2}(-2ac+b^2+bcx^2)}{2ad(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int \frac{(dx)^{m+1}\left(mb^2+cmx^2b+2ac(2-m)\right)}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}\right)}{d}$$

$$\Big\downarrow \text{1600}$$

$$-\frac{\int -\frac{(dx)^m (A(1-m)b^2+aC(m+1)b+c(Ab-2aC)(1-m)x^2-2aAc(3-m))}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}+\frac{B\left(\frac{(dx)^{m+2}(-2ac+b^2+bcx^2)}{2ad(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int \frac{(dx)^{m+1}\left(mb^2+cmx^2b+2ac(2-m)\right)}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}\right)}{d}+\frac{(dx)^{m+1}\left(A(b^2-2ac)+cx^2(Ab-2aC)-abC\right)}{2ad(b^2-4ac)(a+bx^2+cx^4)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(dx)^m \left(A(1-m)b^2+aC(m+1)b+c(Ab-2aC)(1-m)x^2-2aAc(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{B\left(\frac{(dx)^{m+2}\left(-2ac+b^2+bcx^2\right)}{2ad(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int \frac{(dx)^{m+1}\left(mb^2+cmx^2b+2ac(2-m)\right)}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}\right)}{d}+\frac{(dx)^{m+1}\left(A\left(b^2-2ac\right)+cx^2(Ab-2aC)-abC\right)}{2ad\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 1608$$

$$\frac{\frac{c\left(A\left(b(1-m)\sqrt{b^2-4ac}-4ac(3-m)+b^2(1-m)\right)+2aC\left(2b-(1-m)\sqrt{b^2-4ac}\right)\right)\int \frac{2(dx)^m}{2cx^2+b-\sqrt{b^2-4ac}}dx}{2\sqrt{b^2-4ac}}-\frac{c\left(-(1-m)\sqrt{b^2-4ac}(Ab-2aC)-4aAc(3-\cdots\right.}{2\cdots}}{2a\left(b^2-4ac\right)}+\frac{B\left(\frac{(dx)^{m+2}\left(-2ac+b^2+bcx^2\right)}{2ad(b^2-4ac)(a+bx^2+cx^4)}-\frac{\frac{c\left(bm\left(\sqrt{b^2-4ac}+b\right)+4ac(2-m)\right)\int \frac{2(dx)^{m+1}}{2cx^2+b-\sqrt{b^2-4ac}}dx}{2\sqrt{b^2-4ac}}-\frac{c\left(bm\left(b-\sqrt{b^2-4ac}\right)+4ac(2-m)\right)\int \frac{2(dx)^{m+1}}{2cx^2+b+\sqrt{b^2-4ac}}dx}{2\sqrt{b^2-4ac}}}{2a(b^2-4ac)}\right)}{d}+\frac{(dx)^{m+1}\left(A\left(b^2-2ac\right)+cx^2(Ab-2aC)-abC\right)}{2ad\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{c\left(A\left(b(1-m)\sqrt{b^2-4ac}-4ac(3-m)+b^2(1-m)\right)+2aC\left(2b-(1-m)\sqrt{b^2-4ac}\right)\right)\int \frac{(dx)^m}{2cx^2+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{c\left(-(1-m)\sqrt{b^2-4ac}(Ab-2aC)-4aAc(3-\cdots\right.}{\sqrt{\cdots}}}{2a\left(b^2-4ac\right)}+\frac{B\left(\frac{(dx)^{m+2}\left(-2ac+b^2+bcx^2\right)}{2ad(b^2-4ac)(a+bx^2+cx^4)}-\frac{\frac{c\left(bm\left(\sqrt{b^2-4ac}+b\right)+4ac(2-m)\right)\int \frac{(dx)^{m+1}}{2cx^2+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{c\left(bm\left(b-\sqrt{b^2-4ac}\right)+4ac(2-m)\right)\int \frac{(dx)^{m+1}}{2cx^2+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}}{2a(b^2-4ac)}\right)}{d}+\frac{(dx)^{m+1}\left(A\left(b^2-2ac\right)+cx^2(Ab-2aC)-abC\right)}{2ad\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 278$$

3.41. $\int \frac{(dx)^m\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

$$\frac{\frac{c(dx)^{m+1}\left(A\left(b(1-m)\sqrt{b^2-4ac}-4ac(3-m)+b^2(1-m)\right)+2aC\left(2b-(1-m)\sqrt{b^2-4ac}\right)\right)\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(d}{}}{2a\left(b^2-4ac\right)}$$

$$\frac{(dx)^{m+1}\left(A\left(b^2-2ac\right)+cx^2(Ab-2aC)-abC\right)}{2ad\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+$$

$$\frac{B\left(\frac{(dx)^{m+2}\left(-2ac+b^2+bcx^2\right)}{2ad(b^2-4ac)(a+bx^2+cx^4)}-\frac{\frac{c(dx)^{m+2}\left(bm\left(\sqrt{b^2-4ac}+b\right)+4ac(2-m)\right)\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d(m+2)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(dx)^{m+2}\left(bm\left(b-\sqrt{b^2-\right.\right.}{}}{2a(b^2-4ac)}\right.}{d}$$

input
```
Int[((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((d*x)^(1 + m)*(A*(b^2 - 2*a*c) - a*b*C + c*(A*b - 2*a*C)*x^2))/(2*a*(b^2
- 4*a*c)*d*(a + b*x^2 + c*x^4)) + ((c*(2*a*C*(2*b - Sqrt[b^2 - 4*a*c]*(1 -
 m)) + A*(b^2*(1 - m) + b*Sqrt[b^2 - 4*a*c]*(1 - m) - 4*a*c*(3 - m)))*(d*x
)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[
b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m)) - (c
*(4*a*b*C + A*b^2*(1 - m) - Sqrt[b^2 - 4*a*c]*(A*b - 2*a*C)*(1 - m) - 4*a*
A*c*(3 - m))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c]
)*d*(1 + m)))/(2*a*(b^2 - 4*a*c)) + (B*(((d*x)^(2 + m)*(b^2 - 2*a*c + b*c*
x^2))/(2*a*(b^2 - 4*a*c)*d*(a + b*x^2 + c*x^4)) - ((c*(4*a*c*(2 - m) + b*(
b + Sqrt[b^2 - 4*a*c])*m)*(d*x)^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4
 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt
[b^2 - 4*a*c])*d*(2 + m)) - (c*(4*a*c*(2 - m) + b*(b - Sqrt[b^2 - 4*a*c])*
m)*(d*x)^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, (-2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d*(2 + m
)))/(2*a*(b^2 - 4*a*c))))/d
```

3.41.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 1441
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))
  Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

rule 1600
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^m*(a + b*x^2 + c
*x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1))
- a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Int
egerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1608
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (b_.)*(x_)^2 + (c_.)
*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d -
b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d
- b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

3.41. $\int \frac{(dx)^m (A+Bx+Cx^2)}{(a+bx^2+cx^4)^2} \, dx$

3.41.4 Maple [F]

$$\int \frac{(dx)^m\,(C\,x^2+Bx+A)}{(c\,x^4+b\,x^2+a)^2}dx$$

input
```
int((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

output
```
int((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

3.41.5 Fricas [F]

$$\int \frac{(dx)^m\,(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(Cx^2+Bx+A)(dx)^m}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
integral((C*x^2 + B*x + A)*(d*x)^m/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*a*c)*x^
4 + 2*a*b*x^2 + a^2), x)
```

3.41.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m\,(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x)**m*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.41.7 Maxima [F]

$$\int \frac{(dx)^m (A + Bx + Cx^2)}{(a + bx^2 + cx^4)^2} dx = \int \frac{(Cx^2 + Bx + A)(dx)^m}{(cx^4 + bx^2 + a)^2} dx$$

input
```
integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

3.41.8 Giac [F]

$$\int \frac{(dx)^m (A + Bx + Cx^2)}{(a + bx^2 + cx^4)^2} dx = \int \frac{(Cx^2 + Bx + A)(dx)^m}{(cx^4 + bx^2 + a)^2} dx$$

input
```
integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m (A + Bx + Cx^2)}{(a + bx^2 + cx^4)^2} dx = \int \frac{(d\,x)^m (C\,x^2 + B\,x + A)}{(c\,x^4 + b\,x^2 + a)^2} dx$$

input
```
int(((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
int(((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2, x)
```

3.42 $\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

3.42.1 Optimal result

Integrand size = 28, antiderivative size = 356

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} - \frac{bB\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output

```
1/2*B*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(A*b-2*C*a+(2*A*c-C*b
)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(3/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2))*(2*A*c-C*b+(-4*A*b*c+(4*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(
1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*A*c-C*b+(4*A*b*c-(4*a*c+b^2)*C)/(-4*
a*c+b^2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.42.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.06

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$$
$$= \frac{1}{4}\left(\frac{4a(B+Cx)+2x(bx(B+Cx)-A(b+2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(-2Ac\left(-2b+\sqrt{b^2-4ac}\right)+\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(-2Ac\left(2b+\sqrt{b^2-4ac}\right)+\left(b^2+4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{2bB\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}} - \frac{2bB\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

input
```
Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 -
 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ (Sqrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 -
 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqr
t[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

3.42.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {2193, 27, 1434, 1159, 1083, 219, 1598, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$\Big\downarrow \text{ 2193}$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\int \frac{Bx^3}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+B\int \frac{x^3}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{ 1434}$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\int \frac{x^2}{\left(cx^4+bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \text{ 1159}$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{b\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}+\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{ 1083}$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\int \frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)}{b^2-4ac}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

$$\Big\downarrow \text{ 1598}$$

$$\frac{\int \frac{-\left(\left(2Ac-bC\right)x^2\right)+Ab-2aC}{cx^4+bx^2+a}dx}{2\left(b^2-4ac\right)}-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+$$
$$\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

$$\Big\downarrow \text{ 1480}$$

3.42. $\int \frac{x^2\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

$$\frac{-\frac{1}{2}\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{2(b^2-4ac)}$$
$$\frac{x(-2aC+x^2(2Ac-bC)+Ab)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{1}{2}B\left(\frac{2a+bx^2}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{2b\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}\right)$$

$$\Big\downarrow \text{ 218}$$

$$\frac{-\frac{\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2(b^2-4ac)}-$$
$$\frac{x(-2aC+x^2(2Ac-bC)+Ab)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{1}{2}B\left(\frac{2a+bx^2}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{2b\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}\right)$$

input
```
Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*(x*(A*b - 2*a*C + (2*A*c - b*C)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) + (-(((2*A*c - b*C - (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*S
qrt[b - Sqrt[b^2 - 4*a*c]])) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*(b^2 - 4*a*c)) + (B*(
(2*a + b*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (2*b*ArcTanh[(b + 2*c*
x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/2
```

3.42.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

3.42. $\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

3.42.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.00 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.57

method	result
risch	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{x^2 Bb}{2(4ac-b^2)}+\frac{(Ab-2Ca)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\operatorname{RootOf}(c\_Z^4+\_Z^2 b+a)}\frac{\left(\frac{(2Ac-Cb)\_R^2}{4ac-b^2}-\frac{2\_RBb}{4ac-b^2}-\frac{Ab-2Ca}{4ac-b^2}\right)\ln(x-\_ }{2c\_R^3+\_Rb}\right.}{4}$
default	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{x^2 Bb}{2(4ac-b^2)}+\frac{(Ab-2Ca)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{-B\sqrt{-4ac+b^2}\,b\ln(2c\,x^2+\sqrt{-4ac+b^2}+b)+\frac{(-4Abc\sqrt{-4ac+b^2}+8Aa\,c^2-2A\,b^2c+4C}{}}{4c(4ac}\right.}{}$

input
```
int(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2/(4*a*c-b^2)*x^2*B*b+1/2*(A*b-2*C*a)/(
4*a*c-b^2)*x-1/(4*a*c-b^2)*B*a)/(c*x^4+b*x^2+a)+1/4*sum(((2*A*c-C*b)/(4*a*
c-b^2)*_R^2-2/(4*a*c-b^2)*_R*B*b-(A*b-2*C*a)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*
ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.42.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx=\text{Timed out}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.42.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.42.7 Maxima [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(Cx^2+Bx+A)x^2}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*
c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*
x + (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

3.42.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4438 vs. $2(306) = 612$.

Time = 1.68 (sec) , antiderivative size = 4438, normalized size of antiderivative = 12.47

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*
x^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3
*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c
 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*
a*c)*b*c^2)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c
^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*
(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3...
```

3.42.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 3835, normalized size of antiderivative = 10.77

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*
A*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A
*C^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^
3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^
2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^
8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*
z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*
b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*
c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a
^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 819
2*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A
^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B
*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B
*a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*
c^2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c
+ 48*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*
C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c +
 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6
*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4
*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(25...
```

3.42. $\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

3.43 $\int \frac{x(Ax+Bx^2+Cx^3)}{(a+bx^2+cx^4)^2}\,dx$

3.43.1 Optimal result

Integrand size = 30, antiderivative size = 356

$$\int \frac{x(Ax+Bx^2+Cx^3)}{(a+bx^2+cx^4)^2}\,dx = \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} - \frac{bB\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
1/2*B*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(A*b-2*C*a+(2*A*c-C*b
)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(3/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2))*(2*A*c-C*b+(-4*A*b*c+(4*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(
1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*A*c-C*b+(4*A*b*c-(4*a*c+b^2)*C)/(-4*
a*c+b^2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.43.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.06

$$\int \frac{x(Ax+Bx^2+Cx^3)}{(a+bx^2+cx^4)^2}\,dx$$
$$=\frac{1}{4}\left(\frac{4a(B+Cx)+2x(bx(B+Cx)-A(b+2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)}\right.$$
$$+\frac{\sqrt{2}\left(-2Ac\left(-2b+\sqrt{b^2-4ac}\right)+\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{2}\left(-2Ac\left(2b+\sqrt{b^2-4ac}\right)+\left(b^2+4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$
$$\left.+\frac{2bB\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}}-\frac{2bB\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

input
```
Integrate[(x*(A*x + B*x^2 + C*x^3))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 -
 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ (Sqrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 -
 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqr
t[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

3.43.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {9, 2193, 27, 1434, 1159, 1083, 219, 1598, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(Ax+Bx^2+Cx^3\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{x^2\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \ 2193$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\int \frac{Bx^3}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \ 27$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+B\int \frac{x^3}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \ 1434$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\int \frac{x^2}{\left(cx^4+bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \ 1159$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{b\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}+\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \ 1083$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\int \frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)}{b^2-4ac}\right)$$

$$\Big\downarrow \ 219$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

$$\Big\downarrow \ 1598$$

$$\frac{\int \frac{-\left(\left(2Ac-bC\right)x^2\right)+Ab-2aC}{cx^4+bx^2+a}dx}{2\left(b^2-4ac\right)}-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

$$\Big\downarrow \ 1480$$

3.43. $\int \frac{x\left(Ax+Bx^2+Cx^3\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

$$\frac{-\frac{1}{2}\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d}{2\left(b^2-4ac\right)}$$

$$\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2\left(b^2-4ac\right)}-$$

$$\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

input
```
Int[(x*(A*x + B*x^2 + C*x^3))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*(x*(A*b - 2*a*C + (2*A*c - b*C)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) + (-(((2*A*c - b*C - (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*S
qrt[b - Sqrt[b^2 - 4*a*c]])) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*(b^2 - 4*a*c)) + (B*(
(2*a + b*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (2*b*ArcTanh[(b + 2*c*
x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/2
```

3.43.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.43. $\int \frac{x(Ax+Bx^2+Cx^3)}{(a+bx^2+cx^4)^2}\,dx$

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

3.43.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.57

method	result
risch	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{x^2 Bb}{2(4ac-b^2)}+\frac{(Ab-2Ca)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{cx^4+bx^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)}\frac{\left(\frac{(2Ac-Cb)\_R^2}{4ac-b^2}-\frac{2\_R Bb}{4ac-b^2}-\frac{Ab-2Ca}{4ac-b^2}\right)\ln\left(x-\_\right.}{2c\_R^3+\_R b}\right.}{4}$
default	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{x^2 Bb}{2(4ac-b^2)}+\frac{(Ab-2Ca)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{cx^4+bx^2+a}+\frac{2c\left(\frac{-B\sqrt{-4ac+b^2}\,b\ln\left(2cx^2+\sqrt{-4ac+b^2}+b\right)+\frac{\left(-4Abc\sqrt{-4ac+b^2}+8Aac^2-2Ab^2c+4C\right.}{}}{4c(4ac}\right.}{}$

input
```
int(x*(C*x^3+B*x^2+A*x)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2/(4*a*c-b^2)*x^2*B*b+1/2*(A*b-2*C*a)/(
4*a*c-b^2)*x-1/(4*a*c-b^2)*B*a)/(c*x^4+b*x^2+a)+1/4*sum(((2*A*c-C*b)/(4*a*
c-b^2)*_R^2-2/(4*a*c-b^2)*_R*B*b-(A*b-2*C*a)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*
ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.43.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x(Ax + Bx^2 + Cx^3)}{(a + bx^2 + cx^4)^2} dx = \text{Timed out}$$

input
```
integrate(x*(C*x^3+B*x^2+A*x)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.43.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(Ax + Bx^2 + Cx^3)}{(a + bx^2 + cx^4)^2} dx = \text{Timed out}$$

input
```
integrate(x*(C*x**3+B*x**2+A*x)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.43.7 Maxima [F]

$$\int \frac{x(Ax + Bx^2 + Cx^3)}{(a + bx^2 + cx^4)^2} dx = \int \frac{(Cx^3 + Bx^2 + Ax)x}{(cx^4 + bx^2 + a)^2} dx$$

input
```
integrate(x*(C*x^3+B*x^2+A*x)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*
c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*
x + (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

3.43.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4438 vs. $2(306) = 612$.

Time = 1.69 (sec) , antiderivative size = 4438, normalized size of antiderivative = 12.47

$$\int \frac{x(Ax + Bx^2 + Cx^3)}{(a + bx^2 + cx^4)^2} dx = \text{Too large to display}$$

input
```
integrate(x*(C*x^3+B*x^2+A*x)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*
x^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3
*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c
 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*
a*c)*b*c^2)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c
^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*
(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3...
```

3.43.9 Mupad [B] (verification not implemented)

Time = 8.21 (sec) , antiderivative size = 3835, normalized size of antiderivative = 10.77

$$\int \frac{x(Ax + Bx^2 + Cx^3)}{(a + bx^2 + cx^4)^2} dx = \text{Too large to display}$$

input
```
int((x*(A*x + B*x^2 + C*x^3))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*
A*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A
*C^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^
3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^
2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^
8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*
z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*
b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*
c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a
^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 819
2*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A
^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B
*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B
*a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*
c^2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c
+ 48*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*
C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c +
 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6
*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4
*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(25...
```

3.43. $\int \frac{x(Ax+Bx^2+Cx^3)}{(a+bx^2+cx^4)^2}\,dx$

3.44 $\int \frac{Ax^2+Bx^3+Cx^4}{(a+bx^2+cx^4)^2}\,dx$

3.44.1 Optimal result

Integrand size = 31, antiderivative size = 356

$$\int \frac{Ax^2+Bx^3+Cx^4}{(a+bx^2+cx^4)^2}\,dx = \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} - \frac{bB\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
1/2*B*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(A*b-2*C*a+(2*A*c-C*b
)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(3/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2))*(2*A*c-C*b+(-4*A*b*c+(4*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(
1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*A*c-C*b+(4*A*b*c-(4*a*c+b^2)*C)/(-4*
a*c+b^2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.44.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.06

$$\int \frac{Ax^2+Bx^3+Cx^4}{(a+bx^2+cx^4)^2}\,dx$$
$$=\frac{1}{4}\left(\frac{4a(B+Cx)+2x(bx(B+Cx)-A(b+2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)}\right.$$
$$+\frac{\sqrt{2}\left(-2Ac\left(-2b+\sqrt{b^2-4ac}\right)+\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{2}\left(-2Ac\left(2b+\sqrt{b^2-4ac}\right)+\left(b^2+4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$
$$\left.+\frac{2bB\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}}-\frac{2bB\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

input
```
Integrate[(A*x^2 + B*x^3 + C*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 -
 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ (Sqrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 -
 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqr
t[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

3.44.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {2028, 2193, 27, 1434, 1159, 1083, 219, 1598, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{Ax^2+Bx^3+Cx^4}{(a+bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{ 2028}$$

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{ 2193}$$

$$\int \frac{x^2(Cx^2+A)}{(cx^4+bx^2+a)^2}dx+\int \frac{Bx^3}{(cx^4+bx^2+a)^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x^2(Cx^2+A)}{(cx^4+bx^2+a)^2}dx+B\int \frac{x^3}{(cx^4+bx^2+a)^2}dx$$

$$\Big\downarrow \text{ 1434}$$

$$\int \frac{x^2(Cx^2+A)}{(cx^4+bx^2+a)^2}dx+\frac{1}{2}B\int \frac{x^2}{(cx^4+bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{ 1159}$$

$$\int \frac{x^2(Cx^2+A)}{(cx^4+bx^2+a)^2}dx+\frac{1}{2}B\left(\frac{b\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}+\frac{2a+bx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \text{ 1083}$$

$$\int \frac{x^2(Cx^2+A)}{(cx^4+bx^2+a)^2}dx+\frac{1}{2}B\left(\frac{2a+bx^2}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{2b\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{b^2-4ac}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\int \frac{x^2(Cx^2+A)}{(cx^4+bx^2+a)^2}dx+\frac{1}{2}B\left(\frac{2a+bx^2}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}\right)$$

$$\Big\downarrow \text{ 1598}$$

$$\frac{\int \frac{-((2Ac-bC)x^2)+Ab-2aC}{cx^4+bx^2+a}dx}{2(b^2-4ac)}-\frac{x(-2aC+x^2(2Ac-bC)+Ab)}{2(b^2-4ac)(a+bx^2+cx^4)}+$$
$$\frac{1}{2}B\left(\frac{2a+bx^2}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}\right)$$

$$\Big\downarrow \text{ 1480}$$

3.44. $\int \frac{Ax^2+Bx^3+Cx^4}{(a+bx^2+cx^4)^2}dx$

$$\frac{-\frac{1}{2}\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d}{2\left(b^2-4ac\right)}$$

$$\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

$$\downarrow 218$$

$$\frac{-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2\left(b^2-4ac\right)}-$$

$$\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

input
```
Int[(A*x^2 + B*x^3 + C*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*(x*(A*b - 2*a*C + (2*A*c - b*C)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) + (-(((2*A*c - b*C - (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*S
qrt[b - Sqrt[b^2 - 4*a*c]])) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*(b^2 - 4*a*c)) + (B*(
(2*a + b*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (2*b*ArcTanh[(b + 2*c*
x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/2
```

3.44.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2028
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(x_)^(t_.))^(p_.),
x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r))^p*Fx, x] /; FreeQ[
{a, b, c, r, s, t}, x] && IntegerQ[p] && PosQ[s - r] && PosQ[t - r] &&  !(E
qQ[p, 1] && EqQ[u, 1])
```

3.44. $\int \frac{Ax^2+Bx^3+Cx^4}{(a+bx^2+cx^4)^2}\,dx$

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

3.44.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.57

method	result
risch	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{x^2Bb}{2(4ac-b^2)}+\frac{(Ab-2Ca)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{c x^4+b x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)}\frac{\left(\frac{(2Ac-Cb)\_R^2}{4ac-b^2}-\frac{2\_RBb}{4ac-b^2}-\frac{Ab-2Ca}{4ac-b^2}\right)\ln\left(x-\_\right.}{2c\_R^3+\_Rb}\right.}{4}$
default	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{x^2Bb}{2(4ac-b^2)}+\frac{(Ab-2Ca)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{c x^4+b x^2+a}+\frac{2c\left(\frac{-B\sqrt{-4ac+b^2}\,b\ln\left(2c x^2+\sqrt{-4ac+b^2}+b\right)+\frac{\left(-4Abc\sqrt{-4ac+b^2}+8Aac^2-2Ab^2c+4C\right.}{}}{4c\left(4ac\right.}\right.}{}$

input
```
int((C*x^4+B*x^3+A*x^2)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2/(4*a*c-b^2)*x^2*B*b+1/2*(A*b-2*C*a)/(
4*a*c-b^2)*x-1/(4*a*c-b^2)*B*a)/(c*x^4+b*x^2+a)+1/4*sum(((2*A*c-C*b)/(4*a*
c-b^2)*_R^2-2/(4*a*c-b^2)*_R*B*b-(A*b-2*C*a)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*
ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.44.5 Fricas [F(-1)]

Timed out.

$$\int \frac{Ax^2 + Bx^3 + Cx^4}{(a + bx^2 + cx^4)^2} dx = \text{Timed out}$$

input
```
integrate((C*x^4+B*x^3+A*x^2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.44.6 Sympy [F(-1)]

Timed out.

$$\int \frac{Ax^2 + Bx^3 + Cx^4}{(a + bx^2 + cx^4)^2} dx = \text{Timed out}$$

input
```
integrate((C*x**4+B*x**3+A*x**2)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.44.7 Maxima [F]

$$\int \frac{Ax^2 + Bx^3 + Cx^4}{(a + bx^2 + cx^4)^2} dx = \int \frac{Cx^4 + Bx^3 + Ax^2}{(cx^4 + bx^2 + a)^2} dx$$

input
```
integrate((C*x^4+B*x^3+A*x^2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*
c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*
x + (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

3.44.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4438 vs. 2(306) = 612.

Time = 1.51 (sec) , antiderivative size = 4438, normalized size of antiderivative = 12.47

$$\int \frac{Ax^2 + Bx^3 + Cx^4}{(a + bx^2 + cx^4)^2}\, dx = \text{Too large to display}$$

input
```
integrate((C*x^4+B*x^3+A*x^2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*
x^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3
*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c
 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*
a*c)*b*c^2)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c
^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*
(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3...
```

3.44.9 Mupad [B] (verification not implemented)

Time = 8.13 (sec) , antiderivative size = 3835, normalized size of antiderivative = 10.77

$$\int \frac{Ax^2 + Bx^3 + Cx^4}{(a + bx^2 + cx^4)^2}\, dx = \text{Too large to display}$$

input
```
int((A*x^2 + B*x^3 + C*x^4)/(a + b*x^2 + c*x^4)^2,x)
```

output
```
symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*
A*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A
*C^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^
3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^
2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^
8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*
z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*
b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*
c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a
^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 819
2*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A
^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B
*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B
*a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*
c^2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c
+ 48*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*
C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c +
 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6
*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4
*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(25...
```

3.44. $\int \frac{Ax^2+Bx^3+Cx^4}{(a+bx^2+cx^4)^2}\,dx$

3.45 $\int \frac{Ax^3+Bx^4+Cx^5}{x(a+bx^2+cx^4)^2}\,dx$

3.45.1 Optimal result

Integrand size = 34, antiderivative size = 356

$$\int \frac{Ax^3+Bx^4+Cx^5}{x\left(a+bx^2+cx^4\right)^2}\,dx = \frac{B(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b+\sqrt{b^2-4ac}}} - \frac{bB\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

output
```
1/2*B*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(A*b-2*C*a+(2*A*c-C*b
)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(3/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2))*(2*A*c-C*b+(-4*A*b*c+(4*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(
1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*A*c-C*b+(4*A*b*c-(4*a*c+b^2)*C)/(-4*
a*c+b^2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.45.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.06

$$\int \frac{Ax^3 + Bx^4 + Cx^5}{x(a + bx^2 + cx^4)^2}\,dx$$
$$= \frac{1}{4}\left(\frac{4a(B + Cx) + 2x(bx(B + Cx) - A(b + 2cx^2))}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{2}\left(-2Ac\left(-2b + \sqrt{b^2 - 4ac}\right) + \left(-b^2 - 4ac + b\sqrt{b^2 - 4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{2}\left(-2Ac\left(2b + \sqrt{b^2 - 4ac}\right) + \left(b^2 + 4ac + b\sqrt{b^2 - 4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}(b^2 - 4ac)^{3/2}\sqrt{b + \sqrt{b^2 - 4ac}}} + \frac{2bB\log\left(-b + \sqrt{b^2 - 4ac} - 2cx^2\right)}{(b^2 - 4ac)^{3/2}} - \frac{2bB\log\left(b + \sqrt{b^2 - 4ac} + 2cx^2\right)}{(b^2 - 4ac)^{3/2}}\right)$$

input
```
Integrate[(A*x^3 + B*x^4 + C*x^5)/(x*(a + b*x^2 + c*x^4)^2),x]
```

output
```
((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 -
 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ (Sqrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 -
 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqr
t[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

3.45.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {9, 2193, 27, 1434, 1159, 1083, 219, 1598, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{Ax^3+Bx^4+Cx^5}{x\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{x^2\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \ 2193$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\int \frac{Bx^3}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \ 27$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+B\int \frac{x^3}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \ 1434$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\int \frac{x^2}{\left(cx^4+bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \ 1159$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{b\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}+\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \ 1083$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\int \frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)}{b^2-4ac}\right)$$

$$\Big\downarrow \ 219$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

$$\Big\downarrow \ 1598$$

$$\frac{\int \frac{-\left(\left(2Ac-bC\right)x^2\right)+Ab-2aC}{cx^4+bx^2+a}dx}{2\left(b^2-4ac\right)}-\frac{x\left(-2aC+x^2\left(2Ac-bC\right)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+$$
$$\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

$$\Big\downarrow \ 1480$$

3.45. $\int \frac{Ax^3+Bx^4+Cx^5}{x\left(a+bx^2+cx^4\right)^2}\,dx$

$$\frac{-\frac{1}{2}\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d}{2\left(b^2-4ac\right)}$$

$$\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

$$\Big\downarrow \; 218$$

$$\frac{-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2\left(b^2-4ac\right)}-$$

$$\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

input
```
Int[(A*x^3 + B*x^4 + C*x^5)/(x*(a + b*x^2 + c*x^4)^2),x]
```

output
```
-1/2*(x*(A*b - 2*a*C + (2*A*c - b*C)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) + (-(((2*A*c - b*C - (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*S
qrt[b - Sqrt[b^2 - 4*a*c]])) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*(b^2 - 4*a*c)) + (B*(
(2*a + b*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (2*b*ArcTanh[(b + 2*c*
x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/2
```

3.45.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.45. $\int \frac{Ax^3+Bx^4+Cx^5}{x(a+bx^2+cx^4)^2}\,dx$

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

3.45.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.57

method	result
risch	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{x^2Bb}{2(4ac-b^2)}+\frac{(Ab-2Ca)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{cx^4+bx^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)}\frac{\left(\frac{(2Ac-Cb)\_R^2}{4ac-b^2}-\frac{2\_RBb}{4ac-b^2}-\frac{Ab-2Ca}{4ac-b^2}\right)\ln(x-\_\cdots}{2c\_R^3+\_Rb}\right.}{4}$
default	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{x^2Bb}{2(4ac-b^2)}+\frac{(Ab-2Ca)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{cx^4+bx^2+a}+\frac{2c\left(\frac{-B\sqrt{-4ac+b^2}\,b\ln\left(2cx^2+\sqrt{-4ac+b^2}+b\right)+\frac{\left(-4Abc\sqrt{-4ac+b^2}+8Aac^2-2Ab^2c+4C\cdots\right.}{\cdots}}{4c(4ac\cdots}\right.}{\cdots}$

input
```
int((C*x^5+B*x^4+A*x^3)/x/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2/(4*a*c-b^2)*x^2*B*b+1/2*(A*b-2*C*a)/(
4*a*c-b^2)*x-1/(4*a*c-b^2)*B*a)/(c*x^4+b*x^2+a)+1/4*sum(((2*A*c-C*b)/(4*a*
c-b^2)*_R^2-2/(4*a*c-b^2)*_R*B*b-(A*b-2*C*a)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*
ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.45.5 Fricas [F(-1)]

Timed out.

$$\int \frac{Ax^3 + Bx^4 + Cx^5}{x\left(a + bx^2 + cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x^5+B*x^4+A*x^3)/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.45.6 Sympy [F(-1)]

Timed out.

$$\int \frac{Ax^3 + Bx^4 + Cx^5}{x\left(a + bx^2 + cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x**5+B*x**4+A*x**3)/x/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.45.7 Maxima [F]

$$\int \frac{Ax^3 + Bx^4 + Cx^5}{x\left(a + bx^2 + cx^4\right)^2}\,dx = \int \frac{Cx^5 + Bx^4 + Ax^3}{\left(cx^4 + bx^2 + a\right)^2 x}\,dx$$

input
```
integrate((C*x^5+B*x^4+A*x^3)/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*
c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*
x + (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

3.45.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4438 vs. $2(306) = 612$.

Time = 1.42 (sec) , antiderivative size = 4438, normalized size of antiderivative = 12.47

$$\int \frac{Ax^3 + Bx^4 + Cx^5}{x(a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate((C*x^5+B*x^4+A*x^3)/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*
x^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3
*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c
 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*
a*c)*b*c^2)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c
^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*
(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3...
```

3.45.9 Mupad [B] (verification not implemented)

Time = 8.19 (sec) , antiderivative size = 3835, normalized size of antiderivative = 10.77

$$\int \frac{Ax^3 + Bx^4 + Cx^5}{x(a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int((A*x^3 + B*x^4 + C*x^5)/(x*(a + b*x^2 + c*x^4)^2),x)
```

output
```
symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*
A*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A
*C^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^
3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^
2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^
8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*
z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*
b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*
c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a
^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 819
2*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A
^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B
*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B
*a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*
c^2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c
+ 48*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*
C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c +
 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6
*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4
*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(25...
```

3.45. $\int \frac{Ax^3+Bx^4+Cx^5}{x(a+bx^2+cx^4)^2}\,dx$

3.46 $\int \frac{Ax^4+Bx^5+Cx^6}{x^2(a+bx^2+cx^4)^2}\,dx$

3.46.1 Optimal result

Integrand size = 34, antiderivative size = 356

$$\begin{aligned}\int \frac{Ax^4+Bx^5+Cx^6}{x^2(a+bx^2+cx^4)^2}\,dx &= \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)}\\ &\quad - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}\\ &\quad - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}\\ &\quad - \frac{bB\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}\end{aligned}$$

output
```
1/2*B*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(A*b-2*C*a+(2*A*c-C*b
)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(3/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2))*(2*A*c-C*b+(-4*A*b*c+(4*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(
1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*A*c-C*b+(4*A*b*c-(4*a*c+b^2)*C)/(-4*
a*c+b^2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.46.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.06

$$\int \frac{Ax^4+Bx^5+Cx^6}{x^2(a+bx^2+cx^4)^2}\,dx$$
$$=\frac{1}{4}\left(\frac{4a(B+Cx)+2x(bx(B+Cx)-A(b+2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)}\right.$$
$$+\frac{\sqrt{2}\left(-2Ac\left(-2b+\sqrt{b^2-4ac}\right)+\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{2}\left(-2Ac\left(2b+\sqrt{b^2-4ac}\right)+\left(b^2+4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$
$$\left.+\frac{2bB\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}}-\frac{2bB\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

input
```
Integrate[(A*x^4 + B*x^5 + C*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

output
```
((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 -
 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ (Sqrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 -
 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqr
t[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

3.46.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {9, 2193, 27, 1434, 1159, 1083, 219, 1598, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{Ax^4+Bx^5+Cx^6}{x^2\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^2\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \; 2193$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\int \frac{Bx^3}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \; 27$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+B\int \frac{x^3}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \; 1434$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\int \frac{x^2}{\left(cx^4+bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \; 1159$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{b\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}+\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \; 1083$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\int \frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)}{b^2-4ac}\right)$$

$$\Big\downarrow \; 219$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

$$\Big\downarrow \; 1598$$

$$\frac{\int \frac{-\left(\left(2Ac-bC\right)x^2\right)+Ab-2aC}{cx^4+bx^2+a}dx}{2\left(b^2-4ac\right)}-\frac{x\left(-2aC+x^2\left(2Ac-bC\right)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

$$\Big\downarrow \; 1480$$

3.46. $\int \frac{Ax^4+Bx^5+Cx^6}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$

$$\frac{-\frac{1}{2}\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d}{2\left(b^2-4ac\right)}$$
$$\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

$$\downarrow 218$$

$$\frac{-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2\left(b^2-4ac\right)}-$$
$$\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

input
```
Int[(A*x^4 + B*x^5 + C*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

output
```
-1/2*(x*(A*b - 2*a*C + (2*A*c - b*C)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) + (-(((2*A*c - b*C - (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*S
qrt[b - Sqrt[b^2 - 4*a*c]])) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*(b^2 - 4*a*c)) + (B*(
(2*a + b*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (2*b*ArcTanh[(b + 2*c*
x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/2
```

3.46.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.46. $\int \frac{Ax^4+Bx^5+Cx^6}{x^2(a+bx^2+cx^4)^2}\,dx$

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

3.46.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.57

method	result
risch	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{x^2Bb}{2(4ac-b^2)}+\frac{(Ab-2Ca)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{cx^4+bx^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)}\frac{\left(\frac{(2Ac-Cb)\_R^2}{4ac-b^2}-\frac{2\_RBb}{4ac-b^2}-\frac{Ab-2Ca}{4ac-b^2}\right)\ln(x-\_}{2c\_R^3+\_Rb}\right.}{4}$
default	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{x^2Bb}{2(4ac-b^2)}+\frac{(Ab-2Ca)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{cx^4+bx^2+a}+\frac{2c\left(\frac{-B\sqrt{-4ac+b^2}\,b\ln\left(2cx^2+\sqrt{-4ac+b^2}+b\right)+\frac{\left(-4Abc\sqrt{-4ac+b^2}+8Aac^2-2Ab^2c+4C\right.}{}}{4c(4ac}\right.}{}$

input
```
int((C*x^6+B*x^5+A*x^4)/x^2/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2/(4*a*c-b^2)*x^2*B*b+1/2*(A*b-2*C*a)/(
4*a*c-b^2)*x-1/(4*a*c-b^2)*B*a)/(c*x^4+b*x^2+a)+1/4*sum(((2*A*c-C*b)/(4*a*
c-b^2)*_R^2-2/(4*a*c-b^2)*_R*B*b-(A*b-2*C*a)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*
ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.46.5 Fricas [F(-1)]

Timed out.

$$\int \frac{Ax^4 + Bx^5 + Cx^6}{x^2 (a + bx^2 + cx^4)^2} \, dx = \text{Timed out}$$

input
```
integrate((C*x^6+B*x^5+A*x^4)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.46.6 Sympy [F(-1)]

Timed out.

$$\int \frac{Ax^4 + Bx^5 + Cx^6}{x^2 (a + bx^2 + cx^4)^2} \, dx = \text{Timed out}$$

input
```
integrate((C*x**6+B*x**5+A*x**4)/x**2/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.46.7 Maxima [F]

$$\int \frac{Ax^4 + Bx^5 + Cx^6}{x^2 (a + bx^2 + cx^4)^2} \, dx = \int \frac{Cx^6 + Bx^5 + Ax^4}{(cx^4 + bx^2 + a)^2 x^2} \, dx$$

input
```
integrate((C*x^6+B*x^5+A*x^4)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*
c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*
x + (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

3.46.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4438 vs. 2(306) = 612.

Time = 1.50 (sec) , antiderivative size = 4438, normalized size of antiderivative = 12.47

$$\int \frac{Ax^4 + Bx^5 + Cx^6}{x^2 (a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate((C*x^6+B*x^5+A*x^4)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*
x^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3
*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c
 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*
a*c)*b*c^2)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c
^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*
(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3...
```

3.46.9 Mupad [B] (verification not implemented)

Time = 8.30 (sec) , antiderivative size = 3835, normalized size of antiderivative = 10.77

$$\int \frac{Ax^4 + Bx^5 + Cx^6}{x^2 (a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int((A*x^4 + B*x^5 + C*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x)
```

output

```
symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*
A*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A
*C^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^
3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^
2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^
8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*
z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*
b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*
c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a
^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 819
2*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A
^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B
*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B
*a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*
c^2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c
+ 48*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*
C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c +
 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6
*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4
*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(25...
```

3.46. $\int \frac{Ax^4+Bx^5+Cx^6}{x^2(a+bx^2+cx^4)^2}\,dx$

3.47 $\int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$

3.47.1 Optimal result

Integrand size = 30, antiderivative size = 273

$$\int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \frac{(b^2ce-ac^2e-b^3f-bc(cd-2af))\,x^2}{2c^4}+\frac{(c^2d+b^2f-c(be+af))\,x^4}{4c^3}+\frac{(ce-bf)x^6}{6c^2}+\frac{fx^8}{8c} - \frac{(b^4ce-4ab^2c^2e+2a^2c^3e-b^5f-b^3c(cd-5af)+abc^2(3cd-5af))\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^5\sqrt{b^2-4ac}} - \frac{(b^3ce-2abc^2e-b^4f-b^2c(cd-3af)+ac^2(cd-af))\log(a+bx^2+cx^4)}{4c^5}$$

output
```
1/2*(b^2*c*e-a*c^2*e-b^3*f-b*c*(-2*a*f+c*d))*x^2/c^4+1/4*(c^2*d+b^2*f-c*(a
*f+b*e))*x^4/c^3+1/6*(-b*f+c*e)*x^6/c^2+1/8*f*x^8/c-1/4*(b^3*c*e-2*a*b*c^2
*e-b^4*f-b^2*c*(-3*a*f+c*d)+a*c^2*(-a*f+c*d))*ln(c*x^4+b*x^2+a)/c^5-1/2*(b
^4*c*e-4*a*b^2*c^2*e+2*a^2*c^3*e-b^5*f-b^3*c*(-5*a*f+c*d)+a*b*c^2*(-5*a*f+
3*c*d))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^5/(-4*a*c+b^2)^(1/2)
```

3.47.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.95

$$\int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$$

$$= \frac{-12c(-b^2ce+ac^2e+b^3f+bc(cd-2af))\,x^2+6c^2(c^2d+b^2f-c(be+af))\,x^4+4c^3(ce-bf)x^6+3c^4fx}{}$$

input
```
Integrate[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

output
```
(-12*c*(-(b^2*c*e) + a*c^2*e + b^3*f + b*c*(c*d - 2*a*f))*x^2 + 6*c^2*(c^2
*d + b^2*f - c*(b*e + a*f))*x^4 + 4*c^3*(c*e - b*f)*x^6 + 3*c^4*f*x^8 - (1
2*(-(b^4*c*e) + 4*a*b^2*c^2*e - 2*a^2*c^3*e + b^5*f + b^3*c*(c*d - 5*a*f)
+ a*b*c^2*(-3*c*d + 5*a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt
[-b^2 + 4*a*c] + 6*(-(b^3*c*e) + 2*a*b*c^2*e + b^4*f + b^2*c*(c*d - 3*a*f)
 + a*c^2*(-(c*d) + a*f))*Log[a + b*x^2 + c*x^4])/(24*c^5)
```

3.47.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2194, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7\left(d+ex^2+fx^4\right)}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{x^6\left(fx^4+ex^2+d\right)}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{2159}$$

$$\frac{1}{2}\int \left(\frac{fx^6}{c}+\frac{(ce-bf)x^4}{c^2}+\frac{\left(fb^2+c^2d-c(be+af)\right)x^2}{c^3}+\frac{-fb^3+ceb^2-c(cd-2af)b-ac^2e}{c^4}+\frac{-\left(\left(-fb^4+ceb\right.\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(2a^2c^3e-b^3c(cd-5af)-4ab^2c^2e+abc^2(3cd-5af)+b^5(-f)+b^4ce\right)}{c^5\sqrt{b^2-4ac}}+\frac{x^4(-c(af+b\ldots}{2\ldots}\right.$$

input
```
Int[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

output
```
(((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d - 2*a*f))*x^2)/c^4 + ((c^2*d + b^2
*f - c*(b*e + a*f))*x^4)/(2*c^3) + ((c*e - b*f)*x^6)/(3*c^2) + (f*x^8)/(4*
c) - ((b^4*c*e - 4*a*b^2*c^2*e + 2*a^2*c^3*e - b^5*f - b^3*c*(c*d - 5*a*f)
 + a*b*c^2*(3*c*d - 5*a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^5
*Sqrt[b^2 - 4*a*c]) - ((b^3*c*e - 2*a*b*c^2*e - b^4*f - b^2*c*(c*d - 3*a*f
) + a*c^2*(c*d - a*f))*Log[a + b*x^2 + c*x^4])/(2*c^5))/2
```

3.47.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.47.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.21

method	result
default	$\frac{\frac{1}{4}f\,x^8c^3-\frac{1}{3}b\,c^2f\,x^6+\frac{1}{3}c^3e\,x^6-\frac{1}{2}a\,c^2f\,x^4+\frac{1}{2}b^2cf\,x^4-\frac{1}{2}b\,c^2e\,x^4+\frac{1}{2}c^3d\,x^4+2abcf\,x^2-a\,c^2e\,x^2-b^3f\,x^2+b^2ce\,x^2-b\,c^2d\,x^2}{2c^4}+\frac{\left(a^2c^2f-\ldots\right.}{\ldots}$
risch	Expression too large to display

input
```
int(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/c^4*(1/4*f*x^8*c^3-1/3*b*c^2*f*x^6+1/3*c^3*e*x^6-1/2*a*c^2*f*x^4+1/2*b
^2*c*f*x^4-1/2*b*c^2*e*x^4+1/2*c^3*d*x^4+2*a*b*c*f*x^2-a*c^2*e*x^2-b^3*f*x
^2+b^2*c*e*x^2-b*c^2*d*x^2)+1/2/c^4*(1/2*(a^2*c^2*f-3*a*b^2*c*f+2*a*b*c^2*
e-a*c^3*d+b^4*f-b^3*c*e+b^2*c^2*d)/c*ln(c*x^4+b*x^2+a)+2*(-2*a^2*b*c*f+a^2
*c^2*e+a*b^3*f-a*b^2*c*e+a*b*c^2*d-1/2*(a^2*c^2*f-3*a*b^2*c*f+2*a*b*c^2*e-
a*c^3*d+b^4*f-b^3*c*e+b^2*c^2*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)
/(4*a*c-b^2)^(1/2)))
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 900, normalized size of antiderivative = 3.30

$$\int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$$
$$= \left[\frac{3\,(b^2c^4-4\,ac^5)fx^8+4\,((b^2c^4-4\,ac^5)e-(b^3c^3-4\,abc^4)f)x^6+6\,((b^2c^4-4\,ac^5)d-(b^3c^3-4\,abc^4)e+(\cdots}{\cdots}\right.$$

input
```
integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[1/24*(3*(b^2*c^4 - 4*a*c^5)*f*x^8 + 4*((b^2*c^4 - 4*a*c^5)*e - (b^3*c^3 -
 4*a*b*c^4)*f)*x^6 + 6*((b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e +
(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*f)*x^4 - 12*((b^3*c^3 - 4*a*b*c^4)*d -
 (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e + (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^
3)*f)*x^2 + 6*sqrt(b^2 - 4*a*c)*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^
2*c^2 + 2*a^2*c^3)*e + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f)*log((2*c^2*x^4 +
 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2
 + a)) + 6*((b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d - (b^5*c - 6*a*b^3*c^2 +
 8*a^2*b*c^3)*e + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*f)*log(c*
x^4 + b*x^2 + a))/(b^2*c^5 - 4*a*c^6), 1/24*(3*(b^2*c^4 - 4*a*c^5)*f*x^8 +
 4*((b^2*c^4 - 4*a*c^5)*e - (b^3*c^3 - 4*a*b*c^4)*f)*x^6 + 6*((b^2*c^4 - 4
*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*
f)*x^4 - 12*((b^3*c^3 - 4*a*b*c^4)*d - (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)
*e + (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*f)*x^2 + 12*sqrt(-b^2 + 4*a*c)*((
b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e + (b^5 - 5*a*
b^3*c + 5*a^2*b*c^2)*f)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*
a*c)) + 6*((b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d - (b^5*c - 6*a*b^3*c^2 +
8*a^2*b*c^3)*e + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*f)*log(c*x
^4 + b*x^2 + a))/(b^2*c^5 - 4*a*c^6)]
```

3.47.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**7*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.47.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.09

$$
\begin{aligned}
\int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx
&= \frac{3\,c^3fx^8+4\,c^3ex^6-4\,bc^2fx^6+6\,c^3dx^4-6\,bc^2ex^4+6\,b^2cfx^4-6\,ac^2fx^4-12\,bc^2dx^2+12\,b^2cex^2-12\,ac^2}{24\,c^4} \\
&\quad + \frac{(b^2c^2d-ac^3d-b^3ce+2\,abc^2e+b^4f-3\,ab^2cf+a^2c^2f)\log\left(cx^4+bx^2+a\right)}{4\,c^5} \\
&\quad - \frac{(b^3c^2d-3\,abc^3d-b^4ce+4\,ab^2c^2e-2\,a^2c^3e+b^5f-5\,ab^3cf+5\,a^2bc^2f)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^5}
\end{aligned}
$$

input
```
integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/24*(3*c^3*f*x^8 + 4*c^3*e*x^6 - 4*b*c^2*f*x^6 + 6*c^3*d*x^4 - 6*b*c^2*e*
x^4 + 6*b^2*c*f*x^4 - 6*a*c^2*f*x^4 - 12*b*c^2*d*x^2 + 12*b^2*c*e*x^2 - 12
*a*c^2*e*x^2 - 12*b^3*f*x^2 + 24*a*b*c*f*x^2)/c^4 + 1/4*(b^2*c^2*d - a*c^3
*d - b^3*c*e + 2*a*b*c^2*e + b^4*f - 3*a*b^2*c*f + a^2*c^2*f)*log(c*x^4 +
b*x^2 + a)/c^5 - 1/2*(b^3*c^2*d - 3*a*b*c^3*d - b^4*c*e + 4*a*b^2*c^2*e -
2*a^2*c^3*e + b^5*f - 5*a*b^3*c*f + 5*a^2*b*c^2*f)*arctan((2*c*x^2 + b)/sq
rt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^5)
```

3.47.9 Mupad [B] (verification not implemented)

Time = 8.52 (sec) , antiderivative size = 2972, normalized size of antiderivative = 10.89

$$\int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)
```

output
```
x^6*(e/(6*c) - (b*f)/(6*c^2)) - x^4*((b*(e/c - (b*f)/c^2))/(4*c) - d/(4*c)
 + (a*f)/(4*c^2)) - x^2*((a*(e/c - (b*f)/c^2))/(2*c) - (b*((b*(e/c - (b*f)
/c^2))/c - d/c + (a*f)/c^2))/(2*c)) + (f*x^8)/(8*c) - (log(a + b*x^2 + c*x
^4)*(2*b^6*f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5*c*e + 26*a^
2*b^2*c^2*f - 14*a*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e - 16*a^2*b*c^
3*e))/(2*(16*a*c^6 - 4*b^2*c^5)) + (atan((2*c^8*(4*a*c - b^2)*(x^2*(((((4*
a^2*c^8*e - 6*b^3*c^7*d + 6*b^4*c^6*e - 6*b^5*c^5*f + 10*a*b*c^8*d - 16*a*
b^2*c^7*e + 22*a*b^3*c^6*f - 14*a^2*b*c^7*f)/c^8 - (4*b*c^2*(2*b^6*f + 8*a
^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5*c*e + 26*a^2*b^2*c^2*f - 14*a
*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e - 16*a^2*b*c^3*e))/(16*a*c^6 -
4*b^2*c^5))*(b^5*f - 2*a^2*c^3*e + b^3*c^2*d - b^4*c*e - 3*a*b*c^3*d - 5*a
*b^3*c*f + 4*a*b^2*c^2*e + 5*a^2*b*c^2*f))/(8*c^5*(4*a*c - b^2)^(1/2)) - (
b*(b^5*f - 2*a^2*c^3*e + b^3*c^2*d - b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*f +
 4*a*b^2*c^2*e + 5*a^2*b*c^2*f)*(2*b^6*f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a
^3*c^3*f - 2*b^5*c*e + 26*a^2*b^2*c^2*f - 14*a*b^4*c*f - 10*a*b^2*c^3*d +
12*a*b^3*c^2*e - 16*a^2*b*c^3*e))/(2*c^3*(4*a*c - b^2)^(1/2)*(16*a*c^6 - 4
*b^2*c^5)))/a - (b*((((4*a^2*c^8*e - 6*b^3*c^7*d + 6*b^4*c^6*e - 6*b^5*c^5
*f + 10*a*b*c^8*d - 16*a*b^2*c^7*e + 22*a*b^3*c^6*f - 14*a^2*b*c^7*f)/c^8
- (4*b*c^2*(2*b^6*f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5*c*e
+ 26*a^2*b^2*c^2*f - 14*a*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e - 1...
```

3.47. $\int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$

3.48 $\int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$

3.48.1 Optimal result

Integrand size = 30, antiderivative size = 203

$$\begin{aligned}\int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx &= \frac{(c^2d+b^2f-c(be+af))\,x^2}{2c^3}+\frac{(ce-bf)x^4}{4c^2}+\frac{fx^6}{6c}\\ &\quad+\frac{(b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2ac^2(cd-af))\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^4\sqrt{b^2-4ac}}\\ &\quad+\frac{(b^2ce-ac^2e-b^3f-bc(cd-2af))\log\left(a+bx^2+cx^4\right)}{4c^4}\end{aligned}$$

output
```
1/2*(c^2*d+b^2*f-c*(a*f+b*e))*x^2/c^3+1/4*(-b*f+c*e)*x^4/c^2+1/6*f*x^6/c+1
/4*(b^2*c*e-a*c^2*e-b^3*f-b*c*(-2*a*f+c*d))*ln(c*x^4+b*x^2+a)/c^4+1/2*(b^3
*c*e-3*a*b*c^2*e-b^4*f-b^2*c*(-4*a*f+c*d)+2*a*c^2*(-a*f+c*d))*arctanh((2*c
*x^2+b)/(-4*a*c+b^2)^(1/2))/c^4/(-4*a*c+b^2)^(1/2)
```

3.48.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.95

$$\int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$$
$$= \frac{6c(c^2d+b^2f-c(be+af))\,x^2+3c^2(ce-bf)x^4+2c^3fx^6+\frac{6(-b^3ce+3abc^2e+b^4f+b^2c(cd-4af)+2ac^2(-cd+af))\arctan}{\sqrt{-b^2+4ac}}}{12c^4}$$

input
```
Integrate[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

output
```
(6*c*(c^2*d + b^2*f - c*(b*e + a*f))*x^2 + 3*c^2*(c*e - b*f)*x^4 + 2*c^3*f
*x^6 + (6*(-(b^3*c*e) + 3*a*b*c^2*e + b^4*f + b^2*c*(c*d - 4*a*f) + 2*a*c^
2*(-(c*d) + a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*
a*c] - 3*(-(b^2*c*e) + a*c^2*e + b^3*f + b*c*(c*d - 2*a*f))*Log[a + b*x^2
+ c*x^4])/(12*c^4)
```

3.48.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2194, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{x^4(fx^4+ex^2+d)}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{2159}$$

$$\frac{1}{2}\int \left(\frac{fx^4}{c}+\frac{(ce-bf)x^2}{c^2}+\frac{fb^2+c^2d-c(be+af)}{c^3}-\frac{a(fb^2+c^2d-c(be+af))-(-fb^3+ceb^2-c(cd-2af)b}{c^3(cx^4+bx^2+a)}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f)+b^3ce\right)}{c^4\sqrt{b^2-4ac}}+\frac{x^2\left(-c(af+be)+b^2f+c\right.}{c^3}\right.$$

input
```
Int[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

output
```
(((c^2*d + b^2*f - c*(b*e + a*f))*x^2)/c^3 + ((c*e - b*f)*x^4)/(2*c^2) + (
f*x^6)/(3*c) + ((b^3*c*e - 3*a*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*f) + 2*a
*c^2*(c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^4*Sqrt[b^2
- 4*a*c]) + ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d - 2*a*f))*Log[a + b*x^2
 + c*x^4])/(2*c^4))/2
```

3.48.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.48.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.10

method	result
default	$-\frac{-\frac{1}{3}f\,x^6c^2+\frac{1}{2}bcf\,x^4-\frac{1}{2}c^2e\,x^4+acf\,x^2-b^2f\,x^2+bce\,x^2-c^2d\,x^2}{2c^3}+\frac{\frac{\left(2abcf-a\,c^2e-b^3f+b^2ce-b\,c^2d\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(a^2cf-a\,b^2f-\right.}{}}{2}$
risch	Expression too large to display

input
```
int(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/c^3*(-1/3*f*x^6*c^2+1/2*b*c*f*x^4-1/2*c^2*e*x^4+a*c*f*x^2-b^2*f*x^2+b
*c*e*x^2-c^2*d*x^2)+1/2/c^3*(1/2*(2*a*b*c*f-a*c^2*e-b^3*f+b^2*c*e-b*c^2*d)
/c*ln(c*x^4+b*x^2+a)+2*(a^2*c*f-a*b^2*f+a*b*c*e-a*c^2*d-1/2*(2*a*b*c*f-a*c
^2*e-b^3*f+b^2*c*e-b*c^2*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a
*c-b^2)^(1/2)))
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 677, normalized size of antiderivative = 3.33

$$\int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$$
$$= \left[\frac{2\,(b^2c^3-4\,ac^4)fx^6+3\,((b^2c^3-4\,ac^4)e-(b^3c^2-4\,abc^3)f)x^4+6\,((b^2c^3-4\,ac^4)d-(b^3c^2-4\,abc^3)e+(}{} \right.$$

input
```
integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[1/12*(2*(b^2*c^3 - 4*a*c^4)*f*x^6 + 3*((b^2*c^3 - 4*a*c^4)*e - (b^3*c^2 -
 4*a*b*c^3)*f)*x^4 + 6*((b^2*c^3 - 4*a*c^4)*d - (b^3*c^2 - 4*a*b*c^3)*e +
(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*f)*x^2 + 3*sqrt(b^2 - 4*a*c)*((b^2*c^2 -
 2*a*c^3)*d - (b^3*c - 3*a*b*c^2)*e + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*f)*log
((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(
c*x^4 + b*x^2 + a)) - 3*((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 5*a*b^2*c^2 +
4*a^2*c^3)*e + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*f)*log(c*x^4 + b*x^2 + a))/
(b^2*c^4 - 4*a*c^5), 1/12*(2*(b^2*c^3 - 4*a*c^4)*f*x^6 + 3*((b^2*c^3 - 4*a
*c^4)*e - (b^3*c^2 - 4*a*b*c^3)*f)*x^4 + 6*((b^2*c^3 - 4*a*c^4)*d - (b^3*c
^2 - 4*a*b*c^3)*e + (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*f)*x^2 - 6*sqrt(-b^2
 + 4*a*c)*((b^2*c^2 - 2*a*c^3)*d - (b^3*c - 3*a*b*c^2)*e + (b^4 - 4*a*b^2*
c + 2*a^2*c^2)*f)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))
- 3*((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e + (b^5
- 6*a*b^3*c + 8*a^2*b*c^2)*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^4 - 4*a*c^5)]
```

3.48.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.48.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx &= \frac{2\,c^2fx^6+3\,c^2ex^4-3\,bcfx^4+6\,c^2dx^2-6\,bcex^2+6\,b^2fx^2-6\,acfx^2}{12\,c^3} \\ &\quad - \frac{(bc^2d-b^2ce+ac^2e+b^3f-2\,abcf)\log\left(cx^4+bx^2+a\right)}{4\,c^4} \\ &\quad + \frac{(b^2c^2d-2\,ac^3d-b^3ce+3\,abc^2e+b^4f-4\,ab^2cf+2\,a^2c^2f)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^4}\end{aligned}$$

input
```
integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/12*(2*c^2*f*x^6 + 3*c^2*e*x^4 - 3*b*c*f*x^4 + 6*c^2*d*x^2 - 6*b*c*e*x^2
+ 6*b^2*f*x^2 - 6*a*c*f*x^2)/c^3 - 1/4*(b*c^2*d - b^2*c*e + a*c^2*e + b^3*
f - 2*a*b*c*f)*log(c*x^4 + b*x^2 + a)/c^4 + 1/2*(b^2*c^2*d - 2*a*c^3*d - b
^3*c*e + 3*a*b*c^2*e + b^4*f - 4*a*b^2*c*f + 2*a^2*c^2*f)*arctan((2*c*x^2
+ b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^4)
```

3.48.9 Mupad [B] (verification not implemented)

Time = 8.37 (sec) , antiderivative size = 2295, normalized size of antiderivative = 11.31

$$\int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)
```

output
```
x^4*(e/(4*c) - (b*f)/(4*c^2)) - x^2*((b*(e/c - (b*f)/c^2))/(2*c) - d/(2*c)
 + (a*f)/(2*c^2)) + (log(a + b*x^2 + c*x^4)*(2*b^5*f - 8*a^2*c^3*e + 2*b^3
*c^2*d - 2*b^4*c*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*
b*c^2*f))/(2*(16*a*c^5 - 4*b^2*c^4)) + (f*x^6)/(6*c) + (atan((2*c^6*(4*a*c
 - b^2)*(x^2*(((((6*b^2*c^6*d + 4*a^2*c^6*f - 6*b^3*c^5*e + 6*b^4*c^4*f -
4*a*c^7*d + 10*a*b*c^6*e - 16*a*b^2*c^5*f)/c^6 + (4*b*c^2*(2*b^5*f - 8*a^2
*c^3*e + 2*b^3*c^2*d - 2*b^4*c*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c
^2*e + 16*a^2*b*c^2*f))/(16*a*c^5 - 4*b^2*c^4))*(b^4*f + b^2*c^2*d + 2*a^2
*c^2*f - 2*a*c^3*d - b^3*c*e + 3*a*b*c^2*e - 4*a*b^2*c*f))/(8*c^4*(4*a*c -
 b^2)^(1/2)) + (b*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*d - b^3*c*e +
 3*a*b*c^2*e - 4*a*b^2*c*f)*(2*b^5*f - 8*a^2*c^3*e + 2*b^3*c^2*d - 2*b^4*c
*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*b*c^2*f))/(2*c^2
*(4*a*c - b^2)^(1/2)*(16*a*c^5 - 4*b^2*c^4)))/a - (b*((b^7*f^2 + b^3*c^4*d
^2 + b^5*c^2*e^2 - 3*a*b^3*c^3*e^2 + 2*a^2*b*c^4*e^2 - 2*a^3*b*c^3*f^2 - 2
*b^6*c*e*f + 7*a^2*b^3*c^2*f^2 - a*b*c^5*d^2 - 5*a*b^5*c*f^2 - a^2*c^5*d*e
 - 2*b^4*c^3*d*e + a^3*c^4*e*f + 2*b^5*c^2*d*f + 4*a*b^2*c^4*d*e - 6*a*b^3
*c^3*d*f + 3*a^2*b*c^4*d*f + 8*a*b^4*c^2*e*f - 8*a^2*b^2*c^3*e*f)/c^6 + ((
(6*b^2*c^6*d + 4*a^2*c^6*f - 6*b^3*c^5*e + 6*b^4*c^4*f - 4*a*c^7*d + 10*a*
b*c^6*e - 16*a*b^2*c^5*f)/c^6 + (4*b*c^2*(2*b^5*f - 8*a^2*c^3*e + 2*b^3*c^
2*d - 2*b^4*c*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*...
```

3.49 $\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$

3.49.1 Optimal result

Integrand size = 30, antiderivative size = 144

$$\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \frac{(ce-bf)x^2}{2c^2}+\frac{fx^4}{4c} - \frac{(b^2ce-2ac^2e-b^3f-bc(cd-3af))\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}} + \frac{(c^2d+b^2f-c(be+af))\log(a+bx^2+cx^4)}{4c^3}$$

output
```
1/2*(-b*f+c*e)*x^2/c^2+1/4*f*x^4/c+1/4*(c^2*d+b^2*f-c*(a*f+b*e))*ln(c*x^4+
b*x^2+a)/c^3-1/2*(b^2*c*e-2*a*c^2*e-b^3*f-b*c*(-3*a*f+c*d))*arctanh((2*c*x
^2+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)
```

3.49.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.94

$$\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \frac{2c(ce-bf)x^2+c^2fx^4-\frac{2(-b^2ce+2ac^2e+b^3f+bc(cd-3af))\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(c^2d+b^2f-c(be+af))\log(a+b}{4c^3}$$

input
```
Integrate[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

output
```
(2*c*(c*e - b*f)*x^2 + c^2*f*x^4 - (2*(-(b^2*c*e) + 2*a*c^2*e + b^3*f + b*
c*(c*d - 3*a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a
*c] + (c^2*d + b^2*f - c*(b*e + a*f))*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

3.49.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2194, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(d+ex^2+fx^4\right)}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \text{ 2194}$$

$$\frac{1}{2}\int \frac{x^2\left(fx^4+ex^2+d\right)}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{ 2159}$$

$$\frac{1}{2}\int \left(\frac{fx^2}{c}+\frac{ce-bf}{c^2}-\frac{a(ce-bf)-\left(fb^2-ceb+c^2d-acf\right)x^2}{c^2\left(cx^4+bx^2+a\right)}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{c^3\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)\left(-c(af+be)+b^2f+c\right.}{2c^3}\right.$$

input
```
Int[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

output
```
(((c*e - b*f)*x^2)/c^2 + (f*x^4)/(2*c) - ((b^2*c*e - 2*a*c^2*e - b^3*f - b
*c*(c*d - 3*a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2
- 4*a*c]) + ((c^2*d + b^2*f - c*(b*e + a*f))*Log[a + b*x^2 + c*x^4])/(2*c^
3))/2
```

3.49.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.49.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.01

method	result	siz
default	$-\frac{-\frac{1}{2}cf\,x^{4}+bf\,x^{2}-c\,x^{2}e}{2c^{2}}+\frac{\frac{\left(-acf+b^{2}f-ebc+c^{2}d\right)\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2c}+\frac{2\left(abf-ace-\frac{\left(-acf+b^{2}f-ebc+c^{2}d\right)b}{2c}\right)\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2c^{2}}$	14
risch	Expression too large to display	32

input
```
int(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/c^2*(-1/2*c*f*x^4+b*f*x^2-c*x^2*e)+1/2/c^2*(1/2*(-a*c*f+b^2*f-b*c*e+c
^2*d)/c*ln(c*x^4+b*x^2+a)+2*(a*b*f-a*c*e-1/2*(-a*c*f+b^2*f-b*c*e+c^2*d)*b/
c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 473, normalized size of antiderivative = 3.28

$$\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$$
$$= \left[\frac{(b^2c^2-4\,ac^3)fx^4+2\left((b^2c^2-4\,ac^3)e-(b^3c-4\,abc^2)f\right)x^2-(bc^2d-(b^2c-2\,ac^2)e+(b^3-3\,abc)f)\sqrt{b^2}}{}\right.$$

input
```
integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*((b^2*c^2 - 4*a*c^3)*f*x^4 + 2*((b^2*c^2 - 4*a*c^3)*e - (b^3*c - 4*a*
b*c^2)*f)*x^2 - (b*c^2*d - (b^2*c - 2*a*c^2)*e + (b^3 - 3*a*b*c)*f)*sqrt(b
^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(
b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a
*b*c^2)*e + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*f)*log(c*x^4 + b*x^2 + a))/(b^2*
c^3 - 4*a*c^4), 1/4*((b^2*c^2 - 4*a*c^3)*f*x^4 + 2*((b^2*c^2 - 4*a*c^3)*e
- (b^3*c - 4*a*b*c^2)*f)*x^2 + 2*(b*c^2*d - (b^2*c - 2*a*c^2)*e + (b^3 - 3
*a*b*c)*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^
2 - 4*a*c)) + ((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e + (b^4 - 5*a*
b^2*c + 4*a^2*c^2)*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4)]
```

3.49.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.49.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.95

$$\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \frac{cfx^4+2\,cex^2-2\,bfx^2}{4\,c^2} + \frac{(c^2d-bce+b^2f-acf)\log\left(cx^4+bx^2+a\right)}{4\,c^3} - \frac{(bc^2d-b^2ce+2\,ac^2e+b^3f-3\,abcf)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^3}$$

input
```
integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/4*(c*f*x^4 + 2*c*e*x^2 - 2*b*f*x^2)/c^2 + 1/4*(c^2*d - b*c*e + b^2*f - a
*c*f)*log(c*x^4 + b*x^2 + a)/c^3 - 1/2*(b*c^2*d - b^2*c*e + 2*a*c^2*e + b^
3*f - 3*a*b*c*f)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a
*c)*c^3)
```

3.49.9 Mupad [B] (verification not implemented)

Time = 8.38 (sec) , antiderivative size = 1689, normalized size of antiderivative = 11.73

$$\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)
```

output
```
x^2*(e/(2*c) - (b*f)/(2*c^2)) + (f*x^4)/(4*c) - (log(a + b*x^2 + c*x^4)*(2
*b^4*f + 2*b^2*c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b^3*c*e + 8*a*b*c^2*e -
 10*a*b^2*c*f))/(2*(16*a*c^4 - 4*b^2*c^3)) - (atan((2*c^4*(4*a*c - b^2)*(x
^2*(((((6*b^3*c^3*f - 6*b^2*c^4*e + 4*a*c^5*e + 6*b*c^5*d - 10*a*b*c^4*f)/
c^4 + (4*b*c^2*(2*b^4*f + 2*b^2*c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b^3*c*
e + 8*a*b*c^2*e - 10*a*b^2*c*f))/(16*a*c^4 - 4*b^2*c^3))*(b^3*f + 2*a*c^2*
e + b*c^2*d - b^2*c*e - 3*a*b*c*f))/(8*c^3*(4*a*c - b^2)^(1/2)) + (b*(b^3*
f + 2*a*c^2*e + b*c^2*d - b^2*c*e - 3*a*b*c*f)*(2*b^4*f + 2*b^2*c^2*d + 8*
a^2*c^2*f - 8*a*c^3*d - 2*b^3*c*e + 8*a*b*c^2*e - 10*a*b^2*c*f))/(2*c*(4*a
*c - b^2)^(1/2)*(16*a*c^4 - 4*b^2*c^3)))/a - (b*((b^5*f^2 + b*c^4*d^2 + b^
3*c^2*e^2 + 2*a^2*b*c^2*f^2 + a*c^4*d*e - 2*b^4*c*e*f - a*b*c^3*e^2 - 3*a*
b^3*c*f^2 - 2*b^2*c^3*d*e - a^2*c^3*e*f + 2*b^3*c^2*d*f + 4*a*b^2*c^2*e*f
- 3*a*b*c^3*d*f)/c^4 + (((6*b^3*c^3*f - 6*b^2*c^4*e + 4*a*c^5*e + 6*b*c^5*
d - 10*a*b*c^4*f)/c^4 + (4*b*c^2*(2*b^4*f + 2*b^2*c^2*d + 8*a^2*c^2*f - 8*
a*c^3*d - 2*b^3*c*e + 8*a*b*c^2*e - 10*a*b^2*c*f))/(16*a*c^4 - 4*b^2*c^3))
*(2*b^4*f + 2*b^2*c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b^3*c*e + 8*a*b*c^2*
e - 10*a*b^2*c*f))/(2*(16*a*c^4 - 4*b^2*c^3)) - (b*(b^3*f + 2*a*c^2*e + b*
c^2*d - b^2*c*e - 3*a*b*c*f)^2)/(2*c^4*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)
^(1/2)) - ((((8*a^2*c^4*f - 8*a*c^5*d + 8*a*b*c^4*e - 8*a*b^2*c^3*f)/c^4
- (8*a*c^2*(2*b^4*f + 2*b^2*c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b^3*c*e...
```

3.50 $\int \frac{x(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$

3.50.1 Optimal result

Integrand size = 28, antiderivative size = 103

$$\int \frac{x(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \frac{fx^2}{2c} - \frac{(2c^2d-bce+b^2f-2acf)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}} + \frac{(ce-bf)\log(a+bx^2+cx^4)}{4c^2}$$

output
```
1/2*f*x^2/c+1/4*(-b*f+c*e)*ln(c*x^4+b*x^2+a)/c^2-1/2*(-2*a*c*f+b^2*f-b*c*e
+2*c^2*d)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

3.50.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.97

$$\int \frac{x(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \frac{2cfx^2 + \frac{2(2c^2d+b^2f-c(be+2af))\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + (ce-bf)\log(a+bx^2+cx^4)}{4c^2}$$

input
```
Integrate[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

output
```
(2*c*f*x^2 + (2*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*ArcTan[(b + 2*c*x^2)/S
qrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (c*e - b*f)*Log[a + b*x^2 + c*x^4
])/(4*c^2)
```

3.50.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2194, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(d+ex^2+fx^4\right)}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{fx^4+ex^2+d}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{2188}$$

$$\frac{1}{2}\int \left(\frac{f}{c}+\frac{(ce-bf)x^2+cd-af}{c\left(cx^4+bx^2+a\right)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-2acf+b^2f-bce+2c^2d\right)}{c^2\sqrt{b^2-4ac}}+\frac{(ce-bf)\log\left(a+bx^2+cx^4\right)}{2c^2}+\frac{fx^2}{c}\right)$$

input
```
Int[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

output
```
((f*x^2)/c - ((2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*ArcTanh[(b + 2*c*x^2)/Sq
rt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) + ((c*e - b*f)*Log[a + b*x^2 + c
*x^4])/(2*c^2))/2
```

3.50.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.50.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{f\,x^{2}}{2c}+\frac{\frac{(-bf+ec)\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2c}+\frac{2\left(-af+cd-\frac{(-bf+ec)b}{2c}\right)\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2c}$	101
risch	Expression too large to display	1690

input
```
int(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*f*x^2/c+1/2/c*(1/2*(-b*f+c*e)/c*ln(c*x^4+b*x^2+a)+2*(-a*f+c*d-1/2*(-b*
f+c*e)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.09

$$\int \frac{x(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$$
$$=\left[\frac{2\,(b^2c-4\,ac^2)fx^2-(2\,c^2d-bce+(b^2-2\,ac)f)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+(}{4\,(b^2c^2-4\,ac^3)}\right.$$

input
```
integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*(2*(b^2*c - 4*a*c^2)*f*x^2 - (2*c^2*d - b*c*e + (b^2 - 2*a*c)*f)*sqrt
(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqr
t(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((b^2*c - 4*a*c^2)*e - (b^3 - 4*a*b
*c)*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3), 1/4*(2*(b^2*c - 4*a*c^
2)*f*x^2 - 2*(2*c^2*d - b*c*e + (b^2 - 2*a*c)*f)*sqrt(-b^2 + 4*a*c)*arctan
(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + ((b^2*c - 4*a*c^2)*e -
 (b^3 - 4*a*b*c)*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3)]
```

3.50.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(d + ex^2 + fx^4)}{a + bx^2 + cx^4} \, dx = \text{Timed out}$$

input
```
integrate(x*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.50.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(d + ex^2 + fx^4)}{a + bx^2 + cx^4} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.50.8 Giac [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.94

$$\int \frac{x(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \frac{fx^2}{2\,c} + \frac{(ce-bf)\log\left(cx^4+bx^2+a\right)}{4\,c^2} + \frac{(2\,c^2d-bce+b^2f-2\,acf)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^2}$$

input
```
integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/2*f*x^2/c + 1/4*(c*e - b*f)*log(c*x^4 + b*x^2 + a)/c^2 + 1/2*(2*c^2*d -
b*c*e + b^2*f - 2*a*c*f)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b
^2 + 4*a*c)*c^2)
```

3.50.9 Mupad [B] (verification not implemented)

Time = 8.83 (sec) , antiderivative size = 1081, normalized size of antiderivative = 10.50

$$\int \frac{x(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \frac{f\,x^2}{2\,c} + \frac{\ln\left(c\,x^4+b\,x^2+a\right)\,\left(2\,f\,b^3-2\,e\,b^2\,c-8\,a\,f\,b\,c+8\,a\,e\,c^2\right)}{2\,\left(16\,a\,c^3-4\,b^2\,c^2\right)}$$

$$+ \frac{\operatorname{atan}\left(\frac{2\,c^2\,\left(4\,a\,c-b^2\right)\left(x^2\left(\frac{\frac{\left(\frac{6\,f\,b^2\,c^2-6\,e\,b\,c^3+4\,d\,c^4-4\,a\,f\,c^3}{c^2}+\frac{4\,b\,c^2\,\left(2\,f\,b^3-2\,e\,b^2\,c-8\,a\,f\,b\,c+8\,a\,e\,c^2\right)}{16\,a\,c^3-4\,b^2\,c^2}\right)\left(f\,b^2-e\,b\,c+2\,d\,c^2-2\,a\,f\,c\right)}{8\,c^2\,\sqrt{4\,a\,c-b^2}}+\frac{b\,\left(f\,b^2-e\,b\,c \cdots\right.}{\cdots}}{a} \cdots\right.\right.}{\cdots}\right)}{\cdots}$$

input
```
int((x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)
```

output
```
(f*x^2)/(2*c) + (log(a + b*x^2 + c*x^4)*(2*b^3*f + 8*a*c^2*e - 2*b^2*c*e -
 8*a*b*c*f))/(2*(16*a*c^3 - 4*b^2*c^2)) + (atan((2*c^2*(4*a*c - b^2)*(x^2*
(((((4*c^4*d + 6*b^2*c^2*f - 4*a*c^3*f - 6*b*c^3*e)/c^2 + (4*b*c^2*(2*b^3*
f + 8*a*c^2*e - 2*b^2*c*e - 8*a*b*c*f))/(16*a*c^3 - 4*b^2*c^2))*(2*c^2*d +
 b^2*f - 2*a*c*f - b*c*e))/(8*c^2*(4*a*c - b^2)^(1/2)) + (b*(2*c^2*d + b^2
*f - 2*a*c*f - b*c*e)*(2*b^3*f + 8*a*c^2*e - 2*b^2*c*e - 8*a*b*c*f))/(2*(4
*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^2)))/a - (b*((b^3*f^2 + b*c^2*e^2 -
c^3*d*e - a*b*c*f^2 + a*c^2*e*f + b*c^2*d*f - 2*b^2*c*e*f)/c^2 + (((4*c^4*
d + 6*b^2*c^2*f - 4*a*c^3*f - 6*b*c^3*e)/c^2 + (4*b*c^2*(2*b^3*f + 8*a*c^2
*e - 2*b^2*c*e - 8*a*b*c*f))/(16*a*c^3 - 4*b^2*c^2))*(2*b^3*f + 8*a*c^2*e
- 2*b^2*c*e - 8*a*b*c*f))/(2*(16*a*c^3 - 4*b^2*c^2)) - (b*(2*c^2*d + b^2*f
 - 2*a*c*f - b*c*e)^2)/(2*c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2)))
- ((((8*a*c^3*e - 8*a*b*c^2*f)/c^2 - (8*a*c^2*(2*b^3*f + 8*a*c^2*e - 2*b^2
*c*e - 8*a*b*c*f))/(16*a*c^3 - 4*b^2*c^2))*(2*c^2*d + b^2*f - 2*a*c*f - b*
c*e))/(8*c^2*(4*a*c - b^2)^(1/2)) - (a*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e)
*(2*b^3*f + 8*a*c^2*e - 2*b^2*c*e - 8*a*b*c*f))/((4*a*c - b^2)^(1/2)*(16*a
*c^3 - 4*b^2*c^2)))/a + (b*((((8*a*c^3*e - 8*a*b*c^2*f)/c^2 - (8*a*c^2*(2*
b^3*f + 8*a*c^2*e - 2*b^2*c*e - 8*a*b*c*f))/(16*a*c^3 - 4*b^2*c^2))*(2*b^3
*f + 8*a*c^2*e - 2*b^2*c*e - 8*a*b*c*f))/(2*(16*a*c^3 - 4*b^2*c^2)) - (a*b
^2*f^2 + a*c^2*e^2 - 2*a*b*c*e*f)/c^2 + (a*(2*c^2*d + b^2*f - 2*a*c*f -...
```

3.50. $\int \frac{x(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$

3.51 $\int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)}\,dx$

3.51.1 Optimal result

Integrand size = 30, antiderivative size = 97

$$\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)}\,dx=\frac{(bcd-2ace+abf)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2ac\sqrt{b^2-4ac}}+\frac{d\log(x)}{a}-\frac{(cd-af)\log\left(a+bx^2+cx^4\right)}{4ac}$$

output
```
d*ln(x)/a-1/4*(-a*f+c*d)*ln(c*x^4+b*x^2+a)/a/c+1/2*(a*b*f-2*a*c*e+b*c*d)*a
rctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a/c/(-4*a*c+b^2)^(1/2)
```

3.51.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.84

$$\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)}\,dx=\frac{4c\sqrt{b^2-4ac}d\log(x)-\left(bcd+c\sqrt{b^2-4ac}d-2ace+abf-a\sqrt{b^2-4ac}f\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)+}{4ac\sqrt{b^2-4ac}}$$

input
```
Integrate[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)),x]
```

output
```
(4*c*Sqrt[b^2 - 4*a*c]*d*Log[x] - (b*c*d + c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e
 + a*b*f - a*Sqrt[b^2 - 4*a*c]*f)*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2] + (
b*c*d - c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e + a*b*f + a*Sqrt[b^2 - 4*a*c]*f)*L
og[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*a*c*Sqrt[b^2 - 4*a*c])
```

3.51.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2194, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)}dx$$

$$\Big\downarrow \ 2194$$

$$\frac{1}{2}\int \frac{fx^4+ex^2+d}{x^2\left(cx^4+bx^2+a\right)}dx^2$$

$$\Big\downarrow \ 2159$$

$$\frac{1}{2}\int \left(\frac{d}{ax^2}+\frac{-\left((cd-af)x^2\right)-bd+ae}{a\left(cx^4+bx^2+a\right)}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(abf-2ace+bcd)}{ac\sqrt{b^2-4ac}}-\frac{(cd-af)\log\left(a+bx^2+cx^4\right)}{2ac}+\frac{d\log\left(x^2\right)}{a}\right)$$

input
```
Int[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)),x]
```

output
```
(((b*c*d - 2*a*c*e + a*b*f)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a*c
*Sqrt[b^2 - 4*a*c]) + (d*Log[x^2])/a - ((c*d - a*f)*Log[a + b*x^2 + c*x^4]
)/(2*a*c))/2
```

3.51.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.51.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.02

method	result
default	$\frac{d\ln(x)}{a}+\frac{\frac{(af-cd)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(ae-bd-\frac{(af-cd)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2a}$
risch	$\frac{d\ln(x)}{a}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(4a^2c^2-a\,b^2c\right)\_Z^2+\left(-4a^2cf+a\,b^2f+4a\,c^2d-b^2cd\right)\_Z+a^2f^2-abef-2acdf+e^2ac+b^2df-bcde+c^2d^2\right)}\_R\ln\left(\left(\left(\right.\right.\right.}{}$

input
```
int((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
d*ln(x)/a+1/2/a*(1/2*(a*f-c*d)/c*ln(c*x^4+b*x^2+a)+2*(a*e-b*d-1/2*(a*f-c*d
)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 309, normalized size of antiderivative = 3.19

$$\int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)}\,dx$$
$$= \left[\frac{4\,(b^2c-4\,ac^2)d\log(x)+(bcd-2\,ace+abf)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-((b^2c}{4\,(ab^2c-4\,a^2c^2)}\right.$$

input
```
integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*(4*(b^2*c - 4*a*c^2)*d*log(x) + (b*c*d - 2*a*c*e + a*b*f)*sqrt(b^2 -
4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 -
 4*a*c))/(c*x^4 + b*x^2 + a)) - ((b^2*c - 4*a*c^2)*d - (a*b^2 - 4*a^2*c)*f
)*log(c*x^4 + b*x^2 + a))/(a*b^2*c - 4*a^2*c^2), 1/4*(4*(b^2*c - 4*a*c^2)*
d*log(x) + 2*(b*c*d - 2*a*c*e + a*b*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2
 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^2*c - 4*a*c^2)*d - (a*b^2 -
4*a^2*c)*f)*log(c*x^4 + b*x^2 + a))/(a*b^2*c - 4*a^2*c^2)]
```

3.51.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate((f*x**4+e*x**2+d)/x/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.51.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99

$$\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)}\,dx = \frac{d\log\left(x^2\right)}{2\,a} - \frac{(cd-af)\log\left(cx^4+bx^2+a\right)}{4\,ac} - \frac{(bcd-2\,ace+abf)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}ac}$$

input
```
integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/2*d*log(x^2)/a - 1/4*(c*d - a*f)*log(c*x^4 + b*x^2 + a)/(a*c) - 1/2*(b*c
*d - 2*a*c*e + a*b*f)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2
+ 4*a*c)*a*c)
```

3.51.9 Mupad [B] (verification not implemented)

Time = 13.21 (sec) , antiderivative size = 3927, normalized size of antiderivative = 40.48

$$\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)),x)
```

output
```
(d*log(x))/a - (log((b^2*d*f^2 + c^2*d*e^2 - x^2*(b*f - c*e)*(a*f^2 + c*e^
2 - b*e*f - c*d*f) + ((c*d - a*f + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/(a^2*
c^2*(4*a*c - b^2)))^(1/2))*(a*b^2*f^2 - x^2*(b*c^2*e^2 - 3*b^3*f^2 + 5*c^3
*d*e + 11*a*b*c*f^2 - 9*a*c^2*e*f - 7*b*c^2*d*f + 2*b^2*c*e*f) + a*c^2*e^2
 - 4*b*c^2*d*e + 4*b^2*c*d*f + ((c*d - a*f + a*c*(-(a*b*f - 2*a*c*e + b*c*
d)^2/(a^2*c^2*(4*a*c - b^2)))^(1/2))*(2*c*x^2*(6*b^3*f + 10*a*c^2*e + 5*b*
c^2*d - 4*b^2*c*e - 19*a*b*c*f) + 4*b^2*c^2*d - 4*a*b*c^2*e + 4*a*b^2*c*f
+ (b*c*(c*d - a*f + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/(a^2*c^2*(4*a*c - b^
2)))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a))/(4*a*c) - 2*a*b*c*e*f))/(4
*a*c) - 2*b*c*d*e*f)*(b^2*d*f^2 + c^2*d*e^2 - x^2*(b*f - c*e)*(a*f^2 + c*e
^2 - b*e*f - c*d*f) + ((a*f - c*d + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/(a^2
*c^2*(4*a*c - b^2)))^(1/2))*(x^2*(b*c^2*e^2 - 3*b^3*f^2 + 5*c^3*d*e + 11*a
*b*c*f^2 - 9*a*c^2*e*f - 7*b*c^2*d*f + 2*b^2*c*e*f) - a*b^2*f^2 - a*c^2*e^
2 + 4*b*c^2*d*e - 4*b^2*c*d*f + ((a*f - c*d + a*c*(-(a*b*f - 2*a*c*e + b*c
*d)^2/(a^2*c^2*(4*a*c - b^2)))^(1/2))*(2*c*x^2*(6*b^3*f + 10*a*c^2*e + 5*b
*c^2*d - 4*b^2*c*e - 19*a*b*c*f) + 4*b^2*c^2*d - 4*a*b*c^2*e + 4*a*b^2*c*f
 - (b*c*(a*f - c*d + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/(a^2*c^2*(4*a*c - b
^2)))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a))/(4*a*c) + 2*a*b*c*e*f))/(
4*a*c) - 2*b*c*d*e*f))*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(2
*(16*a^2*c^2 - 4*a*b^2*c)) + (atan(((4*a*c - b^2)*(((((a*b*f - 2*a*c*e ...
```

3.51. $\int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)}\,dx$

3.52 $\int \frac{d+ex^2+fx^4}{x^3(a+bx^2+cx^4)} dx$

3.52.1 Optimal result

Integrand size = 30, antiderivative size = 118

$$\int \frac{d+ex^2+fx^4}{x^3(a+bx^2+cx^4)} dx = -\frac{d}{2ax^2} - \frac{(b^2d-abe-2a(cd-af))\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}} - \frac{(bd-ae)\log(x)}{a^2} + \frac{(bd-ae)\log(a+bx^2+cx^4)}{4a^2}$$

output
```
-1/2*d/a/x^2-(-a*e+b*d)*ln(x)/a^2+1/4*(-a*e+b*d)*ln(c*x^4+b*x^2+a)/a^2-1/2
*(b^2*d-a*b*e-2*a*(-a*f+c*d))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/
(-4*a*c+b^2)^(1/2)
```

3.52.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.72

$$\int \frac{d+ex^2+fx^4}{x^3(a+bx^2+cx^4)} dx = \frac{-\frac{2ad}{x^2} + 4(-bd+ae)\log(x) + \frac{\left(b^2d+b\left(\sqrt{b^2-4ac}d-ae\right)+a\left(-2cd-\sqrt{b^2-4ac}e+2af\right)\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}} + \frac{\left(-b^2d+b\left(\sqrt{b^2-4\right.\right.}{}}{4a^2}$$

input
```
Integrate[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)),x]
```

output
```
((-2*a*d)/x^2 + 4*(-(b*d) + a*e)*Log[x] + ((b^2*d + b*(Sqrt[b^2 - 4*a*c]*d
 - a*e) + a*(-2*c*d - Sqrt[b^2 - 4*a*c]*e + 2*a*f))*Log[b - Sqrt[b^2 - 4*a
*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((-(b^2*d) + b*(Sqrt[b^2 - 4*a*c]*d +
a*e) - a*(-2*c*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f))*Log[b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a^2)
```

3.52.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2194, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)}dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{fx^4+ex^2+d}{x^4\left(cx^4+bx^2+a\right)}dx^2$$

$$\Big\downarrow \text{2159}$$

$$\frac{1}{2}\int \left(\frac{d}{ax^4}+\frac{db^2-aeb+c(bd-ae)x^2-a(cd-af)}{a^2\left(cx^4+bx^2+a\right)}+\frac{ae-bd}{a^2x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-abe-2a(cd-af)+b^2d\right)}{a^2\sqrt{b^2-4ac}}+\frac{(bd-ae)\log\left(a+bx^2+cx^4\right)}{2a^2}-\frac{\log\left(x^2\right)(bd-ae)}{a^2}-\frac{d}{ax^2}\right.$$

input
```
Int[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)),x]
```

output
```
(-(d/(a*x^2)) - ((b^2*d - a*b*e - 2*a*(c*d - a*f))*ArcTanh[(b + 2*c*x^2)/S
qrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - ((b*d - a*e)*Log[x^2])/a^2 +
((b*d - a*e)*Log[a + b*x^2 + c*x^4])/(2*a^2))/2
```

3.52.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.52.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.12

method	result
default	$-\frac{d}{2a\,x^{2}}+\frac{(ae-bd)\ln(x)}{a^{2}}+\frac{\frac{(-ace+bcd)\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2c}+\frac{2\left(f\,a^{2}-abe-acd+b^{2}d-\frac{(-ace+bcd)b}{2c}\right)\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2a^{2}}$
risch	$-\frac{d}{2a\,x^{2}}+\frac{\ln(x)e}{a}-\frac{\ln(x)bd}{a^{2}}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(4a^{3}c-a^{2}b^{2}\right)\_Z^{2}+\left(4a^{2}ce-a\,b^{2}e-4abcd+b^{3}d\right)\_Z+a^{2}f^{2}-abef-2acdf+e^{2}ac+b^{2}df-bcde\right.}\right.}{}$

input
```
int((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*d/a/x^2+(a*e-b*d)/a^2*ln(x)+1/2/a^2*(1/2*(-a*c*e+b*c*d)/c*ln(c*x^4+b*
x^2+a)+2*(f*a^2-a*b*e-a*c*d+b^2*d-1/2*(-a*c*e+b*c*d)*b/c)/(4*a*c-b^2)^(1/2
)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 399, normalized size of antiderivative = 3.38

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)}\,dx$$
$$= \left[-\frac{(abe-2\,a^2f-(b^2-2\,ac)d)\sqrt{b^2-4\,ac}x^2\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-((b^3-4\,abc)d-(\cdots}{4\,(a^2b^2-\cdots}\right.$$

input
```
integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/4*((a*b*e - 2*a^2*f - (b^2 - 2*a*c)*d)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^
2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4
+ b*x^2 + a)) - ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^2*log(c*x^4 +
b*x^2 + a) + 4*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^2*log(x) + 2*(a
*b^2 - 4*a^2*c)*d)/((a^2*b^2 - 4*a^3*c)*x^2), 1/4*(2*(a*b*e - 2*a^2*f - (b
^2 - 2*a*c)*d)*sqrt(-b^2 + 4*a*c)*x^2*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*
a*c)/(b^2 - 4*a*c)) + ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^2*log(c*
x^4 + b*x^2 + a) - 4*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^2*log(x)
- 2*(a*b^2 - 4*a^2*c)*d)/((a^2*b^2 - 4*a^3*c)*x^2)]
```

3.52.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)}\,dx = \text{Timed out}$$

input
```
integrate((f*x**4+e*x**2+d)/x**3/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.52.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.11

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)}\,dx = \frac{(bd-ae)\log\left(cx^4+bx^2+a\right)}{4\,a^2} - \frac{(bd-ae)\log\left(x^2\right)}{2\,a^2} + \frac{\left(b^2d-2\,acd-abe+2\,a^2f\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a^2} + \frac{bdx^2-aex^2-ad}{2\,a^2x^2}$$

input
```
integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/4*(b*d - a*e)*log(c*x^4 + b*x^2 + a)/a^2 - 1/2*(b*d - a*e)*log(x^2)/a^2
+ 1/2*(b^2*d - 2*a*c*d - a*b*e + 2*a^2*f)*arctan((2*c*x^2 + b)/sqrt(-b^2 +
 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/2*(b*d*x^2 - a*e*x^2 - a*d)/(a^2*x^2
)
```

3.52.9 Mupad [B] (verification not implemented)

Time = 12.76 (sec) , antiderivative size = 4437, normalized size of antiderivative = 37.60

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)),x)
```

output
```
(log(x)*(a*e - b*d))/a^2 - d/(2*a*x^2) - (log(((c^2*(a*e - b*d)*(a*f - c*d
)^2)/a^3 - ((b*d - a*e + a^2*(-(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)^2/(a^4*
(4*a*c - b^2)))^(1/2))*(((b*d - a*e + a^2*(-(b^2*d + 2*a^2*f - a*b*e - 2*a
*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2))*((2*c^2*x^2*(10*a*c^2*d + 4*a*b^2*f +
b^2*c*d - 10*a^2*c*f - 5*a*b*c*e))/a + (4*b*c^2*(b^2*d + a^2*f - a*b*e - a
*c*d))/a + (b*c^2*(b*d - a*e + a^2*(-(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)^2
/(a^4*(4*a*c - b^2)))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^2))/(4*a^2)
 + (c^2*(a*f - c*d)*(4*b^2*d + a^2*f - 4*a*b*e - a*c*d))/a^2 - (c^2*x^2*(a
*f - c*d)*(a*b*f + 5*a*c*e - 6*b*c*d))/a^2))/(4*a^2) + (c^2*x^2*(a*f - c*d
)^3)/a^3)*((c^2*(a*e - b*d)*(a*f - c*d)^2)/a^3 - ((a*e - b*d + a^2*(-(b^2*
d + 2*a^2*f - a*b*e - 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2))*(((a*e - b*d
+ a^2*(-(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2))*
((2*c^2*x^2*(10*a*c^2*d + 4*a*b^2*f + b^2*c*d - 10*a^2*c*f - 5*a*b*c*e))/a
 + (4*b*c^2*(b^2*d + a^2*f - a*b*e - a*c*d))/a - (b*c^2*(a*e - b*d + a^2*(
-(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2))*(a*b +
3*b^2*x^2 - 10*a*c*x^2))/a^2))/(4*a^2) - (c^2*(a*f - c*d)*(4*b^2*d + a^2*f
 - 4*a*b*e - a*c*d))/a^2 + (c^2*x^2*(a*f - c*d)*(a*b*f + 5*a*c*e - 6*b*c*d
))/a^2))/(4*a^2) + (c^2*x^2*(a*f - c*d)^3)/a^3))*(2*b^3*d - 2*a*b^2*e + 8*
a^2*c*e - 8*a*b*c*d))/(2*(16*a^3*c - 4*a^2*b^2)) - (atan((16*a^6*(4*a*c -
b^2)^(3/2)*(x^2*((((c^5*d^3 - a^3*c^2*f^3 + 3*a^2*c^3*d*f^2 - 3*a*c^4*d...
```

3.53 $\int \frac{d+ex^2+fx^4}{x^5(a+bx^2+cx^4)}\,dx$

3.53.1 Optimal result

Integrand size = 30, antiderivative size = 174

$$\begin{aligned}\int \frac{d+ex^2+fx^4}{x^5(a+bx^2+cx^4)}\,dx = &-\frac{d}{4ax^4}+\frac{bd-ae}{2a^2x^2}\\&+\frac{(b^3d-ab^2e+2a^2ce-ab(3cd-af))\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}}\\&+\frac{(b^2d-abe-a(cd-af))\log(x)}{a^3}\\&-\frac{(b^2d-abe-a(cd-af))\log(a+bx^2+cx^4)}{4a^3}\end{aligned}$$

output
```
-1/4*d/a/x^4+1/2*(-a*e+b*d)/a^2/x^2+(b^2*d-a*b*e-a*(-a*f+c*d))*ln(x)/a^3-1
/4*(b^2*d-a*b*e-a*(-a*f+c*d))*ln(c*x^4+b*x^2+a)/a^3+1/2*(b^3*d-a*b^2*e+2*a
^2*c*e-a*b*(-a*f+3*c*d))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a
*c+b^2)^(1/2)
```

3.53.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.80

$$\int \frac{d+ex^2+fx^4}{x^5(a+bx^2+cx^4)}\,dx =$$
$$-\frac{\frac{a^2d}{x^4}+\frac{2a(-bd+ae)}{x^2}-4(b^2d-abe+a(-cd+af))\log(x)+\frac{\left(b^3d+b^2\left(\sqrt{b^2-4ac}d-ae\right)+ab\left(-3cd-\sqrt{b^2-4ac}e+af\right)+a\left(-c\sqrt{\right.\right.}}{\sqrt{b^2-4a}}}{}$$

input
```
Integrate[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)),x]
```

output
```
-1/4*((a^2*d)/x^4 + (2*a*(-(b*d) + a*e))/x^2 - 4*(b^2*d - a*b*e + a*(-(c*d
) + a*f))*Log[x] + ((b^3*d + b^2*(Sqrt[b^2 - 4*a*c]*d - a*e) + a*b*(-3*c*d
 - Sqrt[b^2 - 4*a*c]*e + a*f) + a*(-(c*Sqrt[b^2 - 4*a*c]*d) + 2*a*c*e + a*
Sqrt[b^2 - 4*a*c]*f))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a
*c] + ((-(b^3*d) + b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) - a*b*(-3*c*d + Sqrt[b^
2 - 4*a*c]*e + a*f) + a*(-(c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e)) + a*Sqrt[b^2 -
 4*a*c]*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/a^3
```

3.53.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2194, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4}{x^5(a+bx^2+cx^4)}\,dx$$
$$\Big\downarrow \text{2194}$$
$$\frac{1}{2}\int \frac{fx^4+ex^2+d}{x^6(cx^4+bx^2+a)}dx^2$$
$$\Big\downarrow \text{2159}$$
$$\frac{1}{2}\int \left(\frac{d}{ax^6}+\frac{-db^3+aeb^2+a(2cd-af)b-c(db^2-aeb-a(cd-af))x^2-a^2ce}{a^3(cx^4+bx^2+a)}+\frac{db^2-aeb-a(cd-af)}{a^3x^2}+\frac{ae}{a}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\log(x^2)(-abe-a(cd-af)+b^2d)}{a^3}-\frac{\log(a+bx^2+cx^4)(-abe-a(cd-af)+b^2d)}{2a^3}+\frac{bd-ae}{a^2x^2}+\frac{\text{arctanh}\Big(\right.$$

input
```
Int[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)),x]
```

output
```
(-1/2*d/(a*x^4) + (b*d - a*e)/(a^2*x^2) + ((b^3*d - a*b^2*e + 2*a^2*c*e -
a*b*(3*c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2
 - 4*a*c]) + ((b^2*d - a*b*e - a*(c*d - a*f))*Log[x^2])/a^3 - ((b^2*d - a*
b*e - a*(c*d - a*f))*Log[a + b*x^2 + c*x^4])/(2*a^3))/2
```

3.53.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.53.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.17

method	result
default	$-\frac{d}{4a\,x^4}-\frac{ae-bd}{2a^2x^2}+\frac{(f\,a^2-abe-acd+b^2d)\ln(x)}{a^3}-\frac{\frac{(a^2cf-abce-a\,c^2d+b^2cd)\ln(c\,x^4+b\,x^2+a)}{2c}+\frac{2\left(a^2bf+a^2ce-a\,b^2e-2abcd+b^3d-\right.}{}}{2a^3}$
risch	$\frac{-\frac{(ae-bd)x^2}{2a^2}-\frac{d}{4a}}{x^4}+\frac{\ln(x)f}{a}-\frac{\ln(x)be}{a^2}-\frac{\ln(x)cd}{a^2}+\frac{\ln(x)b^2d}{a^3}+\frac{\Big(\_R=\text{RootOf}\big((4c\,a^4-a^3b^2)\_Z^2+(4a^3cf-a^2b^2f-4a^2bce-4a^2c^2}{}$

input
```
int((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*d/a/x^4-1/2*(a*e-b*d)/a^2/x^2+(a^2*f-a*b*e-a*c*d+b^2*d)/a^3*ln(x)-1/2
/a^3*(1/2*(a^2*c*f-a*b*c*e-a*c^2*d+b^2*c*d)/c*ln(c*x^4+b*x^2+a)+2*(a^2*b*f
+a^2*c*e-a*b^2*e-2*a*b*c*d+b^3*d-1/2*(a^2*c*f-a*b*c*e-a*c^2*d+b^2*c*d)*b/c
)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 609, normalized size of antiderivative = 3.50

$$\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)}\,dx = \left[\frac{(a^2bf+(b^3-3\,abc)d-(ab^2-2\,a^2c)e)\sqrt{b^2-4\,ac}x^4\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-((b^4-5}{}\right.$$

input
```
integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*((a^2*b*f + (b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*sqrt(b^2 - 4*a*c
)*x^4*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 -
4*a*c))/(c*x^4 + b*x^2 + a)) - ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d - (a*b^3 -
 4*a^2*b*c)*e + (a^2*b^2 - 4*a^3*c)*f)*x^4*log(c*x^4 + b*x^2 + a) + 4*((b^
4 - 5*a*b^2*c + 4*a^2*c^2)*d - (a*b^3 - 4*a^2*b*c)*e + (a^2*b^2 - 4*a^3*c)
*f)*x^4*log(x) + 2*((a*b^3 - 4*a^2*b*c)*d - (a^2*b^2 - 4*a^3*c)*e)*x^2 - (
a^2*b^2 - 4*a^3*c)*d)/((a^3*b^2 - 4*a^4*c)*x^4), 1/4*(2*(a^2*b*f + (b^3 -
3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*sqrt(-b^2 + 4*a*c)*x^4*arctan(-(2*c*x^2
+ b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d
- (a*b^3 - 4*a^2*b*c)*e + (a^2*b^2 - 4*a^3*c)*f)*x^4*log(c*x^4 + b*x^2 + a
) + 4*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d - (a*b^3 - 4*a^2*b*c)*e + (a^2*b^2
- 4*a^3*c)*f)*x^4*log(x) + 2*((a*b^3 - 4*a^2*b*c)*d - (a^2*b^2 - 4*a^3*c)*
e)*x^2 - (a^2*b^2 - 4*a^3*c)*d)/((a^3*b^2 - 4*a^4*c)*x^4)]
```

3.53.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{x^5(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate((f*x**4+e*x**2+d)/x**5/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.53.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^2+fx^4}{x^5(a+bx^2+cx^4)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.18

$$\begin{aligned}\int \frac{d+ex^2+fx^4}{x^5(a+bx^2+cx^4)}\,dx &= -\frac{(b^2d-acd-abe+a^2f)\log(cx^4+bx^2+a)}{4\,a^3} + \frac{(b^2d-acd-abe+a^2f)\log(x^2)}{2\,a^3} \\ &\quad - \frac{(b^3d-3\,abcd-ab^2e+2\,a^2ce+a^2bf)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\sqrt{-b^2+4\,ac}a^3} \\ &\quad - \frac{3\,b^2dx^4-3\,acdx^4-3\,abex^4+3\,a^2fx^4-2\,abdx^2+2\,a^2ex^2+a^2d}{4\,a^3x^4}\end{aligned}$$

input
```
integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/4*(b^2*d - a*c*d - a*b*e + a^2*f)*log(c*x^4 + b*x^2 + a)/a^3 + 1/2*(b^2
*d - a*c*d - a*b*e + a^2*f)*log(x^2)/a^3 - 1/2*(b^3*d - 3*a*b*c*d - a*b^2*
e + 2*a^2*c*e + a^2*b*f)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b
^2 + 4*a*c)*a^3) - 1/4*(3*b^2*d*x^4 - 3*a*c*d*x^4 - 3*a*b*e*x^4 + 3*a^2*f*
x^4 - 2*a*b*d*x^2 + 2*a^2*e*x^2 + a^2*d)/(a^3*x^4)
```

3.53.9 Mupad [B] (verification not implemented)

Time = 14.76 (sec) , antiderivative size = 6187, normalized size of antiderivative = 35.56

$$\int \frac{d+ex^2+fx^4}{x^5\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)),x)
```

output
```
(log(x)*(b^2*d + a^2*f - a*b*e - a*c*d))/a^3 - (d/(4*a) + (x^2*(a*e - b*d)
)/(2*a^2))/x^4 + (log((((((2*c^3*x^2*(b^3*d - a*b^2*e + 5*a^2*b*f - 10*a^
2*c*e + 5*a*b*c*d))/a^2 + (4*b*c^2*(b^3*d - a*b^2*e + a^2*b*f + a^2*c*e -
2*a*b*c*d))/a^2 + (b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*(b^2*d + a^2*f + a
^3*(-(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)^2/(a^6*(4*a*c - b
^2)))^(1/2) - a*b*e - a*c*d))/a^3)*(b^2*d + a^2*f + a^3*(-(b^3*d - a*b^2*e
 + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)^2/(a^6*(4*a*c - b^2)))^(1/2) - a*b*e -
 a*c*d))/(4*a^3) + (c^3*(a*e - b*d)*(4*b^3*d - 4*a*b^2*e + 4*a^2*b*f + a^2
*c*e - 5*a*b*c*d))/a^4 + (c^4*x^2*(a*e - b*d)*(6*b^2*d + 5*a^2*f - 6*a*b*e
 - 5*a*c*d))/a^4)*(b^2*d + a^2*f + a^3*(-(b^3*d - a*b^2*e + a^2*b*f + 2*a^
2*c*e - 3*a*b*c*d)^2/(a^6*(4*a*c - b^2)))^(1/2) - a*b*e - a*c*d))/(4*a^3)
+ (c^4*(a*e - b*d)^2*(b^2*d + a^2*f - a*b*e - a*c*d))/a^6 - (c^5*x^2*(a*e
- b*d)^3)/a^6)*((((c^3*(a*e - b*d)*(4*b^3*d - 4*a*b^2*e + 4*a^2*b*f + a^2*
c*e - 5*a*b*c*d))/a^4 - (((2*c^3*x^2*(b^3*d - a*b^2*e + 5*a^2*b*f - 10*a^2
*c*e + 5*a*b*c*d))/a^2 + (4*b*c^2*(b^3*d - a*b^2*e + a^2*b*f + a^2*c*e - 2
*a*b*c*d))/a^2 - (b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*(a^3*(-(b^3*d - a*b
^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)^2/(a^6*(4*a*c - b^2)))^(1/2) - a^2
*f - b^2*d + a*b*e + a*c*d))/a^3)*(a^3*(-(b^3*d - a*b^2*e + a^2*b*f + 2*a^
2*c*e - 3*a*b*c*d)^2/(a^6*(4*a*c - b^2)))^(1/2) - a^2*f - b^2*d + a*b*e +
a*c*d))/(4*a^3) + (c^4*x^2*(a*e - b*d)*(6*b^2*d + 5*a^2*f - 6*a*b*e - 5...
```

3.54 $\int \frac{d+ex^2+fx^4}{x^7(a+bx^2+cx^4)}\,dx$

3.54.1 Optimal result

Integrand size = 30, antiderivative size = 244

$$\begin{aligned}\int \frac{d+ex^2+fx^4}{x^7(a+bx^2+cx^4)}\,dx &= -\frac{d}{6ax^6}+\frac{bd-ae}{4a^2x^4}-\frac{b^2d-abe-a(cd-af)}{2a^3x^2}\\ &\quad-\frac{(b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd-af))\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4\sqrt{b^2-4ac}}\\ &\quad-\frac{(b^3d-ab^2e+a^2ce-ab(2cd-af))\log(x)}{a^4}\\ &\quad+\frac{(b^3d-ab^2e+a^2ce-ab(2cd-af))\log(a+bx^2+cx^4)}{4a^4}\end{aligned}$$

output
```
-1/6*d/a/x^6+1/4*(-a*e+b*d)/a^2/x^4+1/2*(-b^2*d+a*b*e+a*(-a*f+c*d))/a^3/x^
2-(b^3*d-a*b^2*e+a^2*c*e-a*b*(-a*f+2*c*d))*ln(x)/a^4+1/4*(b^3*d-a*b^2*e+a^
2*c*e-a*b*(-a*f+2*c*d))*ln(c*x^4+b*x^2+a)/a^4-1/2*(b^4*d-a*b^3*e+3*a^2*b*c
*e+2*a^2*c*(-a*f+c*d)-a*b^2*(-a*f+4*c*d))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)
^(1/2))/a^4/(-4*a*c+b^2)^(1/2)
```

3.54.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.70

$$\int \frac{d+ex^2+fx^4}{x^7\left(a+bx^2+cx^4\right)}\,dx$$
$$= \frac{-\frac{2a^3d}{x^6}+\frac{3a^2(bd-ae)}{x^4}+\frac{6a(-b^2d+abe+a(cd-af))}{x^2}-12(b^3d-ab^2e+a^2ce+ab(-2cd+af))\log(x)+\frac{3\left(b^4d+b^3\left(\sqrt{b^2-}\right.\right.}{}}{}$$

input
```
Integrate[(d + e*x^2 + f*x^4)/(x^7*(a + b*x^2 + c*x^4)),x]
```

output
```
((-2*a^3*d)/x^6 + (3*a^2*(b*d - a*e))/x^4 + (6*a*(-(b^2*d) + a*b*e + a*(c*
d - a*f)))/x^2 - 12*(b^3*d - a*b^2*e + a^2*c*e + a*b*(-2*c*d + a*f))*Log[x
] + (3*(b^4*d + b^3*(Sqrt[b^2 - 4*a*c]*d - a*e) + a^2*c*(2*c*d + Sqrt[b^2
- 4*a*c]*e - 2*a*f) + a*b^2*(-4*c*d - Sqrt[b^2 - 4*a*c]*e + a*f) + a*b*(-2
*c*Sqrt[b^2 - 4*a*c]*d + 3*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*Log[b - Sqrt[b^
2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + (3*(-(b^4*d) + b^3*(Sqrt[b^2 -
4*a*c]*d + a*e) - a*b^2*(-4*c*d + Sqrt[b^2 - 4*a*c]*e + a*f) + a^2*c*(-2*c
*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f) + a*b*(-2*c*Sqrt[b^2 - 4*a*c]*d - 3*a*c*
e + a*Sqrt[b^2 - 4*a*c]*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2
 - 4*a*c])/(12*a^4)
```

3.54.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2194, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4}{x^7\left(a+bx^2+cx^4\right)}\,dx$$

$$\Big\downarrow \; 2194$$

$$\frac{1}{2}\int \frac{fx^4+ex^2+d}{x^8\left(cx^4+bx^2+a\right)}dx^2$$

$$\Big\downarrow \; 2159$$

$$\frac{1}{2}\int \left(\frac{d}{ax^8}+\frac{db^4-aeb^3-a(3cd-af)b^2+2a^2ceb+c\left(db^3-aeb^2-a(2cd-af)b+a^2ce\right)x^2+a^2c(cd-af)}{a^4\left(cx^4+bx^2+a\right)}+-$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{-abe-a(cd-af)+b^2d}{a^3x^2}+\frac{bd-ae}{2a^2x^4}-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(3a^2bce+2a^2c(cd-af)-ab^3e-ab^2(4cd-af)\right.}{a^4\sqrt{b^2-4ac}}\right.$$

input
```
Int[(d + e*x^2 + f*x^4)/(x^7*(a + b*x^2 + c*x^4)),x]
```

output
```
(-1/3*d/(a*x^6) + (b*d - a*e)/(2*a^2*x^4) - (b^2*d - a*b*e - a*(c*d - a*f)
)/(a^3*x^2) - ((b^4*d - a*b^3*e + 3*a^2*b*c*e + 2*a^2*c*(c*d - a*f) - a*b^
2*(4*c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a^4*Sqrt[b^2 -
 4*a*c]) - ((b^3*d - a*b^2*e + a^2*c*e - a*b*(2*c*d - a*f))*Log[x^2])/a^4
+ ((b^3*d - a*b^2*e + a^2*c*e - a*b*(2*c*d - a*f))*Log[a + b*x^2 + c*x^4])
/(2*a^4))/2
```

3.54.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.54.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.20

method	result
default	$-\frac{d}{6a\,x^{6}}-\frac{ae-bd}{4a^{2}x^{4}}-\frac{f\,a^{2}-abe-acd+b^{2}d}{2a^{3}x^{2}}+\frac{\left(-a^{2}bf-a^{2}ce+a\,b^{2}e+2abcd-b^{3}d\right)\ln\left(x\right)}{a^{4}}-\frac{\frac{\left(-a^{2}bcf-a^{2}c^{2}e+a\,b^{2}ce+2ab\,c^{2}d-b^{3}cd\right)\ln}{2c}}{}$
risch	$\frac{-\frac{\left(f\,a^{2}-abe-acd+b^{2}d\right)x^{4}}{2a^{3}}-\frac{\left(ae-bd\right)x^{2}}{4a^{2}}-\frac{d}{6a}}{x^{6}}-\frac{\ln\left(x\right)bf}{a^{2}}-\frac{\ln\left(x\right)ce}{a^{2}}+\frac{\ln\left(x\right)b^{2}e}{a^{3}}+\frac{2\ln\left(x\right)bcd}{a^{3}}-\frac{\ln\left(x\right)b^{3}d}{a^{4}}+\left(\_R=\mathrm{RootOf}\left(\left(4c\,a^{5}\right.\right.\right.$

input
```
int((f*x^4+e*x^2+d)/x^7/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/6*d/a/x^6-1/4*(a*e-b*d)/a^2/x^4-1/2*(a^2*f-a*b*e-a*c*d+b^2*d)/a^3/x^2+1
/a^4*(-a^2*b*f-a^2*c*e+a*b^2*e+2*a*b*c*d-b^3*d)*ln(x)-1/2/a^4*(1/2*(-a^2*b
*c*f-a^2*c^2*e+a*b^2*c*e+2*a*b*c^2*d-b^3*c*d)/c*ln(c*x^4+b*x^2+a)+2*(a^3*c
*f-a^2*b^2*f-2*a^2*b*c*e-a^2*c^2*d+a*b^3*e+3*a*b^2*c*d-d*b^4-1/2*(-a^2*b*c
*f-a^2*c^2*e+a*b^2*c*e+2*a*b*c^2*d-b^3*c*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan(
(2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

3.54.5 Fricas [A] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 834, normalized size of antiderivative = 3.42

$$\int \frac{d+ex^2+fx^4}{x^7\left(a+bx^2+cx^4\right)}\,dx$$
$$=\Bigg[-\frac{3\,\sqrt{b^2-4\,ac}\left(\left(b^4-4\,ab^2c+2\,a^2c^2\right)d-\left(ab^3-3\,a^2bc\right)e+\left(a^2b^2-2\,a^3c\right)f\right)x^6\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,c}{cx^4+bx^2+a}\right.}{}$$
$$-\frac{6\,\sqrt{-b^2+4\,ac}\left(\left(b^4-4\,ab^2c+2\,a^2c^2\right)d-\left(ab^3-3\,a^2bc\right)e+\left(a^2b^2-2\,a^3c\right)f\right)x^6\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,a}}{b^2-4\,ac}\right.}{}$$

input
```
integrate((f*x^4+e*x^2+d)/x^7/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/12*(3*sqrt(b^2 - 4*a*c)*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*
a^2*b*c)*e + (a^2*b^2 - 2*a^3*c)*f)*x^6*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 -
 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 3*((b^5 -
 6*a*b^3*c + 8*a^2*b*c^2)*d - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e + (a^2*b
^3 - 4*a^3*b*c)*f)*x^6*log(c*x^4 + b*x^2 + a) + 12*((b^5 - 6*a*b^3*c + 8*a
^2*b*c^2)*d - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e + (a^2*b^3 - 4*a^3*b*c)*
f)*x^6*log(x) + 6*((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 - 4*a^3*
b*c)*e + (a^3*b^2 - 4*a^4*c)*f)*x^4 - 3*((a^2*b^3 - 4*a^3*b*c)*d - (a^3*b^
2 - 4*a^4*c)*e)*x^2 + 2*(a^3*b^2 - 4*a^4*c)*d)/((a^4*b^2 - 4*a^5*c)*x^6),
-1/12*(6*sqrt(-b^2 + 4*a*c)*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*
a^2*b*c)*e + (a^2*b^2 - 2*a^3*c)*f)*x^6*arctan(-(2*c*x^2 + b)*sqrt(-b^2 +
4*a*c)/(b^2 - 4*a*c)) - 3*((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d - (a*b^4 - 5*
a^2*b^2*c + 4*a^3*c^2)*e + (a^2*b^3 - 4*a^3*b*c)*f)*x^6*log(c*x^4 + b*x^2
+ a) + 12*((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d - (a*b^4 - 5*a^2*b^2*c + 4*a^
3*c^2)*e + (a^2*b^3 - 4*a^3*b*c)*f)*x^6*log(x) + 6*((a*b^4 - 5*a^2*b^2*c +
 4*a^3*c^2)*d - (a^2*b^3 - 4*a^3*b*c)*e + (a^3*b^2 - 4*a^4*c)*f)*x^4 - 3*(
(a^2*b^3 - 4*a^3*b*c)*d - (a^3*b^2 - 4*a^4*c)*e)*x^2 + 2*(a^3*b^2 - 4*a^4*
c)*d)/((a^4*b^2 - 4*a^5*c)*x^6)]
```

3.54.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{x^7(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate((f*x**4+e*x**2+d)/x**7/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.54.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^2+fx^4}{x^7(a+bx^2+cx^4)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^4+e*x^2+d)/x^7/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.24

$$\int \frac{d+ex^2+fx^4}{x^7(a+bx^2+cx^4)}\,dx = \frac{(b^3d-2\,abcd-ab^2e+a^2ce+a^2bf)\log(cx^4+bx^2+a)}{4\,a^4} - \frac{(b^3d-2\,abcd-ab^2e+a^2ce+a^2bf)\log(x^2)}{2\,a^4} + \frac{(b^4d-4\,ab^2cd+2\,a^2c^2d-ab^3e+3\,a^2bce+a^2b^2f-2\,a^3cf)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a^4} + \frac{11\,b^3dx^6-22\,abcdx^6-11\,ab^2ex^6+11\,a^2cex^6+11\,a^2bfx^6-6\,ab^2dx^4+6\,a^2cdx^4+6\,a^2bex^4-6\,a^3fx^4}{12\,a^4x^6}$$

input
```
integrate((f*x^4+e*x^2+d)/x^7/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/4*(b^3*d - 2*a*b*c*d - a*b^2*e + a^2*c*e + a^2*b*f)*log(c*x^4 + b*x^2 +
a)/a^4 - 1/2*(b^3*d - 2*a*b*c*d - a*b^2*e + a^2*c*e + a^2*b*f)*log(x^2)/a^
4 + 1/2*(b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e + a^2*b
^2*f - 2*a^3*c*f)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*
a*c)*a^4) + 1/12*(11*b^3*d*x^6 - 22*a*b*c*d*x^6 - 11*a*b^2*e*x^6 + 11*a^2*
c*e*x^6 + 11*a^2*b*f*x^6 - 6*a*b^2*d*x^4 + 6*a^2*c*d*x^4 + 6*a^2*b*e*x^4 -
 6*a^3*f*x^4 + 3*a^2*b*d*x^2 - 3*a^3*e*x^2 - 2*a^3*d)/(a^4*x^6)
```

3.54.9 Mupad [B] (verification not implemented)

Time = 17.78 (sec) , antiderivative size = 9141, normalized size of antiderivative = 37.46

$$\int \frac{d+ex^2+fx^4}{x^7(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4)/(x^7*(a + b*x^2 + c*x^4)),x)
```

output
```
(atan((16*a^12*(4*a*c - b^2)^(3/2)*(x^2*((((a^3*c^8*d^3 - b^6*c^5*d^3 - a^
6*c^5*f^3 + 3*a*b^4*c^6*d^3 - 3*a^4*c^7*d^2*f + 3*a^5*c^6*d*f^2 - 3*a^2*b^
2*c^7*d^3 + a^3*b^3*c^5*e^3 + 3*a*b^5*c^5*d^2*e + 3*a^3*b*c^7*d^2*e + 3*a^
5*b*c^5*e*f^2 - 6*a^2*b^3*c^6*d^2*e - 3*a^2*b^4*c^5*d*e^2 + 3*a^3*b^2*c^6*
d*e^2 - 3*a^2*b^4*c^5*d^2*f + 6*a^3*b^2*c^6*d^2*f - 3*a^4*b^2*c^5*d*f^2 -
3*a^4*b^2*c^5*e^2*f - 6*a^4*b*c^6*d*e*f + 6*a^3*b^3*c^5*d*e*f)/a^9 - (((11
*a^5*b*c^6*d^2 - 5*a^6*b*c^5*e^2 + 6*a^7*b*c^4*f^2 + 6*a^3*b^5*c^4*d^2 - 1
7*a^4*b^3*c^5*d^2 + 6*a^5*b^3*c^4*e^2 - 5*a^6*c^6*d*e + 5*a^7*c^5*e*f - 17
*a^6*b*c^5*d*f - 12*a^4*b^4*c^4*d*e + 22*a^5*b^2*c^5*d*e + 12*a^5*b^3*c^4*
d*f - 12*a^6*b^2*c^4*e*f)/a^9 + (((20*a^9*c^4*f - 20*a^8*c^5*d + 2*a^6*b^4
*c^3*d + 8*a^7*b^2*c^4*d - 2*a^7*b^3*c^3*e + 2*a^8*b^2*c^3*f - 10*a^8*b*c^
4*e)/a^9 + ((40*a^10*b*c^3 - 12*a^9*b^3*c^2)*(2*b^5*d + 2*a^2*b^3*f - 8*a^
3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2
*b^2*c*e))/(2*a^9*(16*a^5*c - 4*a^4*b^2)))*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*
c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b
^2*c*e))/(2*(16*a^5*c - 4*a^4*b^2)))*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e
- 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e
))/(2*(16*a^5*c - 4*a^4*b^2)) + (((((20*a^9*c^4*f - 20*a^8*c^5*d + 2*a^6*b
^4*c^3*d + 8*a^7*b^2*c^4*d - 2*a^7*b^3*c^3*e + 2*a^8*b^2*c^3*f - 10*a^8*b*
c^4*e)/a^9 + ((40*a^10*b*c^3 - 12*a^9*b^3*c^2)*(2*b^5*d + 2*a^2*b^3*f -...
```

3.54. $\int \frac{d+ex^2+fx^4}{x^7(a+bx^2+cx^4)}\,dx$

3.55 $\int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$

3.55.1 Optimal result

Integrand size = 30, antiderivative size = 369

$$\int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \frac{(c^2d+b^2f-c(be+af))\,x}{c^3}+\frac{(ce-bf)x^3}{3c^2}+\frac{fx^5}{5c}$$
$$+\frac{\left(b^2ce-ac^2e-b^3f-bc(cd-2af)-\frac{b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2ac^2(cd-af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\left(b^2ce-ac^2e-b^3f-bc(cd-2af)+\frac{b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2ac^2(cd-af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{7/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
(c^2*d+b^2*f-c*(a*f+b*e))*x/c^3+1/3*(-b*f+c*e)*x^3/c^2+1/5*f*x^5/c+1/2*arc
tan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*c*e-a*c^2*e-b^3*f
-b*c*(-2*a*f+c*d)+(-b^3*c*e+3*a*b*c^2*e+b^4*f+b^2*c*(-4*a*f+c*d)-2*a*c^2*(
-a*f+c*d))/(-4*a*c+b^2)^(1/2))/c^(7/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2
)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*c*e-a*c^
2*e-b^3*f-b*c*(-2*a*f+c*d)+(b^3*c*e-3*a*b*c^2*e-b^4*f-b^2*c*(-4*a*f+c*d)+2
*a*c^2*(-a*f+c*d))/(-4*a*c+b^2)^(1/2))/c^(7/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/
2))^(1/2)
```

3.55.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.24

$$\int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \frac{(c^2d+b^2f-c(be+af))\,x}{c^3}+\frac{(ce-bf)x^3}{3c^2}+\frac{fx^5}{5c}$$
$$-\frac{\left(-b^4f-b^2c\left(cd+\sqrt{b^2-4ac}e-4af\right)+ac^2\left(2cd+\sqrt{b^2-4ac}e-2af\right)+b^3\left(ce+\sqrt{b^2-4ac}f\right)+bc(c\sqrt{\ }\right.}{\sqrt{2}c^{7/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\left(b^4f+b^2c\left(cd-\sqrt{b^2-4ac}e-4af\right)+ac^2\left(-2cd+\sqrt{b^2-4ac}e+2af\right)+b^3\left(-ce+\sqrt{b^2-4ac}f\right)+bc(\right.}{\sqrt{2}c^{7/2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

input
```
Integrate[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

output
```
((c^2*d + b^2*f - c*(b*e + a*f))*x)/c^3 + ((c*e - b*f)*x^3)/(3*c^2) + (f*x
^5)/(5*c) - ((-(b^4*f) - b^2*c*(c*d + Sqrt[b^2 - 4*a*c]*e - 4*a*f) + a*c^2
*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f) + b^3*(c*e + Sqrt[b^2 - 4*a*c]*f) +
 b*c*(c*Sqrt[b^2 - 4*a*c]*d - 3*a*c*e - 2*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b^2
 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b^4*f + b^2*c*(c*d - Sqrt[b^2 -
 4*a*c]*e - 4*a*f) + a*c^2*(-2*c*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f) + b^3*(-
(c*e) + Sqrt[b^2 - 4*a*c]*f) + b*c*(c*Sqrt[b^2 - 4*a*c]*d + 3*a*c*e - 2*a*
Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c
]]])/(Sqrt[2]*c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

3.55.3 Rubi [A] (verified)

Time = 3.02 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$$
$$\Big\downarrow \; 2195$$

$$\int \left(\frac{-c(af+be)+b^2f+c^2d}{c^3}-\frac{a\left(-c(af+be)+b^2f+c^2d\right)-x^2\left(-bc(cd-2af)-ac^2e+b^3(-f)+b^2ce\right)}{c^3\left(a+bx^2+cx^4\right)}+\frac{x^2(\cdots}{\cdots}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f)+b^3ce}{\sqrt{b^2-4ac}}-bc(cd-2af)-ac^2e+b^3(-f)+b^2ce\right)}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f)+b^3ce}{\sqrt{b^2-4ac}}-bc(cd-2af)-ac^2e+b^3(-f)+b^2ce\right)}{\sqrt{2}c^{7/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x\left(-c(af+be)+b^2f+c^2d\right)}{c^3}+\frac{x^3(ce-bf)}{3c^2}+\frac{fx^5}{5c}$$

input
```
Int[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

output
```
((c^2*d + b^2*f - c*(b*e + a*f))*x)/c^3 + ((c*e - b*f)*x^3)/(3*c^2) + (f*x
^5)/(5*c) + ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d - 2*a*f) - (b^3*c*e - 3
*a*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*f) + 2*a*c^2*(c*d - a*f))/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]
*c^(7/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2*c*e - a*c^2*e - b^3*f - b*c*
(c*d - 2*a*f) + (b^3*c*e - 3*a*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*f) + 2*a
*c^2*(c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

3.55.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

3.55.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.44

method	result
risch	$\frac{f x^5}{5c} - \frac{b f x^3}{3c^2} + \frac{e x^3}{3c} - \frac{a f x}{c^2} + \frac{b^2 f x}{c^3} - \frac{b e x}{c^2} + \frac{d x}{c} + \frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2 b+a)} \frac{((2abcf-a c^2 e-b^3 f+b^2 ce-b c^2 d)\_R^2+a^2 \ldots}{2c\_R^3+\_ \ldots}}{2c^3}$
default	$-\frac{-\frac{1}{5} f x^5 c^2+\frac{1}{3} bcf x^3-\frac{1}{3} c^2 e x^3+acfx-b^2 fx+bcex-c^2 dx}{c^3} + \frac{(2\sqrt{-4ac+b^2}\,abcf-\sqrt{-4ac+b^2}\,a c^2 e-b^3 f\sqrt{-4ac+b^2}+b^2 ce\sqrt{-4ac+b^2}-b c^2 \ldots}{2c\sqrt{\ldots}}$

input
```
int(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/5*f*x^5/c-1/3/c^2*b*f*x^3+1/3*e*x^3/c-1/c^2*a*f*x+1/c^3*b^2*f*x-1/c^2*b*
e*x+1/c*d*x+1/2/c^3*sum(((2*a*b*c*f-a*c^2*e-b^3*f+b^2*c*e-b*c^2*d)*_R^2+a^
2*c*f-a*b^2*f+a*b*c*e-a*c^2*d)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_
Z^2*b+a))
```

3.55.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15467 vs. 2(331) = 662.

Time = 39.65 (sec) , antiderivative size = 15467, normalized size of antiderivative = 41.92

$$\int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.55.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.55.7 Maxima [F]

$$\int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \int \frac{(fx^4+ex^2+d)x^4}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/15*(3*c^2*f*x^5 + 5*(c^2*e - b*c*f)*x^3 + 15*(c^2*d - b*c*e + (b^2 - a*c
)*f)*x)/c^3 + integrate(-(a*c^2*d - a*b*c*e + (b*c^2*d - (b^2*c - a*c^2)*e
 + (b^3 - 2*a*b*c)*f)*x^2 + (a*b^2 - a^2*c)*f)/(c*x^4 + b*x^2 + a), x)/c^3
```

3.55.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7235 vs. $2(331) = 662$.

Time = 1.16 (sec) , antiderivative size = 7235, normalized size of antiderivative = 19.61

$$\int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/8*((2*b^5*c^4 - 16*a*b^3*c^5 + 32*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^3*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 8*(b^2 - 4*a*c)*a*b*c^5)
*c^2*d - (2*b^6*c^3 - 18*a*b^4*c^4 + 48*a^2*b^2*c^5 - 32*a^3*c^6 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 9*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 24*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 10*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^3 + 10*(b^2 - 4*
a*c)*a*b^2*c^4 - 8*(b^2 - 4*a*c)*a^2*c^5)*c^2*e + (2*b^7*c^2 - 20*a*b^5...
```

3.55.9 Mupad [B] (verification not implemented)

Time = 10.50 (sec) , antiderivative size = 23332, normalized size of antiderivative = 63.23

$$\int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)
```

output
```
x^3*(e/(3*c) - (b*f)/(3*c^2)) - x*((b*(e/c - (b*f)/c^2))/c - d/c + (a*f)/c
^2) + atan(((((16*a^3*c^6*f - 16*a^2*c^7*d - 20*a^2*b^2*c^5*f + 4*a*b^2*c^
6*d - 4*a*b^3*c^5*e + 16*a^2*b*c^6*e + 4*a*b^4*c^4*f)/c^5 - (2*x*(4*b^3*c^
7 - 16*a*b*c^8)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 + b^6*f^2*(-(4*a*c
- b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 - a*c^5*d^2*(-(4*a*c
- b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 -
2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-(4*a*c - b^2)^3)^(1/2) +
b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3
*f^2 - a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*e^2*(-(4*a*c - b^2)^
3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*
f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f
 + 20*a*b^6*c^2*e*f - 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2
*f^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 3
6*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f + 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3)^
(1/2) - 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f + 76*a
^3*b^2*c^4*e*f + 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c^3*e^2*
(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^
2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(1
/2) - 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 -
 8*a*b^2*c^8)))^(1/2))/c^5)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 + b^...
```

3.55. $\int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$

3.56 $\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4} dx$

3.56.1 Optimal result

Integrand size = 30, antiderivative size = 282

$$\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4} dx = \frac{(ce-bf)x}{c^2} + \frac{fx^3}{3c} + \frac{\left(c^2d-bce+b^2f-acf+\frac{b^2ce-2ac^2e-b^3f-bc(cd-3af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(c^2d-bce+b^2f-acf-\frac{b^2ce-2ac^2e-b^3f-bc(cd-3af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
(-b*f+c*e)*x/c^2+1/3*f*x^3/c+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^
(1/2))^(1/2))*(c^2*d-b*c*e+b^2*f-a*c*f+(b^2*c*e-2*a*c^2*e-b^3*f-b*c*(-3*a*
f+c*d))/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1
/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(c^2*d-b*c*e+b^2
*f-a*c*f+(-b^2*c*e+2*a*c^2*e+b^3*f+b*c*(-3*a*f+c*d))/(-4*a*c+b^2)^(1/2))/c
^(5/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.56.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.29

$$\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$$

$$= \frac{6\sqrt{c}(ce-bf)x+2c^{3/2}fx^3+\frac{3\sqrt{2}\left(-b^3f-bc\left(cd+\sqrt{b^2-4ac}e-3af\right)+b^2\left(ce+\sqrt{b^2-4ac}f\right)+c\left(c\sqrt{b^2-4ac}d-2ace-a\sqrt{b^2-4ac}f\right)\right)\text{arcta}}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}}{6c}$$

input
```
Integrate[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

output
```
(6*Sqrt[c]*(c*e - b*f)*x + 2*c^(3/2)*f*x^3 + (3*Sqrt[2]*(-(b^3*f) - b*c*(c
*d + Sqrt[b^2 - 4*a*c]*e - 3*a*f) + b^2*(c*e + Sqrt[b^2 - 4*a*c]*f) + c*(c
*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e - a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b
^2 - 4*a*c]]) + (3*Sqrt[2]*(b^3*f + b*c*(c*d - Sqrt[b^2 - 4*a*c]*e - 3*a*f
) + b^2*(-(c*e) + Sqrt[b^2 - 4*a*c]*f) + c*(c*Sqrt[b^2 - 4*a*c]*d + 2*a*c*
e - a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(6*c^(5/2))
```

3.56.3 Rubi [A] (verified)

Time = 2.32 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(d+ex^2+fx^4\right)}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \ 2195$$

$$\int \left(-\frac{a(ce-bf)-x^2\left(-acf+b^2f-bce+c^2d\right)}{c^2\left(a+bx^2+cx^4\right)}+\frac{ce-bf}{c^2}+\frac{fx^2}{c}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce}{\sqrt{b^2-4ac}}-acf+b^2f-bce+c^2d\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce}{\sqrt{b^2-4ac}}-acf+b^2f-bce+c^2d\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x(ce-bf)}{c^2}+\frac{fx^3}{3c}$$

input
```
Int[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

output
```
((c*e - b*f)*x)/c^2 + (f*x^3)/(3*c) + ((c^2*d - b*c*e + b^2*f - a*c*f + (b
^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b
- Sqrt[b^2 - 4*a*c]]) + ((c^2*d - b*c*e + b^2*f - a*c*f - (b^2*c*e - 2*a*c
^2*e - b^3*f - b*c*(c*d - 3*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4
*a*c]])
```

3.56.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

3.56.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.35

method	result
risch	$\frac{f\,x^3}{3c}-\frac{bfx}{c^2}+\frac{xe}{c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\left(-acf+b^2f-ebc+c^2d\right)\_R^2+abf-ace\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{2c^2}$
default	$-\frac{-\frac{1}{3}cf\,x^3+bfx-xce}{c^2}+\frac{\left(-acf\sqrt{-4ac+b^2}+b^2f\sqrt{-4ac+b^2}-ebc\sqrt{-4ac+b^2}+c^2d\sqrt{-4ac+b^2}-3abcf+2a\,c^2e+b^3f-b^2ce+b\,c^2d\right)\sqrt{2}\,\arctan\left(\cdots\right.}{2\sqrt{-4ac+b^2}\,c\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$

input
```
int(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*f/c*x^3-1/c^2*b*f*x+1/c*x*e+1/2/c^2*sum(((-a*c*f+b^2*f-b*c*e+c^2*d)*_R
^2+a*b*f-a*c*e)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.56.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9364 vs. 2(246) = 492.

Time = 8.91 (sec) , antiderivative size = 9364, normalized size of antiderivative = 33.21

$$\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.56.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.56.7 Maxima [F]

$$\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \int \frac{(fx^4+ex^2+d)x^2}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/3*(c*f*x^3 + 3*(c*e - b*f)*x)/c^2 - integrate((a*c*e - a*b*f - (c^2*d -
b*c*e + (b^2 - a*c)*f)*x^2)/(c*x^4 + b*x^2 + a), x)/c^2
```

3.56.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5454 vs. $2(246) = 492$.

Time = 1.09 (sec) , antiderivative size = 5454, normalized size of antiderivative = 19.34

$$\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/8*((2*b^4*c^4 - 16*a*b^2*c^5 + 32*a^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^2*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*c^5 - 2*(b^2 - 4*a*c)*b^2*c^4 + 8*(b^2 - 4*a*c)*a*c^5)*c^2*d - (2
*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^4*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*c^2*e + (
2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(...
```

3.56.9 Mupad [B] (verification not implemented)

Time = 9.70 (sec) , antiderivative size = 15674, normalized size of antiderivative = 55.58

$$\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)
```

output
```
x*(e/c - (b*f)/c^2) - atan(((((16*a^2*c^5*e - 4*a*b^2*c^4*e + 4*a*b^3*c^3*
f - 16*a^2*b*c^4*f)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(b^7*f^2 + b^3*c
^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c
 - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 - a
^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3
*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*
c^4*d*f + 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^
2)^3)^(1/2) + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 3*
a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f - 2*b^2*c^2*d*f*
(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*
a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 -
c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c - b^2)^3
)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b^2)^3
)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^2*f^
2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*
c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f
+ 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f +
 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^...
```

3.56. $\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$

3.57 $\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx$

3.57.1 Optimal result

Integrand size = 27, antiderivative size = 219

$$\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx = \frac{fx}{c}+\frac{\left(ce-bf+\frac{2c^2d+b^2f-c(be+2af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(ce-bf-\frac{2c^2d-bce+b^2f-2acf}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
f*x/c+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(c*e-b*f+
(2*c^2*d+b^2*f-c*(2*a*f+b*e))/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a
*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(
1/2))*(c*e-b*f+(2*a*c*f-b^2*f+b*c*e-2*c^2*d)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2
^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.57.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.18

$$\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx = \frac{2\sqrt{c}fx+\frac{\sqrt{2}\left(2c^2d+b\left(b-\sqrt{b^2-4ac}\right)f+c\left(-be+\sqrt{b^2-4ac}e-2af\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\left(2c^2d+b\left(b+\sqrt{b^2-4ac}\right)f-c\left(be+\sqrt{b^2-4a \cdots\right.\right.}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{\cdots}}}}{2c^{3/2}}$$

input
```
Integrate[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4),x]
```

output
```
(2*Sqrt[c]*f*x + (Sqrt[2]*(2*c^2*d + b*(b - Sqrt[b^2 - 4*a*c])*f + c*(-(b*
e) + Sqrt[b^2 - 4*a*c]*e - 2*a*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqr
t[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[
2]*(2*c^2*d + b*(b + Sqrt[b^2 - 4*a*c])*f - c*(b*e + Sqrt[b^2 - 4*a*c]*e +
 2*a*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^
2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*c^(3/2))
```

3.57.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \text{2205}$$

$$\int \left(\frac{-af+x^2(ce-bf)+cd}{c\left(a+bx^2+cx^4\right)}+\frac{f}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-c(2af+be)+b^2f+2c^2d}{\sqrt{b^2-4ac}}-bf+ce\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-2acf+b^2f-bce+2c^2d}{\sqrt{b^2-4ac}}-bf+ce\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{fx}{c}$$

input
```
Int[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4),x]
```

output
```
(f*x)/c + ((c*e - b*f + (2*c^2*d + b^2*f - c*(b*e + 2*a*f))/Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(
3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((c*e - b*f - (2*c^2*d - b*c*e + b^2*f
 - 2*a*c*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

3.57.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

3.57.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.31

method	result
risch	$\frac{fx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left((-bf+ec)\_R^2-af+cd\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{2c}$
default	$\frac{fx}{c}+\frac{\left(-bf\sqrt{-4ac+b^2}+c\sqrt{-4ac+b^2}\,e+2acf-b^2f+ebc-2c^2d\right)\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,c\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(-bf\sqrt{-4ac+b^2}+c\sqrt{-4ac+b\right.}{2}$

input
```
int((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
f*x/c+1/2/c*sum(((-b*f+c*e)*_R^2-a*f+c*d)/(2*_R^3*c+_R*b)*ln(x-_R),_R=Root
Of(_Z^4*c+_Z^2*b+a))
```

3.57.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5788 vs. 2(185) = 370.

Time = 4.34 (sec) , antiderivative size = 5788, normalized size of antiderivative = 26.43

$$\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.57.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate((f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.57.7 Maxima [F]

$$\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx = \int \frac{fx^4+ex^2+d}{cx^4+bx^2+a}\,dx$$

input
```
integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
f*x/c - integrate(-((c*e - b*f)*x^2 + c*d - a*f)/(c*x^4 + b*x^2 + a), x)/c
```

3.57.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4082 vs. $2(185) = 370$.

Time = 1.00 (sec) , antiderivative size = 4082, normalized size of antiderivative = 18.64

$$\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
f*x/c + 1/8*((2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*c^2*
e - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2*f + 2
*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^3*c^4 - 2*b^4*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 +
 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 + sqrt(2)*sqrt(b*c +...
```

3.57.9 Mupad [B] (verification not implemented)

Time = 9.70 (sec) , antiderivative size = 10209, normalized size of antiderivative = 46.62

$$\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4),x)
```

output
```
(f*x)/c - atan(((((4*b^2*c^3*d + 16*a^2*c^3*f - 16*a*c^4*d - 4*a*b^2*c^2*f
)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*(
-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4
*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^
2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2
+ 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*
a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f
- 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a
*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2))/c)*(-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2
*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-
(4*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*
f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^
2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f -
8*a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*
f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 +
 a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2) - (2*x*(2*c^4*d^2 + b^4*f^2 - 2*a*c^3*
e^2 + 2*a^2*c^2*f^2 + b^2*c^2*e^2 - 4*a*c^3*d*f - 2*b*c^3*d*e - 2*b^3*c*e*
f - 4*a*b^2*c*f^2 + 2*b^2*c^2*d*f + 6*a*b*c^2*e*f))/c)*(-(a*b^5*f^2 + b^3*
c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e
^2 + a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^...
```

3.57. $\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx$

3.58 $\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)}\,dx$

3.58.1 Optimal result

Integrand size = 30, antiderivative size = 213

$$\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)}\,dx = -\frac{d}{ax} - \frac{\left(cd-af+\frac{bcd-2ace+abf}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(cd-af-\frac{bcd-2ace+abf}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-d/a/x-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(c*d-a*f
+(a*b*f-2*a*c*e+b*c*d)/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^
2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))
*(c*d-a*f+(-a*b*f+2*a*c*e-b*c*d)/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/c^(1/2)/(b+
(-4*a*c+b^2)^(1/2))^(1/2)
```

3.58.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.19

$$\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)}\,dx = \frac{-\frac{2d}{x} - \frac{\sqrt{2}\left(bcd+c\sqrt{b^2-4ac}d-2ace+abf-a\sqrt{b^2-4ac}f\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(bcd-c\sqrt{b^2-4ac}d-2ace+abf+a\sqrt{b^2-4ac}f\right)\arctan(}{\sqrt{c}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{2a}$$

input
```
Integrate[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)),x]
```

output
```
((-2*d)/x - (Sqrt[2]*(b*c*d + c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e + a*b*f - a*
Sqrt[b^2 - 4*a*c]*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b*
c*d - c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e + a*b*f + a*Sqrt[b^2 - 4*a*c]*f)*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b^2 -
4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a)
```

3.58.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)}dx$$

$$\Big\downarrow \ 2195$$

$$\int \left(\frac{-\left(x^2(cd-af)\right)+ae-bd}{a\left(a+bx^2+cx^4\right)}+\frac{d}{ax^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{abf-2ace+bcd}{\sqrt{b^2-4ac}}-af+cd\right)}{\sqrt{2}a\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{abf-2ace+bcd}{\sqrt{b^2-4ac}}-af+cd\right)}{\sqrt{2}a\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{d}{ax}$$

input
```
Int[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)),x]
```

output
```
-(d/(a*x)) - ((c*d - a*f + (b*c*d - 2*a*c*e + a*b*f)/Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[c]*
Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((c*d - a*f - (b*c*d - 2*a*c*e + a*b*f)/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[2]*a*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

3.58.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

3.58.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.03

method	result
default	$\frac{4c\left(\frac{\left(af\sqrt{-4ac+b^2}-cd\sqrt{-4ac+b^2}+abf-2ace+bcd\right)\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(af\sqrt{-4ac+b^2}-cd\sqrt{-4ac+b^2}-abf+2ace-bcd\right)\sqrt{2}\ \text{arcta}}{8c\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4ac+b\right.}}}\right.}{a}$
risch	Expression too large to display

input
```
int((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
4/a*c*(1/8*(a*f*(-4*a*c+b^2)^(1/2)-c*d*(-4*a*c+b^2)^(1/2)+a*b*f-2*a*c*e+b*
c*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(
c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(a*f*(-4*a*c+b^2)^(1/2)-
c*d*(-4*a*c+b^2)^(1/2)-a*b*f+2*a*c*e-b*c*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)))-d/a/x
```

3.58.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5930 vs. 2(177) = 354.

Time = 1.66 (sec) , antiderivative size = 5930, normalized size of antiderivative = 27.84

$$\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.58.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate((f*x**4+e*x**2+d)/x**2/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.58.7 Maxima [F]

$$\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)}\,dx=\int \frac{fx^4+ex^2+d}{(cx^4+bx^2+a)x^2}\,dx$$

input
```
integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(-((c*d - a*f)*x^2 + b*d - a*e)/(c*x^4 + b*x^2 + a), x)/a - d/(a*
x)
```

3.58.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3984 vs. 2(177) = 354.

Time = 1.36 (sec) , antiderivative size = 3984, normalized size of antiderivative = 18.70

$$\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)}\,dx=\text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-d/(a*x) - 1/8*((2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*a
^2*d - (2*a*b^4*c^2 - 16*a^2*b^2*c^3 + 32*a^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c
^3)*a^2*f + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^4*c^2 - 2*a*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2...
```

3.58.9 Mupad [B] (verification not implemented)

Time = 9.77 (sec) , antiderivative size = 10170, normalized size of antiderivative = 47.75

$$\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)),x)
```

output
```
- atan(((x*(4*a^4*c^4*d^2 - 4*a^5*c^3*e^2 + 4*a^6*c^2*f^2 - 2*a^5*b^2*c*f^
2 - 2*a^3*b^2*c^3*d^2 - 8*a^5*c^3*d*f + 4*a^4*b*c^3*d*e + 4*a^5*b*c^2*e*f)
 + (-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3
*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3
*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f
 + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2
) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e + 2*a*b*c*d*e*(-(
4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2)
*(x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-
(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-
(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*
a*c - b^2)^3)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 -
 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a
^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e -
 2*a*b^4*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^
3*b^4*c - 8*a^4*b^2*c^2)))^(1/2) - 16*a^6*c^3*e - 4*a^4*b^3*c^2*d + 4*a^5*
b^2*c^2*e + 16*a^5*b*c^3*d))*(-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c
 - b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c
 - b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c...
```

3.58. $\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)}\,dx$

3.59 $\int \frac{d+ex^2+fx^4}{x^4(a+bx^2+cx^4)}\,dx$

3.59.1 Optimal result

Integrand size = 30, antiderivative size = 267

$$\begin{aligned}\int \frac{d+ex^2+fx^4}{x^4(a+bx^2+cx^4)}\,dx &= -\frac{d}{3ax^3}+\frac{bd-ae}{a^2x}+\frac{\sqrt{c}\left(bd-ae+\frac{b^2d-abe-2a(cd-af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}}\\ &\quad -\frac{\sqrt{c}\left(b^2d-b\left(\sqrt{b^2-4ac}d+ae\right)-a\left(2cd-\sqrt{b^2-4ac}e-2af\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

output
```
-1/3*d/a/x^3+(-a*e+b*d)/a^2/x+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)
^(1/2))^(1/2))*c^(1/2)*(b*d-a*e+(b^2*d-a*b*e-2*a*(-a*f+c*d))/(-4*a*c+b^2)^
(1/2))/a^2*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/
2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2*d-b*(a*e+d*(-4*a*c+b^2)^(1/2
))-a*(2*c*d-2*a*f-e*(-4*a*c+b^2)^(1/2)))/a^2*2^(1/2)/(-4*a*c+b^2)^(1/2)/(b
+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.59.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.06

$$\int \frac{d+ex^2+fx^4}{x^4(a+bx^2+cx^4)}\,dx$$
$$= \frac{-\frac{2ad}{x^3}+\frac{6bd-6ae}{x}+\frac{3\sqrt{2}\sqrt{c}\left(b^2d+b\left(\sqrt{b^2-4ac}d-ae\right)+a\left(-2cd-\sqrt{b^2-4ac}e+2af\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(-b^2d+b\left(\sqrt{b^2-4ac}\right.\right.}{}}{6a^2}$$

input
```
Integrate[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)),x]
```

output
```
((-2*a*d)/x^3 + (6*b*d - 6*a*e)/x + (3*Sqrt[2]*Sqrt[c]*(b^2*d + b*(Sqrt[b^
2 - 4*a*c]*d - a*e) + a*(-2*c*d - Sqrt[b^2 - 4*a*c]*e + 2*a*f))*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b -
 Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-(b^2*d) + b*(Sqrt[b^2 - 4*a*c]
*d + a*e) - a*(-2*c*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f))*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2
- 4*a*c]]))/(6*a^2)
```

3.59.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4}{x^4(a+bx^2+cx^4)}\,dx$$
$$\Big\downarrow \text{ 2195}$$
$$\int \left(\frac{cx^2(bd-ae)-abe-a(cd-af)+b^2d}{a^2(a+bx^2+cx^4)}+\frac{ae-bd}{a^2x^2}+\frac{d}{ax^4}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-abe-2a(cd-af)+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-a\left(-e\sqrt{b^2-4ac}-2af+2cd\right)-b\left(d\sqrt{b^2-4ac}+ae\right)+b^2d\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{bd-ae}{a^2x}-\frac{d}{3ax^3}$$

input
```
Int[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)),x]
```

output
```
-1/3*d/(a*x^3) + (b*d - a*e)/(a^2*x) + (Sqrt[c]*(b*d - a*e + (b^2*d - a*b*
e - 2*a*(c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c
]*(b^2*d - b*(Sqrt[b^2 - 4*a*c]*d + a*e) - a*(2*c*d - Sqrt[b^2 - 4*a*c]*e
- 2*a*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2
]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

3.59.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

3.59.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.91

method	result
default	$-\frac{d}{3a\,x^3}-\frac{ae-bd}{a^2x}+\frac{4c\left(\frac{\left(-ae\sqrt{-4ac+b^2}+bd\sqrt{-4ac+b^2}-2f\,a^2+abe+2acd-b^2d\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(-ae\sqrt{-4ac+b^2}+bd\right.}{}\right.}{a^2}$
risch	Expression too large to display

input
```
int((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*d/a/x^3-(a*e-b*d)/a^2/x+4/a^2*c*(1/8*(-a*e*(-4*a*c+b^2)^(1/2)+b*d*(-4
*a*c+b^2)^(1/2)-2*f*a^2+a*b*e+2*a*c*d-b^2*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))-1/8*(-a*e*(-4*a*c+b^2)^(1/2)+b*d*(-4*a*c+b^2)^(1/2)+2*f*a^2-a*b*
e-2*a*c*d+b^2*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

3.59.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9850 vs. 2(226) = 452.

Time = 11.66 (sec) , antiderivative size = 9850, normalized size of antiderivative = 36.89

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.59.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)}\,dx = \text{Timed out}$$

input
```
integrate((f*x**4+e*x**2+d)/x**4/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.59.7 Maxima [F]

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)}\,dx=\int \frac{fx^4+ex^2+d}{(cx^4+bx^2+a)x^4}\,dx$$

input
```
integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
-integrate((a*b*e - a^2*f - (b*c*d - a*c*e)*x^2 - (b^2 - a*c)*d)/(c*x^4 +
b*x^2 + a), x)/a^2 + 1/3*(3*(b*d - a*e)*x^2 - a*d)/(a^2*x^3)
```

3.59.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3804 vs. $2(226) = 452$.

Time = 0.99 (sec) , antiderivative size = 3804, normalized size of antiderivative = 14.25

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)}\,dx=\text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/4*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 9*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
5*c - 2*b^6*c + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 1
0*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^4*c^2 + 18*a*b^4*c^2 - 2*b^5*c^2 - 16*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b*c^3 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 48*a
^2*b^2*c^3 + 14*a*b^3*c^3 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
c^4 + 32*a^3*c^4 - 24*a^2*b*c^4 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^5 - 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^4*c + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^2*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 +
 2*(b^2 - 4*a*c)*b^4*c - 10*(b^2 - 4*a*c)*a*b^2*c^2 + 2*(b^2 - 4*a*c)*b^3*
c^2 + 8*(b^2 - 4*a*c)*a^2*c^3 - 6*(b^2 - 4*a*c)*a*b*c^3)*d - (sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*a*b^
5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*...
```

3.59.9 Mupad [B] (verification not implemented)

Time = 10.74 (sec) , antiderivative size = 15505, normalized size of antiderivative = 58.07

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)),x)
```

output
```
atan(((x*(4*a^8*c^5*d^2 - 4*a^9*c^4*e^2 + 4*a^10*c^3*f^2 + 2*a^6*b^4*c^3*d
^2 - 8*a^7*b^2*c^4*d^2 + 2*a^8*b^2*c^3*e^2 - 8*a^9*c^4*d*f + 12*a^8*b*c^4*
d*e - 4*a^9*b*c^3*e*f - 4*a^7*b^3*c^3*d*e + 4*a^8*b^2*c^3*d*f) - (-(b^7*d^
2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a^4*f^2
*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*
c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^
2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)
^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e
 - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) +
 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4
*a*c - b^2)^3)^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c
*e*f - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 2*a^2
*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2)
)/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(x*(32*a^11*b*c^3 - 8*a^
10*b^3*c^2)*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) +
a^4*b^3*f^2 + a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*
b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^
6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^
2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5
*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-...
```

3.59. $\int \frac{d+ex^2+fx^4}{x^4(a+bx^2+cx^4)}\,dx$

3.60 $\int \frac{d+ex^2+fx^4}{x^6(a+bx^2+cx^4)}\,dx$

3.60.1 Optimal result

Integrand size = 30, antiderivative size = 329

$$\begin{aligned}\int \frac{d+ex^2+fx^4}{x^6(a+bx^2+cx^4)}\,dx &= -\frac{d}{5ax^5}+\frac{bd-ae}{3a^2x^3}-\frac{b^2d-abe-a(cd-af)}{a^3x}\\ &\quad-\frac{\sqrt{c}\left(b^2d-abe-a(cd-af)+\frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^3\sqrt{b-\sqrt{b^2-4ac}}}\\ &\quad-\frac{\sqrt{c}\left(b^2d-abe-a(cd-af)-\frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^3\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

output
```
-1/5*d/a/x^5+1/3*(-a*e+b*d)/a^2/x^3+(-b^2*d+a*b*e+a*(-a*f+c*d))/a^3/x-1/2*
arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2*d-a*b*
e-a*(-a*f+c*d)+(b^3*d-a*b^2*e+2*a^2*c*e-a*b*(-a*f+3*c*d))/(-4*a*c+b^2)^(1/
2))/a^3*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2*d-a*b*e-a*(-a*f+c*d)+(-b^3*d+a*
b^2*e-2*a^2*c*e+a*b*(-a*f+3*c*d))/(-4*a*c+b^2)^(1/2))/a^3*2^(1/2)/(b+(-4*a
*c+b^2)^(1/2))^(1/2)
```

3.60.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.20

$$\int \frac{d+ex^2+fx^4}{x^6(a+bx^2+cx^4)}\,dx$$

$$= \frac{-\frac{6a^2d}{x^5}+\frac{10a(bd-ae)}{x^3}+\frac{30(-b^2d+abe+a(cd-af))}{x}-\frac{15\sqrt{2}\sqrt{c}\left(b^3d+b^2\left(\sqrt{b^2-4ac}d-ae\right)+ab\left(-3cd-\sqrt{b^2-4ac}e+af\right)+a\left(-c\sqrt{b^2-4ac}d+2\right.\right.}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}}{}$$

input
```
Integrate[(d + e*x^2 + f*x^4)/(x^6*(a + b*x^2 + c*x^4)),x]
```

output
```
((-6*a^2*d)/x^5 + (10*a*(b*d - a*e))/x^3 + (30*(-(b^2*d) + a*b*e + a*(c*d
- a*f)))/x - (15*Sqrt[2]*Sqrt[c]*(b^3*d + b^2*(Sqrt[b^2 - 4*a*c]*d - a*e)
+ a*b*(-3*c*d - Sqrt[b^2 - 4*a*c]*e + a*f) + a*(-(c*Sqrt[b^2 - 4*a*c]*d) +
 2*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqr
t[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (15*Sq
rt[2]*Sqrt[c]*(b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*b*(-3*c*d + Sqr
t[b^2 - 4*a*c]*e + a*f) + a*(c*Sqrt[b^2 - 4*a*c]*d + 2*a*c*e - a*Sqrt[b^2
- 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqr
t[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(30*a^3)
```

3.60.3 Rubi [A] (verified)

Time = 1.57 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4}{x^6(a+bx^2+cx^4)}\,dx$$

$$\Big\downarrow \ 2195$$

$$\int \left(\frac{-abe-a(cd-af)+b^2d}{a^3x^2}+\frac{ae-bd}{a^2x^4}+\frac{-a^2ce-cx^2\left(-abe-a(cd-af)+b^2d\right)+ab^2e+ab(2cd-af)+b^3(-}{a^3(a+bx^2+cx^4)}\right.$$

$$\Big\downarrow \ 2009$$

$$-\frac{-abe-a(cd-af)+b^2d}{a^3x}+\frac{bd-ae}{3a^2x^3}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2a^2ce-ab^2e-ab(3cd-af)+b^3d}{\sqrt{b^2-4ac}}-abe-a(cd-af)+b^2d\right)}{\sqrt{2}a^3\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{2a^2ce-ab^2e-ab(3cd-af)+b^3d}{\sqrt{b^2-4ac}}-abe-a(cd-af)+b^2d\right)}{\sqrt{2}a^3\sqrt{\sqrt{b^2-4ac}+b}}-\frac{d}{5ax^5}$$

input
```
Int[(d + e*x^2 + f*x^4)/(x^6*(a + b*x^2 + c*x^4)),x]
```

output
```
-1/5*d/(a*x^5) + (b*d - a*e)/(3*a^2*x^3) - (b^2*d - a*b*e - a*(c*d - a*f))
/(a^3*x) - (Sqrt[c]*(b^2*d - a*b*e - a*(c*d - a*f) + (b^3*d - a*b^2*e + 2*
a^2*c*e - a*b*(3*c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^3*Sqrt[b - Sqrt[b^2 - 4*a*c]]) -
 (Sqrt[c]*(b^2*d - a*b*e - a*(c*d - a*f) - (b^3*d - a*b^2*e + 2*a^2*c*e -
a*b*(3*c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^3*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

3.60.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

3.60.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.09

method	result
default	$-\frac{d}{5a\,x^5}-\frac{ae-bd}{3a^2x^3}-\frac{f\,a^2-abe-acd+b^2d}{a^3x}+$ $4c\left(\frac{\left(-f\,a^2\sqrt{-4ac+b^2}+abe\sqrt{-4ac+b^2}+acd\sqrt{-4ac+b^2}-b^2d\sqrt{-4ac+b^2}+a^2bf+2a^2ce-a\,b^2\right.}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right.$
risch	Expression too large to display

input
```
int((f*x^4+e*x^2+d)/x^6/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/5*d/a/x^5-1/3*(a*e-b*d)/a^2/x^3-(a^2*f-a*b*e-a*c*d+b^2*d)/a^3/x+4/a^3*c
*(1/8*(-f*a^2*(-4*a*c+b^2)^(1/2)+a*b*e*(-4*a*c+b^2)^(1/2)+a*c*d*(-4*a*c+b^
2)^(1/2)-b^2*d*(-4*a*c+b^2)^(1/2)+a^2*b*f+2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*
d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(-f*a^2*(-4*a*c+b^2)^(1/2)+a
*b*e*(-4*a*c+b^2)^(1/2)+a*c*d*(-4*a*c+b^2)^(1/2)-b^2*d*(-4*a*c+b^2)^(1/2)-
a^2*b*f-2*a^2*c*e+a*b^2*e+3*a*b*c*d-b^3*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)))
```

3.60.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15830 vs. 2(289) = 578.

Time = 45.94 (sec) , antiderivative size = 15830, normalized size of antiderivative = 48.12

$$\int \frac{d + ex^2 + fx^4}{x^6 (a + bx^2 + cx^4)} \, dx = \text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/x^6/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.60.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d + ex^2 + fx^4}{x^6 (a + bx^2 + cx^4)} \, dx = \text{Timed out}$$

input
```
integrate((f*x**4+e*x**2+d)/x**6/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.60.7 Maxima [F]

$$\int \frac{d+ex^2+fx^4}{x^6\left(a+bx^2+cx^4\right)}\,dx=\int \frac{fx^4+ex^2+d}{(cx^4+bx^2+a)x^6}\,dx$$

input
```
integrate((f*x^4+e*x^2+d)/x^6/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
-integrate((a^2*b*f - (a*b*c*e - a^2*c*f - (b^2*c - a*c^2)*d)*x^2 + (b^3 -
 2*a*b*c)*d - (a*b^2 - a^2*c)*e)/(c*x^4 + b*x^2 + a), x)/a^3 + 1/15*(15*(a
*b*e - a^2*f - (b^2 - a*c)*d)*x^4 - 3*a^2*d + 5*(a*b*d - a^2*e)*x^2)/(a^3*
x^5)
```

3.60.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6710 vs. 2(289) = 578.

Time = 1.35 (sec) , antiderivative size = 6710, normalized size of antiderivative = 20.40

$$\int \frac{d+ex^2+fx^4}{x^6\left(a+bx^2+cx^4\right)}\,dx=\text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/x^6/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/8*((2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^
2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*a^2*d - (2*a*b^5*c^2 - 16*a^2*b^3*c^3 + 3
2*a^3*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*
c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c -
16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 8
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 2*(b^2
- 4*a*c)*a*b^3*c^2 + 8*(b^2 - 4*a*c)*a^2*b*c^3)*a^2*e + (2*a^2*b^4*c^2 ...
```

3.60.9 Mupad [B] (verification not implemented)

Time = 11.57 (sec) , antiderivative size = 23019, normalized size of antiderivative = 69.97

$$\int \frac{d + ex^2 + fx^4}{x^6 (a + bx^2 + cx^4)}\, dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4)/(x^6*(a + b*x^2 + c*x^4)),x)
```

output
```
atan(((x*(4*a^13*c^5*e^2 - 4*a^12*c^6*d^2 - 4*a^14*c^4*f^2 + 2*a^9*b^6*c^3
*d^2 - 12*a^10*b^4*c^4*d^2 + 18*a^11*b^2*c^5*d^2 + 2*a^11*b^4*c^3*e^2 - 8*
a^12*b^2*c^4*e^2 + 2*a^13*b^2*c^3*f^2 + 8*a^13*c^5*d*f - 20*a^12*b*c^5*d*e
 + 12*a^13*b*c^4*e*f - 4*a^10*b^5*c^3*d*e + 20*a^11*b^3*c^4*d*e + 4*a^11*b
^4*c^3*d*f - 16*a^12*b^2*c^4*d*f - 4*a^12*b^3*c^3*e*f) - (-(b^9*d^2 + a^2*
b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^
2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^
2 - a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2
- 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*
(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2
)^3)^(1/2) + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2
*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f - 2*a*b^5*d*e*(
-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c
^3*d*f + 16*a^4*b^4*c*e*f + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5
*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^
3*d*e + 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f - 2*a^
3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2
) - 36*a^5*b^2*c^2*e*f - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a^4*
b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2
) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b^2*c*d*f*(-(4*a*c...
```

3.60. $\int \frac{d+ex^2+fx^4}{x^6(a+bx^2+cx^4)}\,dx$

3.61 $\int \frac{x^7(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

3.61.1 Optimal result

Integrand size = 30, antiderivative size = 320

$$\begin{aligned}\int \frac{x^7(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx &= \frac{(2b^2ce-6ac^2e-3b^3f-bc(cd-11af))\,x^2}{2c^3\,(b^2-4ac)}+\frac{(4c^2d+3b^2f-2c(be+4af))\,x^4}{4c^2\,(b^2-4ac)} \\ &\quad+\frac{x^6(2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} \\ &\quad-\frac{(2b^4ce-12ab^2c^2e+12a^2c^3e-3b^5f-b^3c(cd-20af)+6abc^2(cd-5af))\,\mathrm{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^4\,(b^2-4ac)^{3/2}} \\ &\quad+\frac{(c^2d+3b^2f-2c(be+af))\log(a+bx^2+cx^4)}{4c^4}\end{aligned}$$

output
```
1/2*(2*b^2*c*e-6*a*c^2*e-3*b^3*f-b*c*(-11*a*f+c*d))*x^2/c^3/(-4*a*c+b^2)+1
/4*(4*c^2*d+3*b^2*f-2*c*(4*a*f+b*e))*x^4/c^2/(-4*a*c+b^2)+1/2*x^6*(2*a*c*e
-b*(a*f+c*d)-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x
^2+a)-1/2*(2*b^4*c*e-12*a*b^2*c^2*e+12*a^2*c^3*e-3*b^5*f-b^3*c*(-20*a*f+c*
d)+6*a*b*c^2*(-5*a*f+c*d))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^4/(-4
*a*c+b^2)^(3/2)+1/4*(c^2*d+3*b^2*f-2*c*(a*f+b*e))*ln(c*x^4+b*x^2+a)/c^4
```

3.61.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.97

$$\int \frac{x^7(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{2c(ce-2bf)x^2+c^2fx^4+\frac{2\left(2a^3c^2f+b^3\left(c^2d-bce+b^2f\right)x^2+ab\left(b^3f-3c^3dx^2+bc^2\left(d+4ex^2\right)-b^2c\left(e+5fx^2\right)\right)+a^2c\left(-4b^2f-2c^2\left(d+ex^2\right)\right.}{(b^2-4ac)(a+bx^2+cx^4)}}{}$$

input
```
Integrate[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(2*c*(c*e - 2*b*f)*x^2 + c^2*f*x^4 + (2*(2*a^3*c^2*f + b^3*(c^2*d - b*c*e
+ b^2*f)*x^2 + a*b*(b^3*f - 3*c^3*d*x^2 + b*c^2*(d + 4*e*x^2) - b^2*c*(e +
 5*f*x^2)) + a^2*c*(-4*b^2*f - 2*c^2*(d + e*x^2) + b*c*(3*e + 5*f*x^2))))/
((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*(-2*b^4*c*e + 12*a*b^2*c^2*e - 12
*a^2*c^3*e + 3*b^5*f + b^3*c*(c*d - 20*a*f) + 6*a*b*c^2*(-(c*d) + 5*a*f))*
ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + (c^2*d +
3*b^2*f - 2*c*(b*e + a*f))*Log[a + b*x^2 + c*x^4])/(4*c^4)
```

3.61.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2194, 2175, 27, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \text{ 2194}$$

$$\frac{1}{2}\int \frac{x^6\left(fx^4+ex^2+d\right)}{\left(cx^4+bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \text{ 2175}$$

$$\frac{1}{2}\left(\frac{x^6\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int \frac{x^4\left(3c\left(2ae-\frac{b(cd+af)}{c}\right)-\left(3fb^2-2ceb+4c^2d-8acf\right)x^2\right)}{c\left(cx^4+bx^2+a\right)}dx^2}{b^2-4ac}\right.$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(\frac{x^6\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{x^4\left(\left(3fb^2-2ceb+4c^2d-8acf\right)x^2+3\left(bcd-2ace+abf\right)\right)}{cx^4+bx^2+a}dx^2}{c\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(\frac{\int \frac{x^4\left(\left(3fb^2-2ceb+4c^2d-8acf\right)x^2+3\left(bcd-2ace+abf\right)\right)}{cx^4+bx^2+a}dx^2}{c\left(b^2-4ac\right)}+\frac{x^6\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right.$$

$$\Big\downarrow \; 1200$$

$$\frac{1}{2}\left(\frac{\int \left(-\frac{3fb^3}{c^2}+\frac{2eb^2}{c}-db+\frac{11afb}{c}+\frac{\left(3fb^2-2ceb+4c^2d-8acf\right)x^2}{c}-6ae+\frac{\left(b^2-4ac\right)\left(3fb^2+c^2d-2c(be+af)\right)x^2-a\left(-3fb^3+2ceb^2-c(\cdots\right.}{c^2\left(cx^4+bx^2+a\right)}\right.}{c\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(12a^2c^3e-b^3c(cd-20af)-12ab^2c^2e+6abc^2(cd-5af)-3b^5f+2b^4ce\right)}{c^3\sqrt{b^2-4ac}}+\frac{x^4\left(-8acf+3b^2f-2bce+4c^2d\right)}{2c}+\frac{\left(b^2-4ac\right)\cdots}{\cdots}}{c\left(b^2-4ac\right)}\right.$$

input
```
Int[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((x^6*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x
^2))/(c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (((2*b^2*c*e - 6*a*c^2*e - 3*
b^3*f - b*c*(c*d - 11*a*f))*x^2)/c^2 + ((4*c^2*d - 2*b*c*e + 3*b^2*f - 8*a
*c*f)*x^4)/(2*c) - ((2*b^4*c*e - 12*a*b^2*c^2*e + 12*a^2*c^3*e - 3*b^5*f -
 b^3*c*(c*d - 20*a*f) + 6*a*b*c^2*(c*d - 5*a*f))*ArcTanh[(b + 2*c*x^2)/Sqr
t[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - 4*a*c)*(c^2*d + 3*b^2*f
 - 2*c*(b*e + a*f))*Log[a + b*x^2 + c*x^4])/(2*c^3))/(c*(b^2 - 4*a*c)))/2
```

3.61. $\int \frac{x^7\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

3.61.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2175
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a + b*x + c*x^2, x], R =
 Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], S = Coeff[Polyno
mialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*((R*b - 2*a*S + (2*c*R - b*S)*x)/((p + 1)*(b^2 - 4*a*c))), x
] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2
)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Qx + S*(2*a*e*m + b*d
*(2*p + 3)) - R*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*R - b*S)*(m + 2*p + 3)*x
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Inte
gerQ[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] &&
 RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.61. $\int \frac{x^7(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

3.61.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.35

method	result
default	$\frac{(-cf\,x^2+2bf-ec)^2}{4c^4f}+\frac{-\frac{(5a^2b\,c^2f-2a^2c^3e-5a\,b^3cf+4a\,b^2c^2e-3ab\,c^3d+b^5f-b^4ec+b^3c^2d)x^2}{c(4ac-b^2)}-\frac{a(2a^2c^2f-4a\,b^2cf+3ab\,c^2e-2a\,c^3d+b^4f-b}{c(4ac-b^2)}}{c\,x^4+b\,x^2+a}$
risch	Expression too large to display

input
```
int(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-c*f*x^2+2*b*f-c*e)^2/c^4/f+1/2/c^3*((-(5*a^2*b*c^2*f-2*a^2*c^3*e-5*a
*b^3*c*f+4*a*b^2*c^2*e-3*a*b*c^3*d+b^5*f-b^4*c*e+b^3*c^2*d)/c/(4*a*c-b^2)*
x^2-a*(2*a^2*c^2*f-4*a*b^2*c*f+3*a*b*c^2*e-2*a*c^3*d+b^4*f-b^3*c*e+b^2*c^2
*d)/c/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/(4*a*c-b^2)*(1/2*(-8*a^2*c^2*f+14*a*b
^2*c*f-8*a*b*c^2*e+4*a*c^3*d-3*b^4*f+2*b^3*c*e-b^2*c^2*d)/c*ln(c*x^4+b*x^2
+a)+2*(11*a^2*b*c*f-6*a^2*c^2*e-3*a*b^3*f+2*a*b^2*c*e-a*b*c^2*d-1/2*(-8*a^
2*c^2*f+14*a*b^2*c*f-8*a*b*c^2*e+4*a*c^3*d-3*b^4*f+2*b^3*c*e-b^2*c^2*d)*b/
c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))
```

3.61.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1043 vs. 2(306) = 612.

Time = 0.53 (sec) , antiderivative size = 2111, normalized size of antiderivative = 6.60

$$\int \frac{x^7(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*f*x^8 + (2*(b^4*c^3 - 8*a*b^2*c
^4 + 16*a^2*c^5)*e - 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*f)*x^6 + (2*
(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e - (4*b^6*c - 33*a*b^4*c^2 + 72*a^
2*b^2*c^3 - 16*a^3*c^4)*f)*x^4 + 2*((b^5*c^2 - 7*a*b^3*c^3 + 12*a^2*b*c^4)
*d - (b^6*c - 9*a*b^4*c^2 + 26*a^2*b^2*c^3 - 24*a^3*c^4)*e + (b^7 - 11*a*b
^5*c + 41*a^2*b^3*c^2 - 52*a^3*b*c^3)*f)*x^2 - (((b^3*c^3 - 6*a*b*c^4)*d -
 2*(b^4*c^2 - 6*a*b^2*c^3 + 6*a^2*c^4)*e + (3*b^5*c - 20*a*b^3*c^2 + 30*a^
2*b*c^3)*f)*x^4 + ((b^4*c^2 - 6*a*b^2*c^3)*d - 2*(b^5*c - 6*a*b^3*c^2 + 6*
a^2*b*c^3)*e + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*f)*x^2 + (a*b^3*c^2 -
 6*a^2*b*c^3)*d - 2*(a*b^4*c - 6*a^2*b^2*c^2 + 6*a^3*c^3)*e + (3*a*b^5 - 2
0*a^2*b^3*c + 30*a^3*b*c^2)*f)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^
2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) +
2*(a*b^4*c^2 - 6*a^2*b^2*c^3 + 8*a^3*c^4)*d - 2*(a*b^5*c - 7*a^2*b^3*c^2 +
 12*a^3*b*c^3)*e + 2*(a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^2 - 8*a^4*c^3)*f
+ (((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d - 2*(b^5*c^2 - 8*a*b^3*c^3 + 16
*a^2*b*c^4)*e + (3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*f)*
x^4 + ((b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d - 2*(b^6*c - 8*a*b^4*c^2 +
 16*a^2*b^2*c^3)*e + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*
f)*x^2 + (a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d - 2*(a*b^5*c - 8*a^2*b
^3*c^2 + 16*a^3*b*c^3)*e + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 3...
```

3.61.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^7(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**7*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.61.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.30

$$\int \frac{x^7(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx =$$
$$-\frac{(b^3c^2d-6\,abc^3d-2\,b^4ce+12\,ab^2c^2e-12\,a^2c^3e+3\,b^5f-20\,ab^3cf+30\,a^2bc^2f)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(b^2c^4-4\,ac^5)\sqrt{-b^2+4\,ac}}$$
$$-\frac{b^2c^3dx^4-4\,ac^4dx^4-2\,b^3c^2ex^4+8\,abc^3ex^4+3\,b^4cfx^4-14\,ab^2c^2fx^4+8\,a^2c^3fx^4-b^3c^2dx^2+2\,abc^3dx}{4\,(b^2c^4-4\,ac^5)(cx}$$
$$+\frac{(c^2d-2\,bce+3\,b^2f-2\,acf)\log(cx^4+bx^2+a)}{4\,c^4}+\frac{c^2fx^4+2\,c^2ex^2-4\,bcfx^2}{4\,c^4}$$

input
```
integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(b^3*c^2*d - 6*a*b*c^3*d - 2*b^4*c*e + 12*a*b^2*c^2*e - 12*a^2*c^3*e
+ 3*b^5*f - 20*a*b^3*c*f + 30*a^2*b*c^2*f)*arctan((2*c*x^2 + b)/sqrt(-b^2
+ 4*a*c))/((b^2*c^4 - 4*a*c^5)*sqrt(-b^2 + 4*a*c)) - 1/4*(b^2*c^3*d*x^4 -
4*a*c^4*d*x^4 - 2*b^3*c^2*e*x^4 + 8*a*b*c^3*e*x^4 + 3*b^4*c*f*x^4 - 14*a*b
^2*c^2*f*x^4 + 8*a^2*c^3*f*x^4 - b^3*c^2*d*x^2 + 2*a*b*c^3*d*x^2 + 4*a^2*c
^3*e*x^2 + b^5*f*x^2 - 4*a*b^3*c*f*x^2 - 2*a^2*b*c^2*f*x^2 - a*b^2*c^2*d +
 2*a^2*b*c^2*e + a*b^4*f - 6*a^2*b^2*c*f + 4*a^3*c^2*f)/((b^2*c^4 - 4*a*c^
5)*(c*x^4 + b*x^2 + a)) + 1/4*(c^2*d - 2*b*c*e + 3*b^2*f - 2*a*c*f)*log(c*
x^4 + b*x^2 + a)/c^4 + 1/4*(c^2*f*x^4 + 2*c^2*e*x^2 - 4*b*c*f*x^2)/c^4
```

3.61.9 Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 3499, normalized size of antiderivative = 10.93

$$\int \frac{x^7(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
x^2*(e/(2*c^2) - (b*f)/c^3) - ((2*a^3*c^2*f - 2*a^2*c^3*d + a*b^4*f - a*b^
3*c*e + a*b^2*c^2*d + 3*a^2*b*c^2*e - 4*a^2*b^2*c*f)/(2*c*(4*a*c - b^2)) +
 (x^2*(b^5*f - 2*a^2*c^3*e + b^3*c^2*d - b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c
*f + 4*a*b^2*c^2*e + 5*a^2*b*c^2*f))/(2*c*(4*a*c - b^2)))/(a*c^3 + c^4*x^4
 + b*c^3*x^2) - (log(a + b*x^2 + c*x^4)*(6*b^8*f - 128*a^3*c^5*d + 2*b^6*c
^2*d + 256*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d - 192*a^2*b^3*c^3*e +
336*a^2*b^4*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f - 24*a*b^4*c^3*d + 48
*a*b^5*c^2*e + 256*a^3*b*c^4*e))/(2*(256*a^3*c^7 - 4*b^6*c^4 + 48*a*b^4*c^
5 - 192*a^2*b^2*c^6)) + (f*x^4)/(4*c^2) + (atan(((8*a*c^7*(4*a*c - b^2)^3
- 2*b^2*c^6*(4*a*c - b^2)^3)*(((((16*a^2*c^5*f - 8*a*c^6*d + 16*a*b*c^5*e
- 24*a*b^2*c^4*f)/c^6 - (8*a*c^2*(6*b^8*f - 128*a^3*c^5*d + 2*b^6*c^2*d +
256*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d - 192*a^2*b^3*c^3*e + 336*a^2
*b^4*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f - 24*a*b^4*c^3*d + 48*a*b^5*
c^2*e + 256*a^3*b*c^4*e))/(256*a^3*c^7 - 4*b^6*c^4 + 48*a*b^4*c^5 - 192*a^
2*b^2*c^6))*(3*b^5*f - 12*a^2*c^3*e + b^3*c^2*d - 2*b^4*c*e - 6*a*b*c^3*d
- 20*a*b^3*c*f + 12*a*b^2*c^2*e + 30*a^2*b*c^2*f))/(8*c^4*(4*a*c - b^2)^(3
/2)) - (a*(3*b^5*f - 12*a^2*c^3*e + b^3*c^2*d - 2*b^4*c*e - 6*a*b*c^3*d -
20*a*b^3*c*f + 12*a*b^2*c^2*e + 30*a^2*b*c^2*f)*(6*b^8*f - 128*a^3*c^5*d +
 2*b^6*c^2*d + 256*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d - 192*a^2*b^3*
c^3*e + 336*a^2*b^4*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f - 24*a*b^4...
```

3.62 $\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

3.62.1 Optimal result

Integrand size = 30, antiderivative size = 236

$$\begin{aligned}\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx &= \frac{(2c^2d+2b^2f-c(be+6af))\,x^2}{2c^2\,(b^2-4ac)} + \frac{x^4(2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} \\ &\quad - \frac{(12a^2c^2f-b^3(ce-2bf)-2ac(2c^2d-3bce+6b^2f))\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\,(b^2-4ac)^{3/2}} \\ &\quad + \frac{(ce-2bf)\log(a+bx^2+cx^4)}{4c^3}\end{aligned}$$

output
```
1/2*(2*c^2*d+2*b^2*f-c*(6*a*f+b*e))*x^2/c^2/(-4*a*c+b^2)+1/2*x^4*(2*a*c*e-
b*(a*f+c*d)-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x^
2+a)-1/2*(12*a^2*c^2*f-b^3*(-2*b*f+c*e)-2*a*c*(6*b^2*f-3*b*c*e+2*c^2*d))*a
rctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(3/2)+1/4*(-2*b*f+
c*e)*ln(c*x^4+b*x^2+a)/c^3
```

3.62.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00

$$\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$
$$=\frac{2cfx^2-\frac{2(b^2(c^2d-bce+b^2f)x^2+a^2c(-3bf+2c(e+fx^2))+a(b^3f-2c^3dx^2+bc^2(d+3ex^2)-b^2c(e+4fx^2)))}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{2(12a^2c^2f+b^3(-ce+2bf)-2a\cdots}{(\cdots}}{4c^3}$$

input
```
Integrate[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(2*c*f*x^2 - (2*(b^2*(c^2*d - b*c*e + b^2*f)*x^2 + a^2*c*(-3*b*f + 2*c*(e
+ f*x^2)) + a*(b^3*f - 2*c^3*d*x^2 + b*c^2*(d + 3*e*x^2) - b^2*c*(e + 4*f*
x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (2*(12*a^2*c^2*f + b^3*(-(c*
e) + 2*b*f) - 2*a*c*(2*c^2*d - 3*b*c*e + 6*b^2*f))*ArcTan[(b + 2*c*x^2)/Sq
rt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + (c*e - 2*b*f)*Log[a + b*x^2 + c*
x^4])/(4*c^3)
```

3.62.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2194, 2175, 27, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\left(d+ex^2+fx^4\right)}{(a+bx^2+cx^4)^2}\,dx$$
$$\Big\downarrow \ 2194$$
$$\frac{1}{2}\int \frac{x^4\left(fx^4+ex^2+d\right)}{(cx^4+bx^2+a)^2}dx^2$$
$$\Big\downarrow \ 2175$$
$$\frac{1}{2}\left(\frac{x^4\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int \frac{x^2\left(2c\left(2ae-\frac{b(cd+af)}{c}\right)-\left(2fb^2-ceb+2c^2d-6acf\right)x^2\right)}{c(cx^4+bx^2+a)}dx\cdots}{b^2-4ac}\cdots\right.$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\left(\frac{x^4\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{x^2\left(\left(2fb^2-ceb+2c^2d-6acf\right)x^2+2\left(bcd-2ace+abf\right)\right)}{cx^4+bx^2+a}dx^2}{c\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\left(\frac{\int \frac{x^2\left(\left(2fb^2-ceb+2c^2d-6acf\right)x^2+2\left(bcd-2ace+abf\right)\right)}{cx^4+bx^2+a}dx^2}{c\left(b^2-4ac\right)}+\frac{x^4\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right.$$

$$\Big\downarrow \ 1200$$

$$\frac{1}{2}\left(\frac{\int \left(\frac{2fb^2}{c}-eb+2cd-6af-\frac{a\left(2fb^2-ceb+2c^2d-6acf\right)-\left(b^2-4ac\right)\left(ce-2bf\right)x^2}{c\left(cx^4+bx^2+a\right)}\right)dx^2}{c\left(b^2-4ac\right)}+\frac{x^4\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2\left(-2\right.\right.}{c\left(b^2-4ac\right)\left(a+b\right.}\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(12a^2c^2f-2ac\left(6b^2f-3bce+2c^2d\right)-\left(b^3\left(ce-2bf\right)\right)\right)}{c^2\sqrt{b^2-4ac}}+\frac{\left(b^2-4ac\right)\left(ce-2bf\right)\log\left(a+bx^2+cx^4\right)}{2c^2}+x^2\left(-6af+\frac{2b^2f}{c}\right.}{c\left(b^2-4ac\right)}\right.$$

input
```
Int[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((x^4*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x
^2))/(c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((2*c*d - b*e - 6*a*f + (2*b^
2*f)/c)*x^2 - ((12*a^2*c^2*f - b^3*(c*e - 2*b*f) - 2*a*c*(2*c^2*d - 3*b*c*
e + 6*b^2*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*
a*c]) + ((b^2 - 4*a*c)*(c*e - 2*b*f)*Log[a + b*x^2 + c*x^4])/(2*c^2))/(c*(
b^2 - 4*a*c)))/2
```

3.62. $\int \frac{x^5\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

3.62.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2175
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a + b*x + c*x^2, x], R =
 Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], S = Coeff[Polyno
mialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*((R*b - 2*a*S + (2*c*R - b*S)*x)/((p + 1)*(b^2 - 4*a*c))), x
] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2
)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Qx + S*(2*a*e*m + b*d
*(2*p + 3)) - R*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*R - b*S)*(m + 2*p + 3)*x
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Inte
gerQ[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] &&
 RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.62.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.31

method	result
default	$\frac{f\,x^{2}}{2c^{2}}-\frac{\frac{-\frac{\left(2a^{2}c^{2}f-4a\,b^{2}cf+3ab\,c^{2}e-2a\,c^{3}d+b^{4}f-b^{3}ce+b^{2}c^{2}d\right)x^{2}}{c\left(4ac-b^{2}\right)}+\frac{a\left(3abcf-2a\,c^{2}e-b^{3}f+b^{2}ce-b\,c^{2}d\right)}{c\left(4ac-b^{2}\right)}}{c\,x^{4}+b\,x^{2}+a}+\frac{\frac{\left(8abcf-4a\,c^{2}e-2b^{3}f+b^{2}ce\right)\ln\left(c\,x\right.}{2c}}{}}{2c^{2}}$
risch	Expression too large to display

input
```
int(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*f*x^2/c^2-1/2/c^2*((-(2*a^2*c^2*f-4*a*b^2*c*f+3*a*b*c^2*e-2*a*c^3*d+b^
4*f-b^3*c*e+b^2*c^2*d)/c/(4*a*c-b^2)*x^2+a*(3*a*b*c*f-2*a*c^2*e-b^3*f+b^2*
c*e-b*c^2*d)/c/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/(4*a*c-b^2)*(1/2*(8*a*b*c*f-
4*a*c^2*e-2*b^3*f+b^2*c*e)/c*ln(c*x^4+b*x^2+a)+2*(6*a^2*c*f-2*a*b^2*f+a*b*
c*e-2*a*c^2*d-1/2*(8*a*b*c*f-4*a*c^2*e-2*b^3*f+b^2*c*e)*b/c)/(4*a*c-b^2)^(
1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))
```

3.62.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 716 vs. 2(224) = 448.

Time = 0.35 (sec) , antiderivative size = 1455, normalized size of antiderivative = 6.17

$$\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx=\text{Too large to display}$$

input
```
integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*f*x^6 + 2*(b^5*c - 8*a*b^3*c^
2 + 16*a^2*b*c^3)*f*x^4 - 2*((b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d - (b^5*
c - 7*a*b^3*c^2 + 12*a^2*b*c^3)*e + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24
*a^3*c^3)*f)*x^2 + (4*a^2*c^3*d + (4*a*c^4*d + (b^3*c^2 - 6*a*b*c^3)*e - 2
*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*f)*x^4 + (4*a*b*c^3*d + (b^4*c - 6*a*b^
2*c^2)*e - 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*f)*x^2 + (a*b^3*c - 6*a^2*b*c
^2)*e - 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*f)*sqrt(b^2 - 4*a*c)*log((2*c^
2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4
+ b*x^2 + a)) - 2*(a*b^3*c^2 - 4*a^2*b*c^3)*d + 2*(a*b^4*c - 6*a^2*b^2*c^2
 + 8*a^3*c^3)*e - 2*(a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2)*f + (((b^4*c^2 -
8*a*b^2*c^3 + 16*a^2*c^4)*e - 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*f)*x^
4 + ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e - 2*(b^6 - 8*a*b^4*c + 16*a^2*
b^2*c^2)*f)*x^2 + (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e - 2*(a*b^5 - 8*
a^2*b^3*c + 16*a^3*b*c^2)*f)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^3 - 8*a^2*b^
2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^4 + (b^5*c^3 -
 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^2), 1/4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*
c^4)*f*x^6 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*f*x^4 - 2*((b^4*c^2 -
6*a*b^2*c^3 + 8*a^2*c^4)*d - (b^5*c - 7*a*b^3*c^2 + 12*a^2*b*c^3)*e + (b^6
 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*f)*x^2 + 2*(4*a^2*c^3*d + (4*a
*c^4*d + (b^3*c^2 - 6*a*b*c^3)*e - 2*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*...
```

3.62.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.62.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.15

$$\begin{aligned}&\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx\\&=\frac{fx^2}{2\,c^2}-\frac{(4\,ac^3d+b^3ce-6\,abc^2e-2\,b^4f+12\,ab^2cf-12\,a^2c^2f)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(b^2c^3-4\,ac^4)\sqrt{-b^2+4\,ac}}\\&-\frac{b^2cex^4-4\,ac^2ex^4-2\,b^3fx^4+8\,abcfx^4+2\,b^2cdx^2-4\,ac^2dx^2-b^3ex^2+2\,abcex^2+4\,a^2cfx^2+2\,abcd-}{4\,(cx^4+bx^2+a)(b^2c^2-4\,ac^3)}\\&+\frac{(ce-2\,bf)\log(cx^4+bx^2+a)}{4\,c^3}\end{aligned}$$

input
```
integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*f*x^2/c^2 - 1/2*(4*a*c^3*d + b^3*c*e - 6*a*b*c^2*e - 2*b^4*f + 12*a*b^
2*c*f - 12*a^2*c^2*f)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^3 -
 4*a*c^4)*sqrt(-b^2 + 4*a*c)) - 1/4*(b^2*c*e*x^4 - 4*a*c^2*e*x^4 - 2*b^3*f
*x^4 + 8*a*b*c*f*x^4 + 2*b^2*c*d*x^2 - 4*a*c^2*d*x^2 - b^3*e*x^2 + 2*a*b*c
*e*x^2 + 4*a^2*c*f*x^2 + 2*a*b*c*d - a*b^2*e + 2*a^2*b*f)/((c*x^4 + b*x^2
+ a)*(b^2*c^2 - 4*a*c^3)) + 1/4*(c*e - 2*b*f)*log(c*x^4 + b*x^2 + a)/c^3
```

3.62.9 Mupad [B] (verification not implemented)

Time = 8.79 (sec) , antiderivative size = 2450, normalized size of antiderivative = 10.38

$$\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
((a*(b^3*f + 2*a*c^2*e + b*c^2*d - b^2*c*e - 3*a*b*c*f))/(2*c*(4*a*c - b^2
)) + (x^2*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*d - b^3*c*e + 3*a*b*c
^2*e - 4*a*b^2*c*f))/(2*c*(4*a*c - b^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) +
(f*x^2)/(2*c^2) + (log(a + b*x^2 + c*x^4)*(4*b^7*f + 128*a^3*c^4*e - 2*b^6
*c*e - 96*a^2*b^2*c^3*e + 192*a^2*b^3*c^2*f - 48*a*b^5*c*f + 24*a*b^4*c^2*
e - 256*a^3*b*c^3*f))/(2*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2
*b^2*c^5)) - (atan(((8*a*c^5*(4*a*c - b^2)^3 - 2*b^2*c^4*(4*a*c - b^2)^3)*
(x^2*(((((24*a^2*c^5*f - 6*b^3*c^4*e + 12*b^4*c^3*f - 8*a*c^6*d + 28*a*b*c
^5*e - 56*a*b^2*c^4*f)/(4*a*c^5 - b^2*c^4) + ((8*b^3*c^6 - 32*a*b*c^7)*(4*
b^7*f + 128*a^3*c^4*e - 2*b^6*c*e - 96*a^2*b^2*c^3*e + 192*a^2*b^3*c^2*f -
 48*a*b^5*c*f + 24*a*b^4*c^2*e - 256*a^3*b*c^3*f))/(2*(4*a*c^5 - b^2*c^4)*
(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)))*(2*b^4*f + 12
*a^2*c^2*f - 4*a*c^3*d - b^3*c*e + 6*a*b*c^2*e - 12*a*b^2*c*f))/(8*c^3*(4*
a*c - b^2)^(3/2)) + ((8*b^3*c^6 - 32*a*b*c^7)*(2*b^4*f + 12*a^2*c^2*f - 4*
a*c^3*d - b^3*c*e + 6*a*b*c^2*e - 12*a*b^2*c*f)*(4*b^7*f + 128*a^3*c^4*e -
 2*b^6*c*e - 96*a^2*b^2*c^3*e + 192*a^2*b^3*c^2*f - 48*a*b^5*c*f + 24*a*b^
4*c^2*e - 256*a^3*b*c^3*f))/(16*c^3*(4*a*c - b^2)^(3/2)*(4*a*c^5 - b^2*c^4
)*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)))/(a*(4*a*c -
 b^2)) + (b*((4*b^5*f^2 + b^3*c^2*e^2 + 12*a^2*b*c^2*f^2 + 2*a*c^4*d*e - 4
*b^4*c*e*f - 5*a*b*c^3*e^2 - 20*a*b^3*c*f^2 - 6*a^2*c^3*e*f + 20*a*b^2*...
```

3.63 $\int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

3.63.1 Optimal result

Integrand size = 30, antiderivative size = 165

$$\int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \frac{x^2(2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{(4ac^2e+b^3f-2bc(cd+3af))\,\mathrm{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\,(b^2-4ac)^{3/2}} + \frac{f\log(a+bx^2+cx^4)}{4c^2}$$

output
```
1/2*x^2*(2*a*c*e-b*(a*f+c*d)-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x^2)/c/(-4*a*c
+b^2)/(c*x^4+b*x^2+a)+1/2*(4*a*c^2*e+b^3*f-2*b*c*(3*a*f+c*d))*arctanh((2*c
*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(3/2)+1/4*f*ln(c*x^4+b*x^2+a)
/c^2
```

3.63.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.06

$$\int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{\frac{2(-2a^2cf+b(c^2d-bce+b^2f)x^2+a(b^2f+2c^2(d+ex^2)-bc(e+3fx^2)))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{2(4ac^2e+b^3f-2bc(cd+3af))\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} + f\log(a+b}{4c^2}$$

input
```
Integrate[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((2*(-2*a^2*c*f + b*(c^2*d - b*c*e + b^2*f)*x^2 + a*(b^2*f + 2*c^2*(d + e*
x^2) - b*c*(e + 3*f*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*(4*a*
c^2*e + b^3*f - 2*b*c*(c*d + 3*a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*
c]])/(-b^2 + 4*a*c)^(3/2) + f*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

3.63.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {2194, 2175, 27, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{x^2\left(fx^4+ex^2+d\right)}{\left(cx^4+bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \text{2175}$$

$$\frac{1}{2}\left(\frac{x^2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int \frac{c\left(2ae-\frac{b(cd+af)}{c}\right)-\left(b^2-4ac\right)fx^2}{c\left(cx^4+bx^2+a\right)}dx^2}{b^2-4ac}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{x^2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int \frac{-\left(\left(b^2-4ac\right)fx^2\right)+2ace-b(cd+af)}{cx^4+bx^2+a}dx^2}{c\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{x^2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\frac{\left(-2bc(3af+cd)+4ac^2e+b^3f\right)\int \frac{1}{cx^4+bx^2+a}dx^2}{2c}-\frac{f\left(b^2-4ac\right)\cdots}{}}{c\left(b^2-4ac\right)}\right.$$

$\Big\downarrow$ 1083

$$\frac{1}{2}\left(\frac{x^2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-\frac{f\left(b^2-4ac\right)\int\frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}-\frac{\left(-2bc(3af+cd)+4ac^2e+\cdots\right.}{\cdots}}{c\left(b^2-4ac\right)}\right.$$

$\Big\downarrow$ 219

$$\frac{1}{2}\left(\frac{x^2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-\frac{f\left(b^2-4ac\right)\int\frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(\cdots}{c\sqrt{\cdots}}}{c\left(b^2-4ac\right)}\right.$$

$\Big\downarrow$ 1103

$$\frac{1}{2}\left(\frac{x^2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-2bc(3af+cd)+4ac^2e+b^3f\right)}{c\sqrt{b^2-4ac}}-\cdots}{c\left(b^2-4ac\right)}\right.$$

input
```
Int[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((x^2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x
^2))/(c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (-(((4*a*c^2*e + b^3*f - 2*b*
c*(c*d + 3*a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4
*a*c])) - ((b^2 - 4*a*c)*f*Log[a + b*x^2 + c*x^4])/(2*c))/(c*(b^2 - 4*a*c)
))/2
```

3.63.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2175
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a + b*x + c*x^2, x], R =
 Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], S = Coeff[Polyno
mialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*((R*b - 2*a*S + (2*c*R - b*S)*x)/((p + 1)*(b^2 - 4*a*c))), x
] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2
)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Qx + S*(2*a*e*m + b*d
*(2*p + 3)) - R*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*R - b*S)*(m + 2*p + 3)*x
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Inte
gerQ[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] &&
 RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.63. $\int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

3.63.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.38

method	result
default	$\frac{\frac{\left(3abcf-2a\,c^2e-b^3f+b^2ce-b\,c^2d\right)x^2}{\left(4ac-b^2\right)c^2}+\frac{a\left(2acf-b^2f+ebc-2c^2d\right)}{\left(4ac-b^2\right)c^2}}{2c\,x^4+2b\,x^2+2a}+\frac{\frac{\left(4acf-b^2f\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(-abf+2ace-bcd-\frac{\left(4acf-b^2f\right)b}{2c}\right)\text{arcta}}{\sqrt{4ac-b^2}}}{2c(4ac-b^2)}$
risch	Expression too large to display

input
```
int(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*((3*a*b*c*f-2*a*c^2*e-b^3*f+b^2*c*e-b*c^2*d)/(4*a*c-b^2)/c^2*x^2+a*(2*
a*c*f-b^2*f+b*c*e-2*c^2*d)/(4*a*c-b^2)/c^2)/(c*x^4+b*x^2+a)+1/2/c/(4*a*c-b
^2)*(1/2*(4*a*c*f-b^2*f)/c*ln(c*x^4+b*x^2+a)+2*(-a*b*f+2*a*c*e-b*c*d-1/2*(
4*a*c*f-b^2*f)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)
))
```

3.63.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. 2(155) = 310.

Time = 0.33 (sec) , antiderivative size = 970, normalized size of antiderivative = 5.88

$$\int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \left[\frac{2\left(\left(b^3c^2-4\,abc^3\right)d-\left(b^4c-6\,ab^2c^2+8\,a^2c^3\right)e+\left(b^5-7\,ab^3c+12\,a^2bc^2\right)f\right)x^2-\left(2\,abc^2d-4\,a^2c^2e+(2\,b\right.}{}\right.$$

input
```
integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*e + (
b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*f)*x^2 - (2*a*b*c^2*d - 4*a^2*c^2*e + (2*b
*c^3*d - 4*a*c^3*e - (b^3*c - 6*a*b*c^2)*f)*x^4 + (2*b^2*c^2*d - 4*a*b*c^2
*e - (b^4 - 6*a*b^2*c)*f)*x^2 - (a*b^3 - 6*a^2*b*c)*f)*sqrt(b^2 - 4*a*c)*l
og((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))
/(c*x^4 + b*x^2 + a)) + 4*(a*b^2*c^2 - 4*a^2*c^3)*d - 2*(a*b^3*c - 4*a^2*b
*c^2)*e + 2*(a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*f + ((b^4*c - 8*a*b^2*c^2 +
16*a^2*c^3)*f*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*f*x^2 + (a*b^4 - 8*a^
2*b^2*c + 16*a^3*c^2)*f)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^
3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a
*b^3*c^3 + 16*a^2*b*c^4)*x^2), 1/4*(2*((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c -
6*a*b^2*c^2 + 8*a^2*c^3)*e + (b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*f)*x^2 - 2*(
2*a*b*c^2*d - 4*a^2*c^2*e + (2*b*c^3*d - 4*a*c^3*e - (b^3*c - 6*a*b*c^2)*f
)*x^4 + (2*b^2*c^2*d - 4*a*b*c^2*e - (b^4 - 6*a*b^2*c)*f)*x^2 - (a*b^3 - 6
*a^2*b*c)*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(
b^2 - 4*a*c)) + 4*(a*b^2*c^2 - 4*a^2*c^3)*d - 2*(a*b^3*c - 4*a^2*b*c^2)*e
+ 2*(a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*f + ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c
^3)*f*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*f*x^2 + (a*b^4 - 8*a^2*b^2*c
+ 16*a^3*c^2)*f)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a
^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*...
```

3.63.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.63.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(d + ex^2 + fx^4)}{(a + bx^2 + cx^4)^2} dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.16

$$\int \frac{x^3(d + ex^2 + fx^4)}{(a + bx^2 + cx^4)^2} dx = \frac{(2\,bc^2d - 4\,ac^2e - b^3f + 6\,abcf)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(b^2c^2 - 4\,ac^3)\sqrt{-b^2 + 4\,ac}} + \frac{f\log\left(cx^4 + bx^2 + a\right)}{4\,c^2} + \frac{2\,ac^2d - abce + ab^2f - 2\,a^2cf + (bc^2d - b^2ce + 2\,ac^2e + b^3f - 3\,abcf)x^2}{2\,(cx^4 + bx^2 + a)(b^2 - 4\,ac)c^2}$$

input
```
integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(2*b*c^2*d - 4*a*c^2*e - b^3*f + 6*a*b*c*f)*arctan((2*c*x^2 + b)/sqrt(
-b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^3)*sqrt(-b^2 + 4*a*c)) + 1/4*f*log(c*x^4
+ b*x^2 + a)/c^2 + 1/2*(2*a*c^2*d - a*b*c*e + a*b^2*f - 2*a^2*c*f + (b*c^2
*d - b^2*c*e + 2*a*c^2*e + b^3*f - 3*a*b*c*f)*x^2)/((c*x^4 + b*x^2 + a)*(b
^2 - 4*a*c)*c^2)
```

3.63.9 Mupad [B] (verification not implemented)

Time = 9.66 (sec) , antiderivative size = 1651, normalized size of antiderivative = 10.01

$$\int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
- ((a*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e))/(2*c^2*(4*a*c - b^2)) + (x^2*(b
^3*f + 2*a*c^2*e + b*c^2*d - b^2*c*e - 3*a*b*c*f))/(2*c^2*(4*a*c - b^2)))/
(a + b*x^2 + c*x^4) - (log(a + b*x^2 + c*x^4)*(2*b^6*f - 128*a^3*c^3*f + 9
6*a^2*b^2*c^2*f - 24*a*b^4*c*f))/(2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^
3 - 192*a^2*b^2*c^4)) - (atan(((8*a*c^3*(4*a*c - b^2)^3 - 2*b^2*c^2*(4*a*c
 - b^2)^3)*((((8*a*f + (8*a*c^2*(2*b^6*f - 128*a^3*c^3*f + 96*a^2*b^2*c^2*
f - 24*a*b^4*c*f))/(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c
^4))*(b^3*f + 4*a*c^2*e - 2*b*c^2*d - 6*a*b*c*f))/(8*c^2*(4*a*c - b^2)^(3/
2)) + (a*(2*b^6*f - 128*a^3*c^3*f + 96*a^2*b^2*c^2*f - 24*a*b^4*c*f)*(b^3*
f + 4*a*c^2*e - 2*b*c^2*d - 6*a*b*c*f))/((4*a*c - b^2)^(3/2)*(256*a^3*c^5
- 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))/(a*(4*a*c - b^2)) - x^2*((
(((6*b^3*c^2*f + 8*a*c^4*e - 4*b*c^4*d - 28*a*b*c^3*f)/(4*a*c^3 - b^2*c^2)
 + ((8*b^3*c^4 - 32*a*b*c^5)*(2*b^6*f - 128*a^3*c^3*f + 96*a^2*b^2*c^2*f -
 24*a*b^4*c*f))/(2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4
*c^3 - 192*a^2*b^2*c^4)))*(b^3*f + 4*a*c^2*e - 2*b*c^2*d - 6*a*b*c*f))/(8*
c^2*(4*a*c - b^2)^(3/2)) + ((8*b^3*c^4 - 32*a*b*c^5)*(2*b^6*f - 128*a^3*c^
3*f + 96*a^2*b^2*c^2*f - 24*a*b^4*c*f)*(b^3*f + 4*a*c^2*e - 2*b*c^2*d - 6*
a*b*c*f))/(16*c^2*(4*a*c - b^2)^(3/2)*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4
*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))/(a*(4*a*c - b^2)) + (b*((b^3*
f^2 - 5*a*b*c*f^2 + 2*a*c^2*e*f - b*c^2*d*f)/(4*a*c^3 - b^2*c^2) + (((6...
```

3.64 $\int \frac{x(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

3.64.1 Optimal result

Integrand size = 28, antiderivative size = 123

$$\int \frac{x(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \frac{2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{(2cd-be+2af)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
1/2*(2*a*c*e-b*(a*f+c*d)-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x^2)/c/(-4*a*c+b^2
)/(c*x^4+b*x^2+a)+(2*a*f-b*e+2*c*d)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2)
)/(-4*a*c+b^2)^(3/2)
```

3.64.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.06

$$\int \frac{x(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \frac{abf+2c^2dx^2+b^2fx^2+bc(d-ex^2)-2ac(e+fx^2)}{2c\,(-b^2+4ac)\,(a+bx^2+cx^4)} - \frac{(-2cd+be-2af)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

input
```
Integrate[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(a*b*f + 2*c^2*d*x^2 + b^2*f*x^2 + b*c*(d - e*x^2) - 2*a*c*(e + f*x^2))/(2
*c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) - ((-2*c*d + b*e - 2*a*f)*ArcTan[(b
 + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

3.64.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2194, 2191, 27, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{fx^4+ex^2+d}{(cx^4+bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{2}\left(\frac{c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2(-2acf+b^2f-bce+2c^2d)}{c(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int \frac{2cd-be+2af}{cx^4+bx^2+a}dx^2}{b^2-4ac}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2(-2acf+b^2f-bce+2c^2d)}{c(b^2-4ac)(a+bx^2+cx^4)}-\frac{(2af-be+2cd)\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{2(2af-be+2cd)\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{b^2-4ac}+\frac{c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2(-2acf+b^2f-bce+2c^2d)}{c(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{2\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(2af-be+2cd)}{(b^2-4ac)^{3/2}}+\frac{c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2(-2acf+b^2f-bce+2c^2d)}{c(b^2-4ac)(a+bx^2+cx^4)}\right)$$

input
```
Int[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^2)/(
c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*(2*c*d - b*e + 2*a*f)*ArcTanh[(b
 + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2))/2
```

3.64.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.64.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.13

method	result
default	$\frac{-\frac{(2acf-b^2f+ebc-2c^2d)x^2}{c(4ac-b^2)}+\frac{abf-2ace+bcd}{c(4ac-b^2)}}{2cx^4+2bx^2+2a}+\frac{(2af-be+2cd)\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$
risch	$\frac{-\frac{(2acf-b^2f+ebc-2c^2d)x^2}{2c(4ac-b^2)}+\frac{abf-2ace+bcd}{2c(4ac-b^2)}}{cx^4+bx^2+a}+\frac{\ln\left(\left(\left((-4ac+b^2)^{\frac{3}{2}}+4abc-b^3\right)x^2+8ca^2-2b^2a\right)af}{(-4ac+b^2)^{\frac{3}{2}}}-\frac{\ln\left(\left(\left((-4ac+b^2)^{\frac{3}{2}}+4abc-b^3\right.\right.\right.}{2(-4ac+b^2}$

input
```
int(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-(2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c/(4*a*c-b^2)*x^2+1/c*(a*b*f-2*a*c*e+b
*c*d)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+(2*a*f-b*e+2*c*d)/(4*a*c-b^2)^(3/2)*arc
tan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

3.64.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(117) = 234.

Time = 0.29 (sec) , antiderivative size = 650, normalized size of antiderivative = 5.28

$$\int \frac{x(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$
$$=\left[-\frac{(2\,(b^2c^2-4\,ac^3)d-(b^3c-4\,abc^2)e+(b^4-6\,ab^2c+8\,a^2c^2)f)x^2+((2\,c^3d-bc^2e+2\,ac^2f)x^4+2\,ac^2d}{2\,(ab^4c-8\,a^2b^2c^2}\right.$$
$$-\frac{(2\,(b^2c^2-4\,ac^3)d-(b^3c-4\,abc^2)e+(b^4-6\,ab^2c+8\,a^2c^2)f)x^2-2\,((2\,c^3d-bc^2e+2\,ac^2f)x^4+2\,ac^2}{2\,(ab^4c-8\,a^2b^2c^2+16\,}$$

input
```
integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/2*((2*(b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e + (b^4 - 6*a*b^2*c
 + 8*a^2*c^2)*f)*x^2 + ((2*c^3*d - b*c^2*e + 2*a*c^2*f)*x^4 + 2*a*c^2*d -
a*b*c*e + 2*a^2*c*f + (2*b*c^2*d - b^2*c*e + 2*a*b*c*f)*x^2)*sqrt(b^2 - 4*
a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4
*a*c))/(c*x^4 + b*x^2 + a)) + (b^3*c - 4*a*b*c^2)*d - 2*(a*b^2*c - 4*a^2*c
^2)*e + (a*b^3 - 4*a^2*b*c)*f)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^
4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^
3)*x^2), -1/2*((2*(b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e + (b^4 - 6
*a*b^2*c + 8*a^2*c^2)*f)*x^2 - 2*((2*c^3*d - b*c^2*e + 2*a*c^2*f)*x^4 + 2*
a*c^2*d - a*b*c*e + 2*a^2*c*f + (2*b*c^2*d - b^2*c*e + 2*a*b*c*f)*x^2)*sqr
t(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) +
(b^3*c - 4*a*b*c^2)*d - 2*(a*b^2*c - 4*a^2*c^2)*e + (a*b^3 - 4*a^2*b*c)*f)
/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c
^4)*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2)]
```

3.64.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.64.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.11

$$\int \frac{x(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = -\frac{(2\,cd-be+2\,af)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} - \frac{2\,c^2dx^2-bcex^2+b^2fx^2-2\,acfx^2+bcd-2\,ace+abf}{2\,(cx^4+bx^2+a)(b^2c-4\,ac^2)}$$

input
```
integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-(2*c*d - b*e + 2*a*f)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*
a*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*c^2*d*x^2 - b*c*e*x^2 + b^2*f*x^2 - 2*a*
c*f*x^2 + b*c*d - 2*a*c*e + a*b*f)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2))
```

3.64.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.78

$$\int \frac{x(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \frac{\frac{a\,b\,f-2\,a\,c\,e+b\,c\,d}{2\,c\,(4\,a\,c-b^2)}+\frac{x^2\,(f\,b^2-e\,b\,c+2\,d\,c^2-2\,a\,f\,c)}{2\,c\,(4\,a\,c-b^2)}}{c\,x^4+b\,x^2+a} + \frac{\mathrm{atan}\left(\frac{(4\,a\,c-b^2)^4\left(x^2\left(\frac{(2\,c^3\,d+2\,a\,c^2\,f-b\,c^2\,e)\,(2\,a\,f-b\,e+2\,c\,d)}{a\,(4\,a\,c-b^2)^{7/2}}+\frac{(2\,b^3\,c^2-8\,a\,b\,c^3)\,(b^3-4\,a\,b\,c)\,(2\,a\,f-b\,e+2\,c\,d)^2}{2\,a\,(4\,a\,c-b^2)^{13/2}}\right)-\frac{2\,c^2\,(b^3-4\,a\,b\,c)\,(2\,a\,f-b\,e+\cdots}{(4\,a\,c-b^2)^{11/2}}\cdots}{8\,a^2\,c^2\,f^2-8\,a\,b\,c^2\,e\,f+16\,a\,c^3\,d\,f+2\,b^2\,c^2\,e^2-8\,b\,c^3\,d\,e+8\,c^4\,d^2}\right.\cdots}{(4\,a\,c-b^2)^{3/2}}$$

input
```
int((x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
((a*b*f - 2*a*c*e + b*c*d)/(2*c*(4*a*c - b^2)) + (x^2*(2*c^2*d + b^2*f - 2
*a*c*f - b*c*e))/(2*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) + (atan(((4*a*c
- b^2)^4*(x^2*(((2*c^3*d + 2*a*c^2*f - b*c^2*e)*(2*a*f - b*e + 2*c*d))/(a*
(4*a*c - b^2)^(7/2)) + ((2*b^3*c^2 - 8*a*b*c^3)*(b^3 - 4*a*b*c)*(2*a*f - b
*e + 2*c*d)^2)/(2*a*(4*a*c - b^2)^(13/2))) - (2*c^2*(b^3 - 4*a*b*c)*(2*a*f
 - b*e + 2*c*d)^2)/(4*a*c - b^2)^(11/2)))/(8*c^4*d^2 + 8*a^2*c^2*f^2 + 2*b
^2*c^2*e^2 + 16*a*c^3*d*f - 8*b*c^3*d*e - 8*a*b*c^2*e*f))*(2*a*f - b*e + 2
*c*d))/(4*a*c - b^2)^(3/2)
```

3.65 $\int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)^2}\,dx$

3.65.1 Optimal result

Integrand size = 30, antiderivative size = 166

$$\int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)^2}\,dx = \frac{b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{(b^3d+4a^2ce-2ab(3cd+af))\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2(b^2-4ac)^{3/2}} + \frac{d\log(x)}{a^2} - \frac{d\log(a+bx^2+cx^4)}{4a^2}$$

output
```
1/2*(b^2*d-a*b*e-2*a*(-a*f+c*d)+(a*b*f-2*a*c*e+b*c*d)*x^2)/a/(-4*a*c+b^2)/
(c*x^4+b*x^2+a)+1/2*(b^3*d+4*a^2*c*e-2*a*b*(a*f+3*c*d))*arctanh((2*c*x^2+b
)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)+d*ln(x)/a^2-1/4*d*ln(c*x^4+b*
x^2+a)/a^2
```

3.65.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.61

$$\int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)^2}\,dx = -\frac{-\frac{2a(b^2d+b(-ae+cdx^2+afx^2)+2a(af-c(d+ex^2)))}{(b^2-4ac)(a+bx^2+cx^4)} - 4d\log(x) + \frac{\left(b^3d+b^2\sqrt{b^2-4ac}\,d+4ac\left(-\sqrt{b^2-4ac}\,d+ae\right)-2ab(3cd+af)\right)\log\left(b\right.}{(b^2-4ac)^{3/2}}}{4a^2}$$

input
```
Integrate[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)^2),x]
```

output
```
-1/4*((-2*a*(b^2*d + b*(-(a*e) + c*d*x^2 + a*f*x^2) + 2*a*(a*f - c*(d + e*
x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - 4*d*Log[x] + ((b^3*d + b^2*S
qrt[b^2 - 4*a*c]*d + 4*a*c*(-(Sqrt[b^2 - 4*a*c]*d) + a*e) - 2*a*b*(3*c*d +
 a*f))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((-(b^3
*d) + b^2*Sqrt[b^2 - 4*a*c]*d - 4*a*c*(Sqrt[b^2 - 4*a*c]*d + a*e) + 2*a*b*
(3*c*d + a*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/
a^2
```

3.65.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2194, 2177, 25, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \text{ 2194}$$

$$\frac{1}{2}\int \frac{fx^4+ex^2+d}{x^2\left(cx^4+bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \text{ 2177}$$

$$\frac{1}{2}\left(\frac{a\left(\frac{b^2d}{a}+2af-be-2cd\right)+x^2(abf-2ace+bcd)}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{(bcd-2ace+abf)x^2+a\left(\frac{b^2}{a}-4c\right)d}{ax^2\left(cx^4+bx^2+a\right)}dx^2}{b^2-4ac}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{\int \frac{(bcd-2ace+abf)x^2+\left(b^2-4ac\right)d}{ax^2\left(cx^4+bx^2+a\right)}dx^2}{b^2-4ac}+\frac{a\left(\frac{b^2d}{a}+2af-be-2cd\right)+x^2(abf-2ace+bcd)}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\frac{\int \frac{(bcd-2ace+abf)x^2+\left(b^2-4ac\right)d}{x^2\left(cx^4+bx^2+a\right)}dx^2}{a\left(b^2-4ac\right)}+\frac{a\left(\frac{b^2d}{a}+2af-be-2cd\right)+x^2(abf-2ace+bcd)}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \ 1200$$

$$\frac{1}{2}\left(\frac{\int \left(\frac{-db^3+a(5cd+af)b-c(b^2-4ac)dx^2-2a^2ce}{a(cx^4+bx^2+a)}-\frac{(4ac-b^2)d}{ax^2}\right)dx^2}{a\left(b^2-4ac\right)}+\frac{a\left(\frac{b^2d}{a}+2af-be-2cd\right)+x^2(abf-2ace+bcd)}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(4a^2ce-2ab(af+3cd)+b^3d\right)}{a\sqrt{b^2-4ac}}+\frac{d\log\left(x^2\right)\left(b^2-4ac\right)}{a}-\frac{d(b^2-4ac)\log\left(a+bx^2+cx^4\right)}{2a}}{a\left(b^2-4ac\right)}+\frac{a\left(\frac{b^2d}{a}+2af-be-2cd\right)}{a\left(b^2-4ac\right)}\right.$$

input
```
Int[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)^2),x]
```

output
```
((a*((b^2*d)/a - 2*c*d - b*e + 2*a*f) + (b*c*d - 2*a*c*e + a*b*f)*x^2)/(a*
(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (((b^3*d + 4*a^2*c*e - 2*a*b*(3*c*d +
 a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + (
(b^2 - 4*a*c)*d*Log[x^2])/a - ((b^2 - 4*a*c)*d*Log[a + b*x^2 + c*x^4])/(2*
a))/(a*(b^2 - 4*a*c)))/2
```

3.65.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.65.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.37

method	result
default	$\frac{d\ln(x)}{a^2}+\frac{\frac{-\frac{a(abf-2ace+bcd)x^2}{4ac-b^2}-\frac{a\left(2f\,a^2-abe-2acd+b^2d\right)}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\frac{\left(-4a\,c^2d+b^2cd\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(-a^2bf+2a^2ce-5abcd+b^3d-\frac{\left(-4a\,c^2d+\right.}{2c}\right.}{\sqrt{4ac-b^2}}}{4ac-b^2}}{2a^2}$
risch	$\frac{-\frac{(abf-2ace+bcd)x^2}{2a\left(4ac-b^2\right)}-\frac{2f\,a^2-abe-2acd+b^2d}{2a\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{d\ln(x)}{a^2}+\frac{\left(\right.}{\_R=\mathrm{RootOf}\left(\left(64a^5c^3-48a^4b^2c^2+12c\,b^4a^3-b^6a^2\right)\_Z^2+\left(64c^3a^3d-48a^2b^2c^2\right.\right.}$

input
```
int((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
d*ln(x)/a^2+1/2/a^2*((-a*(a*b*f-2*a*c*e+b*c*d)/(4*a*c-b^2)*x^2-a*(2*a^2*f-
a*b*e-2*a*c*d+b^2*d)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/(4*a*c-b^2)*(1/2*(-4*a
*c^2*d+b^2*c*d)/c*ln(c*x^4+b*x^2+a)+2*(-a^2*b*f+2*a^2*c*e-5*a*b*c*d+b^3*d-
1/2*(-4*a*c^2*d+b^2*c*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-
b^2)^(1/2))))
```

3.65.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(156) = 312$.

Time = 0.97 (sec) , antiderivative size = 1103, normalized size of antiderivative = 6.64

$$\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e + (a^2*b^
3 - 4*a^3*b*c)*f)*x^2 + (4*a^3*c*e - 2*a^3*b*f + (4*a^2*c^2*e - 2*a^2*b*c*
f + (b^3*c - 6*a*b*c^2)*d)*x^4 + (4*a^2*b*c*e - 2*a^2*b^2*f + (b^4 - 6*a*b
^2*c)*d)*x^2 + (a*b^3 - 6*a^2*b*c)*d)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2
*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 +
 a)) + 2*(a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*d - 2*(a^2*b^3 - 4*a^3*b*c)*e +
 4*(a^3*b^2 - 4*a^4*c)*f - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^4 + (b^
5 - 8*a*b^3*c + 16*a^2*b*c^2)*d*x^2 + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*d
)*log(c*x^4 + b*x^2 + a) + 4*((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^4 + (
b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d*x^2 + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)
*d)*log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c
^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), 1/4*(2
*((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e + (a^2*b^3 - 4*a
^3*b*c)*f)*x^2 + 2*(4*a^3*c*e - 2*a^3*b*f + (4*a^2*c^2*e - 2*a^2*b*c*f + (
b^3*c - 6*a*b*c^2)*d)*x^4 + (4*a^2*b*c*e - 2*a^2*b^2*f + (b^4 - 6*a*b^2*c)
*d)*x^2 + (a*b^3 - 6*a^2*b*c)*d)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*
sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + 2*(a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*d
 - 2*(a^2*b^3 - 4*a^3*b*c)*e + 4*(a^3*b^2 - 4*a^4*c)*f - ((b^4*c - 8*a*b^2*
c^2 + 16*a^2*c^3)*d*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d*x^2 + (a*b^4
 - 8*a^2*b^2*c + 16*a^3*c^2)*d)*log(c*x^4 + b*x^2 + a) + 4*((b^4*c - 8*a...
```

3.65.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x**4+e*x**2+d)/x/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.65.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.65.8 Giac [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.35

$$\int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)^2}\,dx = -\frac{(b^3d-6\,abcd+4\,a^2ce-2\,a^2bf)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(a^2b^2-4\,a^3c)\sqrt{-b^2+4\,ac}} - \frac{d\log\left(cx^4+bx^2+a\right)}{4\,a^2} + \frac{d\log\left(x^2\right)}{2\,a^2} + \frac{b^2cdx^4-4\,ac^2dx^4+b^3dx^2-2\,abcdx^2-4\,a^2cex^2+2\,a^2bfx^2+3\,ab^2d-8\,a^2cd-2\,a^2be+4\,a^3f}{4\,(cx^4+bx^2+a)(a^2b^2-4\,a^3c)}$$

input
```
integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(b^3*d - 6*a*b*c*d + 4*a^2*c*e - 2*a^2*b*f)*arctan((2*c*x^2 + b)/sqrt
(-b^2 + 4*a*c))/((a^2*b^2 - 4*a^3*c)*sqrt(-b^2 + 4*a*c)) - 1/4*d*log(c*x^4
 + b*x^2 + a)/a^2 + 1/2*d*log(x^2)/a^2 + 1/4*(b^2*c*d*x^4 - 4*a*c^2*d*x^4
+ b^3*d*x^2 - 2*a*b*c*d*x^2 - 4*a^2*c*e*x^2 + 2*a^2*b*f*x^2 + 3*a*b^2*d -
8*a^2*c*d - 2*a^2*b*e + 4*a^3*f)/((c*x^4 + b*x^2 + a)*(a^2*b^2 - 4*a^3*c))
```

3.65.9 Mupad [B] (verification not implemented)

Time = 16.31 (sec) , antiderivative size = 8706, normalized size of antiderivative = 52.45

$$\int \frac{d + ex^2 + fx^4}{x\left(a + bx^2 + cx^4\right)^2} \, dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)^2),x)
```

output
```
(d*log(x))/a^2 - ((b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)/(2*a*(4*a*c - b^2))
+ (x^2*(a*b*f - 2*a*c*e + b*c*d))/(2*a*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
 - (log((((d + a^2*(-(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)^2/(a^4*(4
*a*c - b^2)^3))^(1/2))*(((d + a^2*(-(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b
*c*d)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*((2*c^2*x^2*(20*a^2*c^2*e + 4*a*b^3*
f - b^3*c*d + 10*a*b*c^2*d - 8*a*b^2*c*e - 10*a^2*b*c*f))/(a*(4*a*c - b^2)
) + (b*c^2*(d + a^2*(-(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)^2/(a^4*(
4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^2 - (4*b*c^2*(b^
3*d - a^2*b*f + 2*a^2*c*e - 5*a*b*c*d))/(a*(4*a*c - b^2))))/(4*a^2) + (c^2
*(a^3*b^2*f^2 - 4*b^4*c*d^2 + 4*a^3*c^2*e^2 + 17*a*b^2*c^2*d^2 - 4*a*b^4*d
*f - 36*a^2*b*c^2*d*e + 18*a^2*b^2*c*d*f + 8*a*b^3*c*d*e - 4*a^3*b*c*e*f))
/(a^2*(4*a*c - b^2)^2) - (c^2*x^2*(a^2*b^3*f^2 + 6*b^3*c^2*d^2 + 4*a^2*b*c
^2*e^2 - 20*a*b*c^3*d^2 + 40*a^2*c^3*d*e - 14*a*b^2*c^2*d*e - 20*a^2*b*c^2
*d*f - 4*a^2*b^2*c*e*f + 7*a*b^3*c*d*f))/(a^2*(4*a*c - b^2)^2)))/(4*a^2) -
 (c^2*x^2*(a*b*f - 2*a*c*e + b*c*d)^3)/(a^3*(4*a*c - b^2)^3) + (c^2*d*(a*b
*f - 2*a*c*e + b*c*d)^2)/(a^3*(4*a*c - b^2)^2))*(((d - a^2*(-(b^3*d - 2*a^
2*b*f + 4*a^2*c*e - 6*a*b*c*d)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*(((d - a^2*
(-(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)^2/(a^4*(4*a*c - b^2)^3))^(1/
2))*((2*c^2*x^2*(20*a^2*c^2*e + 4*a*b^3*f - b^3*c*d + 10*a*b*c^2*d - 8*a*b
^2*c*e - 10*a^2*b*c*f))/(a*(4*a*c - b^2)) + (b*c^2*(d - a^2*(-(b^3*d - ...
```

3.66 $\int \frac{d+ex^2+fx^4}{x^3(a+bx^2+cx^4)^2}\,dx$

3.66.1 Optimal result

Integrand size = 30, antiderivative size = 234

$$\begin{aligned}\int \frac{d+ex^2+fx^4}{x^3(a+bx^2+cx^4)^2}\,dx &= -\frac{d}{2a^2x^2}-\frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)+c(b^2d-abe-2a(cd-af))\,x^2}{2a^2(b^2-4ac)\,(a+bx^2+cx^4)}\\ &\quad-\frac{(2b^4d-12ab^2cd-ab^3e+6a^2bce+4a^2c(3cd-af))\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3(b^2-4ac)^{3/2}}\\ &\quad-\frac{(2bd-ae)\log(x)}{a^3}+\frac{(2bd-ae)\log(a+bx^2+cx^4)}{4a^3}\end{aligned}$$

output
```
-1/2*d/a^2/x^2+1/2*(-b^3*d+a*b^2*e-2*a^2*c*e+a*b*(-a*f+3*c*d)-c*(b^2*d-a*b
*e-2*a*(-a*f+c*d))*x^2)/a^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*(2*b^4*d-12*a
*b^2*c*d-a*b^3*e+6*a^2*b*c*e+4*a^2*c*(-a*f+3*c*d))*arctanh((2*c*x^2+b)/(-4
*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)-(-a*e+2*b*d)*ln(x)/a^3+1/4*(-a*e+2
*b*d)*ln(c*x^4+b*x^2+a)/a^3
```

3.66.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.72

$$\int \frac{d+ex^2+fx^4}{x^3 (a+bx^2+cx^4)^2} \, dx$$
$$= \frac{-\frac{2ad}{x^2} - \frac{2a(b^3d+b^2(-ae+cdx^2)+ab(af-c(3d+ex^2))+2ac(-cdx^2+a(e+fx^2)))}{(b^2-4ac)(a+bx^2+cx^4)} + 4(-2bd+ae)\log(x) + \frac{(2b^4d+b^3(2\sqrt{b^2-4ac}d-}{}}{}$$

input
```
Integrate[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

output
```
((-2*a*d)/x^2 - (2*a*(b^3*d + b^2*(-(a*e) + c*d*x^2) + a*b*(a*f - c*(3*d +
 e*x^2)) + 2*a*c*(-(c*d*x^2) + a*(e + f*x^2))))/((b^2 - 4*a*c)*(a + b*x^2
+ c*x^4)) + 4*(-2*b*d + a*e)*Log[x] + ((2*b^4*d + b^3*(2*Sqrt[b^2 - 4*a*c]
*d - a*e) + 2*a*b*c*(-4*Sqrt[b^2 - 4*a*c]*d + 3*a*e) - a*b^2*(12*c*d + Sqr
t[b^2 - 4*a*c]*e) + 4*a^2*c*(3*c*d + Sqrt[b^2 - 4*a*c]*e - a*f))*Log[b - S
qrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((-2*b^4*d + b^3*(2*Sqr
t[b^2 - 4*a*c]*d + a*e) - 2*a*b*c*(4*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + a*b^2*
(12*c*d - Sqrt[b^2 - 4*a*c]*e) + 4*a^2*c*(-3*c*d + Sqrt[b^2 - 4*a*c]*e + a
*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(4*a^3)
```

3.66.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2194, 2177, 25, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4}{x^3 (a+bx^2+cx^4)^2} \, dx$$
$$\Big\downarrow \text{2194}$$
$$\frac{1}{2}\int \frac{fx^4+ex^2+d}{x^4 (cx^4+bx^2+a)^2}dx^2$$
$$\Big\downarrow \text{2177}$$

$$\frac{1}{2}\left(-\frac{\int -\frac{-\frac{c\left(db^2-aeb-2a(cd-af)\right)x^4}{a^2}-\frac{\left(b^2-4ac\right)(bd-ae)x^2}{a^2}+\left(\frac{b^2}{a}-4c\right)d}{x^4(cx^4+bx^2+a)}dx^2}{b^2-4ac}-\frac{a^2\left(\frac{b^3d}{a^2}-\frac{b(be+3cd)}{a}+bf+2ce\right)+cx^2(-abe-2a(}{a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right.$$

$\downarrow$ 25

$$\frac{1}{2}\left(\frac{\int \frac{-\frac{c\left(db^2-aeb-2a(cd-af)\right)x^4}{a^2}-\frac{\left(b^2-4ac\right)(bd-ae)x^2}{a^2}+\left(\frac{b^2}{a}-4c\right)d}{x^4(cx^4+bx^2+a)}dx^2}{b^2-4ac}-\frac{a^2\left(\frac{b^3d}{a^2}-\frac{b(be+3cd)}{a}+bf+2ce\right)+cx^2(-abe-2a(cd-}{a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right.$$

$\downarrow$ 2159

$$\frac{1}{2}\left(\frac{\int\left(-\frac{(4ac-b^2)d}{a^2x^4}+\frac{2db^4-aeb^3-10acdb^2+5a^2ceb+c(b^2-4ac)(2bd-ae)x^2+2a^2c(3cd-af)}{a^3(cx^4+bx^2+a)}-\frac{(4ac-b^2)(ae-2bd)}{a^3x^2}\right)dx^2}{b^2-4ac}-\frac{a^2\left(\frac{b^3d}{a^2}-\frac{b(}{}\right.}{}\right.$$

$\downarrow$ 2009

$$\frac{1}{2}\left(\frac{-\frac{\log(x^2)(b^2-4ac)(2bd-ae)}{a^3}+\frac{(b^2-4ac)(2bd-ae)\log(a+bx^2+cx^4)}{2a^3}-\frac{d(b^2-4ac)}{a^2x^2}-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(6a^2bce+4a^2c(3cd-af)-ab^3}{a^3\sqrt{b^2-4ac}}}{b^2-4ac}\right.$$

input
```
Int[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

output
```
(-((a^2*((b^3*d)/a^2 + 2*c*e - (b*(3*c*d + b*e))/a + b*f) + c*(b^2*d - a*b
*e - 2*a*(c*d - a*f))*x^2)/(a^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4))) + (-((
(b^2 - 4*a*c)*d)/(a^2*x^2)) - ((2*b^4*d - 12*a*b^2*c*d - a*b^3*e + 6*a^2*b
*c*e + 4*a^2*c*(3*c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a
^3*Sqrt[b^2 - 4*a*c]) - ((b^2 - 4*a*c)*(2*b*d - a*e)*Log[x^2])/a^3 + ((b^2
 - 4*a*c)*(2*b*d - a*e)*Log[a + b*x^2 + c*x^4])/(2*a^3))/(b^2 - 4*a*c))/2
```

3.66. $\int \frac{d+ex^2+fx^4}{x^3(a+bx^2+cx^4)^2}\,dx$

3.66.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.66.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.35

method	result
default	$-\frac{d}{2a^2x^2}+\frac{(ae-2bd)\ln(x)}{a^3}+\frac{\frac{ac\left(2f\,a^2-abe-2acd+b^2d\right)x^2}{4ac-b^2}+\frac{a\left(a^2bf+2a^2ce-a\,b^2e-3abcd+b^3d\right)}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\left(-4a^2c^2e+a\,b^2ce+8ab\,c^2d-2b^3cd\right)\ln}{2c}$
risch	$\frac{\frac{c\left(2f\,a^2-abe-6acd+2b^2d\right)x^4}{2a^2\left(4ac-b^2\right)}+\frac{\left(a^2bf+2a^2ce-a\,b^2e-7abcd+2b^3d\right)x^2}{2\left(4ac-b^2\right)a^2}-\frac{d}{2a}}{x^2(c\,x^4+b\,x^2+a)}+\frac{\ln(x)e}{a^2}-\frac{2\ln(x)bd}{a^3}+\Big(\_R=\operatorname{RootOf}\Big(\Big(64a^6c^3-48b^2a^5c^2+$

input
```
int((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*d/a^2/x^2+(a*e-2*b*d)/a^3*ln(x)+1/2/a^3*((a*c*(2*a^2*f-a*b*e-2*a*c*d+
b^2*d)/(4*a*c-b^2)*x^2+a*(a^2*b*f+2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)/(4*a*
c-b^2))/(c*x^4+b*x^2+a)+1/(4*a*c-b^2)*(1/2*(-4*a^2*c^2*e+a*b^2*c*e+8*a*b*c
^2*d-2*b^3*c*d)/c*ln(c*x^4+b*x^2+a)+2*(2*a^3*c*f-5*a^2*b*c*e-6*a^2*c^2*d+a
*b^3*e+10*a*b^2*c*d-2*d*b^4-1/2*(-4*a^2*c^2*e+a*b^2*c*e+8*a*b*c^2*d-2*b^3*
c*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))
```

3.66.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 870 vs. $2(222) = 444$.

Time = 2.08 (sec) , antiderivative size = 1764, normalized size of antiderivative = 7.54

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d - (a^2*b^3*c - 4*a^3*
b*c^2)*e + 2*(a^3*b^2*c - 4*a^4*c^2)*f)*x^4 + 2*((2*a*b^5 - 15*a^2*b^3*c +
 28*a^3*b*c^2)*d - (a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*e + (a^3*b^3 - 4*a^
4*b*c)*f)*x^2 + ((4*a^3*c^2*f - 2*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d + (a
*b^3*c - 6*a^2*b*c^2)*e)*x^6 + (4*a^3*b*c*f - 2*(b^5 - 6*a*b^3*c + 6*a^2*b
*c^2)*d + (a*b^4 - 6*a^2*b^2*c)*e)*x^4 + (4*a^4*c*f - 2*(a*b^4 - 6*a^2*b^2
*c + 6*a^3*c^2)*d + (a^2*b^3 - 6*a^3*b*c)*e)*x^2)*sqrt(b^2 - 4*a*c)*log((2
*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x
^4 + b*x^2 + a)) + 2*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*d - ((2*(b^5*c -
 8*a*b^3*c^2 + 16*a^2*b*c^3)*d - (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e)
*x^6 + (2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d - (a*b^5 - 8*a^2*b^3*c + 16
*a^3*b*c^2)*e)*x^4 + (2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d - (a^2*b^4
- 8*a^3*b^2*c + 16*a^4*c^2)*e)*x^2)*log(c*x^4 + b*x^2 + a) + 4*((2*(b^5*c
- 8*a*b^3*c^2 + 16*a^2*b*c^3)*d - (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e
)*x^6 + (2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d - (a*b^5 - 8*a^2*b^3*c + 1
6*a^3*b*c^2)*e)*x^4 + (2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d - (a^2*b^4
 - 8*a^3*b^2*c + 16*a^4*c^2)*e)*x^2)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 +
 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^4 + (a^4*b^4 -
 8*a^5*b^2*c + 16*a^6*c^2)*x^2), -1/4*(2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*
a^3*c^3)*d - (a^2*b^3*c - 4*a^3*b*c^2)*e + 2*(a^3*b^2*c - 4*a^4*c^2)*f)...
```

3.66.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((f*x**4+e*x**2+d)/x**3/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.66.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.20

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$$
$$=\frac{\left(2\,b^4d-12\,ab^2cd+12\,a^2c^2d-ab^3e+6\,a^2bce-4\,a^3cf\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(a^3b^2-4\,a^4c)\sqrt{-b^2+4\,ac}}$$
$$-\frac{2\,b^2cdx^4-6\,ac^2dx^4-abcex^4+2\,a^2cfx^4+2\,b^3dx^2-7\,abcdx^2-ab^2ex^2+2\,a^2cex^2+a^2bfx^2+ab^2d-}{2\,(cx^6+bx^4+ax^2)(a^2b^2-4\,a^3c)}$$
$$+\frac{(2\,bd-ae)\log\left(cx^4+bx^2+a\right)}{4\,a^3}-\frac{(2\,bd-ae)\log\left(x^2\right)}{2\,a^3}$$

input
```
integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(2*b^4*d - 12*a*b^2*c*d + 12*a^2*c^2*d - a*b^3*e + 6*a^2*b*c*e - 4*a^3
*c*f)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^3*b^2 - 4*a^4*c)*sqrt(-
b^2 + 4*a*c)) - 1/2*(2*b^2*c*d*x^4 - 6*a*c^2*d*x^4 - a*b*c*e*x^4 + 2*a^2*c
*f*x^4 + 2*b^3*d*x^2 - 7*a*b*c*d*x^2 - a*b^2*e*x^2 + 2*a^2*c*e*x^2 + a^2*b
*f*x^2 + a*b^2*d - 4*a^2*c*d)/((c*x^6 + b*x^4 + a*x^2)*(a^2*b^2 - 4*a^3*c)
) + 1/4*(2*b*d - a*e)*log(c*x^4 + b*x^2 + a)/a^3 - 1/2*(2*b*d - a*e)*log(x
^2)/a^3
```

3.66.9 Mupad [B] (verification not implemented)

Time = 17.16 (sec) , antiderivative size = 11879, normalized size of antiderivative = 50.76

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)^2),x)
```

output
```
((x^2*(2*b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 7*a*b*c*d))/(2*a^2*(4*a*c
 - b^2)) - d/(2*a) + (c*x^4*(2*b^2*d + 2*a^2*f - a*b*e - 6*a*c*d))/(2*a^2*
(4*a*c - b^2)))/(a*x^2 + b*x^4 + c*x^6) + (log(x)*(a*e - 2*b*d))/a^3 + (lo
g((((((a*e - 2*b*d + a^3*(-(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f -
 12*a*b^2*c*d + 6*a^2*b*c*e)^2/(a^6*(4*a*c - b^2)^3))^(1/2))*((2*c^3*x^2*(
2*b^4*d - 60*a^2*c^2*d - 8*a^2*b^2*f - a*b^3*e + 20*a^3*c*f + 4*a*b^2*c*d
+ 10*a^2*b*c*e))/(a^2*(4*a*c - b^2)) + (4*b*c^2*(2*b^4*d + 6*a^2*c^2*d - a
*b^3*e - 2*a^3*c*f - 10*a*b^2*c*d + 5*a^2*b*c*e))/(a^2*(4*a*c - b^2)) + (b
*c^2*(a*e - 2*b*d + a^3*(-(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f -
12*a*b^2*c*d + 6*a^2*b*c*e)^2/(a^6*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*x
^2 - 10*a*c*x^2))/a^3))/(4*a^3) + (c^3*(4*a^5*c*f^2 - 16*b^6*d^2 - 4*a^2*b
^4*e^2 + 36*a^3*c^3*d^2 + 17*a^3*b^2*c*e^2 + 16*a*b^5*d*e - 216*a^2*b^2*c^
2*d^2 + 116*a*b^4*c*d^2 - 16*a^2*b^4*d*f + 8*a^3*b^3*e*f - 24*a^4*c^2*d*f
- 92*a^2*b^3*c*d*e + 108*a^3*b*c^2*d*e + 72*a^3*b^2*c*d*f - 36*a^4*b*c*e*f
))/(a^4*(4*a*c - b^2)^2) - (2*c^4*x^2*(12*b^5*d^2 + 2*a^4*b*f^2 + 3*a^2*b^
3*e^2 + 138*a^2*b*c^2*d^2 - 12*a*b^4*d*e + 20*a^4*c*e*f - 82*a*b^3*c*d^2 -
 10*a^3*b*c*e^2 + 14*a^2*b^3*d*f - 60*a^3*c^2*d*e - 7*a^3*b^2*e*f + 61*a^2
*b^2*c*d*e - 52*a^3*b*c*d*f))/(a^4*(4*a*c - b^2)^2))*(a*e - 2*b*d + a^3*(-
(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e
)^2/(a^6*(4*a*c - b^2)^3))^(1/2)))/(4*a^3) + (c^4*(a*e - 2*b*d)*(2*b^2*...
```

3.67 $\int \frac{d+ex^2+fx^4}{x^5(a+bx^2+cx^4)^2}\,dx$

3.67.1 Optimal result

Integrand size = 30, antiderivative size = 329

$$\begin{aligned}\int \frac{d+ex^2+fx^4}{x^5(a+bx^2+cx^4)^2}\,dx &= -\frac{d}{4a^2x^4}+\frac{2bd-ae}{2a^3x^2}\\ &+\frac{b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd-af)+c(b^3d-ab^2e+2a^2ce-ab(3cd-af))x^2}{2a^3(b^2-4ac)(a+bx^2+cx^4)}\\ &+\frac{(3b^5d-2ab^4e+12a^2b^2ce-12a^3c^2e+6a^2bc(5cd-af)-ab^3(20cd-af))\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4(b^2-4ac)^{3/2}}\\ &+\frac{(3b^2d-2abe-a(2cd-af))\log(x)}{a^4}-\frac{(3b^2d-2abe-a(2cd-af))\log(a+bx^2+cx^4)}{4a^4}\end{aligned}$$

output
```
-1/4*d/a^2/x^4+1/2*(-a*e+2*b*d)/a^3/x^2+1/2*(b^4*d-a*b^3*e+3*a^2*b*c*e+2*a
^2*c*(-a*f+c*d)-a*b^2*(-a*f+4*c*d)+c*(b^3*d-a*b^2*e+2*a^2*c*e-a*b*(-a*f+3*
c*d))*x^2)/a^3/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(3*b^5*d-2*a*b^4*e+12*a^2*
b^2*c*e-12*a^3*c^2*e+6*a^2*b*c*(-a*f+5*c*d)-a*b^3*(-a*f+20*c*d))*arctanh((
2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(3/2)+(3*b^2*d-2*a*b*e-a*(
-a*f+2*c*d))*ln(x)/a^4-1/4*(3*b^2*d-2*a*b*e-a*(-a*f+2*c*d))*ln(c*x^4+b*x^2
+a)/a^4
```

3.67.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 592, normalized size of antiderivative = 1.80

$$\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)^2}\,dx =$$

$$-\frac{\frac{a^2d}{x^4}+\frac{2a(-2bd+ae)}{x^2}+\frac{2a\left(-b^4d+b^3\left(ae-cdx^2\right)+ab^2\left(4cd-af+cex^2\right)-abc\left(3ae-3cdx^2+afx^2\right)+2a^2c\left(af-c\left(d+ex^2\right)\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-4(3b^2d-2a}{}$$

input
```
Integrate[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)^2),x]
```

output
```
-1/4*((a^2*d)/x^4 + (2*a*(-2*b*d + a*e))/x^2 + (2*a*(-(b^4*d) + b^3*(a*e -
 c*d*x^2) + a*b^2*(4*c*d - a*f + c*e*x^2) - a*b*c*(3*a*e - 3*c*d*x^2 + a*f
*x^2) + 2*a^2*c*(a*f - c*(d + e*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
) - 4*(3*b^2*d - 2*a*b*e + a*(-2*c*d + a*f))*Log[x] + ((3*b^5*d + b^4*(3*S
qrt[b^2 - 4*a*c]*d - 2*a*e) + 2*a^2*b*c*(15*c*d + 4*Sqrt[b^2 - 4*a*c]*e -
3*a*f) + a*b^3*(-20*c*d - 2*Sqrt[b^2 - 4*a*c]*e + a*f) - 4*a^2*c*(-2*c*Sqr
t[b^2 - 4*a*c]*d + 3*a*c*e + a*Sqrt[b^2 - 4*a*c]*f) + a*b^2*(-14*c*Sqrt[b^
2 - 4*a*c]*d + 12*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*Log[b - Sqrt[b^2 - 4*a*c
] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((-3*b^5*d + b^4*(3*Sqrt[b^2 - 4*a*c]*
d + 2*a*e) - a*b^3*(-20*c*d + 2*Sqrt[b^2 - 4*a*c]*e + a*f) + 2*a^2*b*c*(-1
5*c*d + 4*Sqrt[b^2 - 4*a*c]*e + 3*a*f) + 4*a^2*c*(2*c*Sqrt[b^2 - 4*a*c]*d
+ 3*a*c*e - a*Sqrt[b^2 - 4*a*c]*f) + a*b^2*(-2*c*(7*Sqrt[b^2 - 4*a*c]*d +
6*a*e) + a*Sqrt[b^2 - 4*a*c]*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^
2 - 4*a*c)^(3/2))/a^4
```

3.67.3 Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2194, 2177, 25, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)^2}\,dx$$

$$\Big\downarrow \; 2194$$

$$\frac{1}{2}\int \frac{fx^4+ex^2+d}{x^6\left(cx^4+bx^2+a\right)^2}dx^2$$

$\Big\downarrow$ 2177

$$\frac{1}{2}\left(\frac{cx^2\left(2a^2ce-ab^2e-ab(3cd-af)+b^3d\right)+3a^2bce+2a^2c(cd-af)-ab^3e-ab^2(4cd-af)+b^4d}{a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{c\left(db^3-\right.}{}}{}\right.$$

$\Big\downarrow$ 25

$$\frac{1}{2}\left(\frac{\int \frac{\frac{c\left(db^3-aeb^2-a(3cd-af)b+2a^2ce\right)x^6}{a^3}+\frac{\left(b^2-4ac\right)\left(db^2-aeb-a(cd-af)\right)x^4}{a^3}-\frac{\left(b^2-4ac\right)(bd-ae)x^2}{a^2}+\left(\frac{b^2}{a}-4c\right)d}{x^6\left(cx^4+bx^2+a\right)}dx^2}{b^2-4ac}+\frac{cx^2\left(2a^2ce-ab^2e-a\right.}{}\right.$$

$\Big\downarrow$ 2159

$$\frac{1}{2}\left(\frac{\int\left(-\frac{\left(4ac-b^2\right)d}{a^2x^6}+\frac{-3db^5+2aeb^4+a(17cd-af)b^3-10a^2ceb^2-a^2c(19cd-5af)b-c\left(b^2-4ac\right)\left(3db^2-2aeb-a(2cd-af)\right)x^2+6a^3c^2e}{a^4\left(cx^4+bx^2+a\right)}+\frac{\left(b^2-4\right.}{}\right.}{b^2-4ac}\right.$$

$\Big\downarrow$ 2009

$$\frac{1}{2}\left(\frac{cx^2\left(2a^2ce-ab^2e-ab(3cd-af)+b^3d\right)+3a^2bce+2a^2c(cd-af)-ab^3e-ab^2(4cd-af)+b^4d}{a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\log\left(x^2\right)\left(b^2-\right.}{}\right.$$

input

```
Int[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)^2),x]
```

output

```
((b^4*d - a*b^3*e + 3*a^2*b*c*e + 2*a^2*c*(c*d - a*f) - a*b^2*(4*c*d - a*f
) + c*(b^3*d - a*b^2*e + 2*a^2*c*e - a*b*(3*c*d - a*f))*x^2)/(a^3*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)) + (-1/2*((b^2 - 4*a*c)*d)/(a^2*x^4) + ((b^2 - 4
*a*c)*(2*b*d - a*e))/(a^3*x^2) + ((3*b^5*d - 2*a*b^4*e + 12*a^2*b^2*c*e -
12*a^3*c^2*e + 6*a^2*b*c*(5*c*d - a*f) - a*b^3*(20*c*d - a*f))*ArcTanh[(b
+ 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a^4*Sqrt[b^2 - 4*a*c]) + ((b^2 - 4*a*c)*(3
*b^2*d - 2*a*b*e - a*(2*c*d - a*f))*Log[x^2])/a^4 - ((b^2 - 4*a*c)*(3*b^2*
d - 2*a*b*e - a*(2*c*d - a*f))*Log[a + b*x^2 + c*x^4])/(2*a^4))/(b^2 - 4*a
*c))/2
```

3.67. $\int \frac{d+ex^2+fx^4}{x^5(a+bx^2+cx^4)^2}\,dx$

3.67.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.67.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.42

method	result
default	$-\frac{d}{4a^2x^4}-\frac{ae-2bd}{2a^3x^2}+\frac{\left(f\,a^2-2abe-2acd+3b^2d\right)\ln(x)}{a^4}-\frac{\frac{ac\left(a^2bf+2a^2ce-a\,b^2e-3abcd+b^3d\right)x^2}{4ac-b^2}-\frac{a\left(2a^3cf-a^2b^2f-3a^2bce-2a^2c^2d+\cdots\right.}{4ac-b^2}}{c\,x^4+b\,x^2+a}$
risch	Expression too large to display

input
```
int((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*d/a^2/x^4-1/2*(a*e-2*b*d)/a^3/x^2+(a^2*f-2*a*b*e-2*a*c*d+3*b^2*d)/a^4
*ln(x)-1/2/a^4*((a*c*(a^2*b*f+2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)/(4*a*c-b^
2)*x^2-a*(2*a^3*c*f-a^2*b^2*f-3*a^2*b*c*e-2*a^2*c^2*d+a*b^3*e+4*a*b^2*c*d-
b^4*d)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/(4*a*c-b^2)*(1/2*(4*a^3*c^2*f-a^2*b^
2*c*f-8*a^2*b*c^2*e-8*a^2*c^3*d+2*a*b^3*c*e+14*a*b^2*c^2*d-3*b^4*c*d)/c*ln
(c*x^4+b*x^2+a)+2*(5*a^3*b*c*f+6*a^3*c^2*e-a^2*b^3*f-10*a^2*b^2*c*e-19*a^2
*b*c^2*d+2*a*b^4*e+17*a*b^3*c*d-3*b^5*d-1/2*(4*a^3*c^2*f-a^2*b^2*c*f-8*a^2
*b*c^2*e-8*a^2*c^3*d+2*a*b^3*c*e+14*a*b^2*c^2*d-3*b^4*c*d)*b/c)/(4*a*c-b^2
)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))
```

3.67.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1272 vs. $2(315) = 630$.

Time = 4.47 (sec) , antiderivative size = 2567, normalized size of antiderivative = 7.80

$$\int \frac{d+ex^2+fx^4}{x^5 (a+bx^2+cx^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*((3*a*b^5*c - 23*a^2*b^3*c^2 + 44*a^3*b*c^3)*d - 2*(a^2*b^4*c - 7*
a^3*b^2*c^2 + 12*a^4*c^3)*e + (a^3*b^3*c - 4*a^4*b*c^2)*f)*x^6 + ((6*a*b^6
 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c^3)*d - 2*(2*a^2*b^5 - 15*a^3*
b^3*c + 28*a^4*b*c^2)*e + 2*(a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*f)*x^4 + (
3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d - 2*(a^3*b^4 - 8*a^4*b^2*c + 16
*a^5*c^2)*e)*x^2 + (((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*d - 2*(a*b^4*
c - 6*a^2*b^2*c^2 + 6*a^3*c^3)*e + (a^2*b^3*c - 6*a^3*b*c^2)*f)*x^8 + ((3*
b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*d - 2*(a*b^5 - 6*a^2*b^3*c + 6*a^3*b*c^
2)*e + (a^2*b^4 - 6*a^3*b^2*c)*f)*x^6 + ((3*a*b^5 - 20*a^2*b^3*c + 30*a^3*
b*c^2)*d - 2*(a^2*b^4 - 6*a^3*b^2*c + 6*a^4*c^2)*e + (a^3*b^3 - 6*a^4*b*c)
*f)*x^4)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c
*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (a^3*b^4 - 8*a^4*b^2*c
 + 16*a^5*c^2)*d - (((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4
)*d - 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*e + (a^2*b^4*c - 8*a^3*b^
2*c^2 + 16*a^4*c^3)*f)*x^8 + ((3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^
3*b*c^3)*d - 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*e + (a^2*b^5 - 8*a^3
*b^3*c + 16*a^4*b*c^2)*f)*x^6 + ((3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2
 - 32*a^4*c^3)*d - 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*e + (a^3*b^4 -
8*a^4*b^2*c + 16*a^5*c^2)*f)*x^4)*log(c*x^4 + b*x^2 + a) + 4*(((3*b^6*c -
26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*d - 2*(a*b^5*c - 8*a^2*b^3*...
```

3.67.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x**4+e*x**2+d)/x**5/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.67.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.58

$$\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)^2}\,dx =$$

$$-\frac{\left(3\,b^5d-20\,ab^3cd+30\,a^2bc^2d-2\,ab^4e+12\,a^2b^2ce-12\,a^3c^2e+a^2b^3f-6\,a^3bcf\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\left(a^4b^2-4\,a^5c\right)\sqrt{-b^2+4\,ac}}$$

$$+\frac{3\,b^4cdx^4-14\,ab^2c^2dx^4+8\,a^2c^3dx^4-2\,ab^3cex^4+8\,a^2bc^2ex^4+a^2b^2cfx^4-4\,a^3c^2fx^4+3\,b^5dx^2-12\,ab^3}{}$$

$$-\frac{\left(3\,b^2d-2\,acd-2\,abe+a^2f\right)\log\left(cx^4+bx^2+a\right)}{4\,a^4}$$

$$+\frac{\left(3\,b^2d-2\,acd-2\,abe+a^2f\right)\log\left(x^2\right)}{2\,a^4}$$

$$-\frac{9\,b^2dx^4-6\,acdx^4-6\,abex^4+3\,a^2fx^4-4\,abdx^2+2\,a^2ex^2+a^2d}{4\,a^4x^4}$$

input
```
integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(3*b^5*d - 20*a*b^3*c*d + 30*a^2*b*c^2*d - 2*a*b^4*e + 12*a^2*b^2*c*e
 - 12*a^3*c^2*e + a^2*b^3*f - 6*a^3*b*c*f)*arctan((2*c*x^2 + b)/sqrt(-b^2
+ 4*a*c))/((a^4*b^2 - 4*a^5*c)*sqrt(-b^2 + 4*a*c)) + 1/4*(3*b^4*c*d*x^4 -
14*a*b^2*c^2*d*x^4 + 8*a^2*c^3*d*x^4 - 2*a*b^3*c*e*x^4 + 8*a^2*b*c^2*e*x^4
 + a^2*b^2*c*f*x^4 - 4*a^3*c^2*f*x^4 + 3*b^5*d*x^2 - 12*a*b^3*c*d*x^2 + 2*
a^2*b*c^2*d*x^2 - 2*a*b^4*e*x^2 + 6*a^2*b^2*c*e*x^2 + 4*a^3*c^2*e*x^2 + a^
2*b^3*f*x^2 - 2*a^3*b*c*f*x^2 + 5*a*b^4*d - 22*a^2*b^2*c*d + 12*a^3*c^2*d
- 4*a^2*b^3*e + 14*a^3*b*c*e + 3*a^3*b^2*f - 8*a^4*c*f)/((a^4*b^2 - 4*a^5*
c)*(c*x^4 + b*x^2 + a)) - 1/4*(3*b^2*d - 2*a*c*d - 2*a*b*e + a^2*f)*log(c*
x^4 + b*x^2 + a)/a^4 + 1/2*(3*b^2*d - 2*a*c*d - 2*a*b*e + a^2*f)*log(x^2)/
a^4 - 1/4*(9*b^2*d*x^4 - 6*a*c*d*x^4 - 6*a*b*e*x^4 + 3*a^2*f*x^4 - 4*a*b*d
*x^2 + 2*a^2*e*x^2 + a^2*d)/(a^4*x^4)
```

3.67.9 Mupad [B] (verification not implemented)

Time = 24.98 (sec) , antiderivative size = 15905, normalized size of antiderivative = 48.34

$$\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)^2),x)
```

output
```
(log(x)*(3*b^2*d + a^2*f - 2*a*b*e - 2*a*c*d))/a^4 - (log(((((((4*b*c^2*(3
*b^5*d + a^2*b^3*f - 6*a^3*c^2*e - 2*a*b^4*e - 17*a*b^3*c*d - 5*a^3*b*c*f
+ 19*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(a^3*(4*a*c - b^2)) - (b*c^2*(a*b + 3*
b^2*x^2 - 10*a*c*x^2)*(a^4*(-(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4
*e - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e)^2/(a^8*
(4*a*c - b^2)^3))^(1/2) + 3*b^2*d + a^2*f - 2*a*b*e - 2*a*c*d))/a^4 + (2*c
^3*x^2*(3*b^5*d + a^2*b^3*f + 60*a^3*c^2*e - 2*a*b^4*e + 4*a*b^3*c*d - 10*
a^3*b*c*f - 70*a^2*b*c^2*d - 4*a^2*b^2*c*e))/(a^3*(4*a*c - b^2)))*(a^4*(-(
3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*
f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e)^2/(a^8*(4*a*c - b^2)^3))^(1/2) + 3*b^
2*d + a^2*f - 2*a*b*e - 2*a*c*d))/(4*a^4) + (c^3*(36*b^8*d^2 + 16*a^2*b^6*
e^2 + 4*a^4*b^4*f^2 - 36*a^5*c^3*e^2 - 116*a^3*b^4*c*e^2 - 17*a^5*b^2*c*f^
2 - 48*a*b^7*d*e + 778*a^2*b^4*c^2*d^2 - 473*a^3*b^2*c^3*d^2 + 216*a^4*b^2
*c^2*e^2 - 309*a*b^6*c*d^2 + 24*a^2*b^6*d*f - 16*a^3*b^5*e*f + 380*a^2*b^5
*c*d*e + 324*a^4*b*c^3*d*e - 154*a^3*b^4*c*d*f + 92*a^4*b^3*c*e*f - 108*a^
5*b*c^2*e*f - 832*a^3*b^3*c^2*d*e + 230*a^4*b^2*c^2*d*f))/(a^6*(4*a*c - b^
2)^2) + (c^4*x^2*(54*b^7*d^2 + 24*a^2*b^5*e^2 + 6*a^4*b^3*f^2 - 440*a^3*b*
c^3*d^2 - 164*a^3*b^3*c*e^2 + 276*a^4*b*c^2*e^2 - 72*a*b^6*d*e + 1011*a^2*
b^3*c^2*d^2 - 441*a*b^5*c*d^2 - 20*a^5*b*c*f^2 + 36*a^2*b^5*d*f + 240*a^4*
c^3*d*e - 24*a^3*b^4*e*f - 120*a^5*c^2*e*f + 540*a^2*b^4*c*d*e - 207*a^...
```

3.67. $\int \frac{d+ex^2+fx^4}{x^5(a+bx^2+cx^4)^2}\,dx$

3.68 $\int \frac{x^6\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

3.68.1 Optimal result

Integrand size = 30, antiderivative size = 550

$$\begin{aligned}\int \frac{x^6(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx &= \frac{(ce-2bf)x}{c^3}+\frac{fx^3}{3c^2} \\ &+\frac{x\left(a(b^2ce-2ac^2e-b^3f-bc(cd-3af))+\left(b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2ac^2(cd-af)\right)x^2\right)}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} \\ &-\frac{\left(3b^3ce-13abc^2e-5b^4f-b^2c(cd-24af)+2ac^2(3cd-7af)-\frac{3b^4ce-19ab^2c^2e+20a^2c^3e-5b^5f-b^3c(cd-34af)+4}{\sqrt{b^2-4ac}}\right.}{2\sqrt{2}c^{7/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}} \\ &-\frac{\left(3b^3ce-13abc^2e-5b^4f-b^2c(cd-24af)+2ac^2(3cd-7af)+\frac{3b^4ce-19ab^2c^2e+20a^2c^3e-5b^5f-b^3c(cd-34af)+4}{\sqrt{b^2-4ac}}\right.}{2\sqrt{2}c^{7/2}\left(b^2-4ac\right)\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

output
```
(-2*b*f+c*e)*x/c^3+1/3*f*x^3/c^2+1/2*x*(a*(b^2*c*e-2*a*c^2*e-b^3*f-b*c*(-3
*a*f+c*d))+(b^3*c*e-3*a*b*c^2*e-b^4*f-b^2*c*(-4*a*f+c*d)+2*a*c^2*(-a*f+c*d
))*x^2)/c^3/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(
-4*a*c+b^2)^(1/2))^(1/2))*(3*b^3*c*e-13*a*b*c^2*e-5*b^4*f-b^2*c*(-24*a*f+c
*d)+2*a*c^2*(-7*a*f+3*c*d)+(-3*b^4*c*e+19*a*b^2*c^2*e-20*a^2*c^3*e+5*b^5*f
+b^3*c*(-34*a*f+c*d)-4*a*b*c^2*(-13*a*f+2*c*d))/(-4*a*c+b^2)^(1/2))/c^(7/2
)/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c
^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(3*b^3*c*e-13*a*b*c^2*e-5*b^4*f-b^2*c
*(-24*a*f+c*d)+2*a*c^2*(-7*a*f+3*c*d)+(3*b^4*c*e-19*a*b^2*c^2*e+20*a^2*c^3
*e-5*b^5*f-b^3*c*(-34*a*f+c*d)+4*a*b*c^2*(-13*a*f+2*c*d))/(-4*a*c+b^2)^(1/
2))/c^(7/2)/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.68.2 Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 648, normalized size of antiderivative = 1.18

$$\int \frac{x^6(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{12\sqrt{c}(ce-2bf)x+4c^{3/2}fx^3-\frac{6\sqrt{c}x\left(b^2\left(c^2d-bce+b^2f\right)x^2+a^2c\left(-3bf+2c\left(e+fx^2\right)\right)+a\left(b^3f-2c^3dx^2+bc^2\left(d+3ex^2\right)-b^2c\left(e+4fx^2\right)\right)\right)}{(b^2-4ac)(a+bx^2+cx^4)}}{}$$

input
```
Integrate[(x^6*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(12*Sqrt[c]*(c*e - 2*b*f)*x + 4*c^(3/2)*f*x^3 - (6*Sqrt[c]*x*(b^2*(c^2*d -
 b*c*e + b^2*f)*x^2 + a^2*c*(-3*b*f + 2*c*(e + f*x^2)) + a*(b^3*f - 2*c^3*
d*x^2 + b*c^2*(d + 3*e*x^2) - b^2*c*(e + 4*f*x^2))))/((b^2 - 4*a*c)*(a + b
*x^2 + c*x^4)) + (3*Sqrt[2]*(-5*b^5*f + a*b*c^2*(8*c*d + 13*Sqrt[b^2 - 4*a
*c]*e - 52*a*f) - b^3*c*(c*d + 3*Sqrt[b^2 - 4*a*c]*e - 34*a*f) + b^4*(3*c*
e + 5*Sqrt[b^2 - 4*a*c]*f) + b^2*c*(c*Sqrt[b^2 - 4*a*c]*d - 19*a*c*e - 24*
a*Sqrt[b^2 - 4*a*c]*f) + 2*a*c^2*(-3*c*Sqrt[b^2 - 4*a*c]*d + 10*a*c*e + 7*
a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a
*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*(5*b
^5*f + b^3*c*(c*d - 3*Sqrt[b^2 - 4*a*c]*e - 34*a*f) + a*b*c^2*(-8*c*d + 13
*Sqrt[b^2 - 4*a*c]*e + 52*a*f) + b^4*(-3*c*e + 5*Sqrt[b^2 - 4*a*c]*f) + b^
2*c*(c*Sqrt[b^2 - 4*a*c]*d + 19*a*c*e - 24*a*Sqrt[b^2 - 4*a*c]*f) - 2*a*c^
2*(3*c*Sqrt[b^2 - 4*a*c]*d + 10*a*c*e - 7*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt
[b + Sqrt[b^2 - 4*a*c]]))/(12*c^(7/2))
```

3.68.3 Rubi [A] (verified)

Time = 6.90 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2197, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$\Big\downarrow \; 2197$$

$$\frac{x\left(a\left(-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)+x^2\left(-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f)+b^3ce\right)\right)}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\frac{\int \frac{2a\left(4a-\frac{b^2}{c}\right)fx^6-\frac{2a\left(b^2-4ac\right)(ce-bf)x^4}{c^2}+\frac{a\left(-fb^4+ceb^3-c(cd-6af)b^2-5ac^2eb+6ac^2(cd-af)\right)x^2}{c^3}+\frac{a^2\left(-fb^3+ceb^2-c(cd-3af)b-2ac^2e\right)}{c^3}}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 2205$$

$$\frac{x\left(a\left(-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)+x^2\left(-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f)+b^3ce\right)\right)}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\frac{\int \left(-\frac{2a(b^2-4ac)fx^2}{c^2}-\frac{2a(b^2-4ac)(ce-2bf)}{c^3}-\frac{-((-5fb^3+3ceb^2-c(cd-19af)b-10ac^2e)a^2)-(-5fb^4+3ceb^3-c(cd-24af)b^2-13ac^2eb+2ac}{c^3(cx^4+bx^2+a)}\right.}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{x\left(a\left(-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)+x^2\left(-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f)+b^3ce\right)\right)}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\frac{a\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{20a^2c^3e-b^3c(cd-34af)-19ab^2c^2e+4abc^2(2cd-13af)-5b^5f+3b^4ce}{\sqrt{b^2-4ac}}-b^2c(cd-24af)-13abc^2e+2ac^2(3cd-7af)-5b^4f+3b^3c\right.}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

input
```
Int[(x^6*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(x*(a*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f)) + (b^3*c*e - 3*a*b
*c^2*e - b^4*f - b^2*c*(c*d - 4*a*f) + 2*a*c^2*(c*d - a*f))*x^2))/(2*c^3*(
b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((-2*a*(b^2 - 4*a*c)*(c*e - 2*b*f)*x)/
c^3 - (2*a*(b^2 - 4*a*c)*f*x^3)/(3*c^2) + (a*(3*b^3*c*e - 13*a*b*c^2*e - 5
*b^4*f - b^2*c*(c*d - 24*a*f) + 2*a*c^2*(3*c*d - 7*a*f) - (3*b^4*c*e - 19*
a*b^2*c^2*e + 20*a^2*c^3*e - 5*b^5*f - b^3*c*(c*d - 34*a*f) + 4*a*b*c^2*(2
*c*d - 13*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqr
t[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (a*(3*b^
3*c*e - 13*a*b*c^2*e - 5*b^4*f - b^2*c*(c*d - 24*a*f) + 2*a*c^2*(3*c*d - 7
*a*f) + (3*b^4*c*e - 19*a*b^2*c^2*e + 20*a^2*c^3*e - 5*b^5*f - b^3*c*(c*d
- 34*a*f) + 4*a*b*c^2*(2*c*d - 13*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b + Sqrt[b
^2 - 4*a*c]]))/(2*a*(b^2 - 4*a*c))
```

3.68. $\int \frac{x^6\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

3.68.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

3.68.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.59

method	result
risch	$\frac{f x^3}{3c^2} - \frac{2bfx}{c^3} + \frac{xe}{c^2} + \frac{\frac{\left(2a^2c^2f-4a\,b^2cf+3ab\,c^2e-2a\,c^3d+b^4f-b^3ce+b^2c^2d\right)x^3}{8ac-2b^2} - \frac{a\left(3abcf-2a\,c^2e-b^3f+b^2ce-b\,c^2d\right)x}{2\left(4ac-b^2\right)}}{c^3\left(c\,x^4+b\,x^2+a\right)} + \frac{\_R=\text{RootOf}\left(\right.}{}$
default	$-\frac{-\frac{1}{3}cf\,x^3+2bfx-xce}{c^3} + \frac{\frac{\frac{\left(2a^2c^2f-4a\,b^2cf+3ab\,c^2e-2a\,c^3d+b^4f-b^3ce+b^2c^2d\right)x^3}{8ac-2b^2} - \frac{a\left(3abcf-2a\,c^2e-b^3f+b^2ce-b\,c^2d\right)x}{2\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a} + \frac{2c\left(\frac{\left(-14a^2c^2\right.}{}\right.}{}}{}$

input
```
int(x^6*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*f*x^3/c^2-2/c^3*b*f*x+1/c^2*x*e+(1/2*(2*a^2*c^2*f-4*a*b^2*c*f+3*a*b*c^
2*e-2*a*c^3*d+b^4*f-b^3*c*e+b^2*c^2*d)/(4*a*c-b^2)*x^3-1/2*a*(3*a*b*c*f-2*
a*c^2*e-b^3*f+b^2*c*e-b*c^2*d)/(4*a*c-b^2)*x)/c^3/(c*x^4+b*x^2+a)+1/4/c^3*
sum((-(14*a^2*c^2*f-24*a*b^2*c*f+13*a*b*c^2*e-6*a*c^3*d+5*b^4*f-3*b^3*c*e+
b^2*c^2*d)/(4*a*c-b^2)*_R^2+a*(19*a*b*c*f-10*a*c^2*e-5*b^3*f+3*b^2*c*e-b*c
^2*d)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.68.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18909 vs. $2(506) = 1012$.

Time = 80.29 (sec) , antiderivative size = 18909, normalized size of antiderivative = 34.38

$$\int \frac{x^6(d + ex^2 + fx^4)}{(a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate(x^6*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.68.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^6(d + ex^2 + fx^4)}{(a + bx^2 + cx^4)^2} \, dx = \text{Timed out}$$

input
```
integrate(x**6*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.68.7 Maxima [F]

$$\int \frac{x^6(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(fx^4+ex^2+d)x^6}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^6*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(((b^2*c^2 - 2*a*c^3)*d - (b^3*c - 3*a*b*c^2)*e + (b^4 - 4*a*b^2*c +
2*a^2*c^2)*f)*x^3 + (a*b*c^2*d - (a*b^2*c - 2*a^2*c^2)*e + (a*b^3 - 3*a^2*
b*c)*f)*x)/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^4 + (b^3*c^3 - 4
*a*b*c^4)*x^2) + 1/2*integrate((a*b*c^2*d + ((b^2*c^2 - 6*a*c^3)*d - (3*b^
3*c - 13*a*b*c^2)*e + (5*b^4 - 24*a*b^2*c + 14*a^2*c^2)*f)*x^2 - (3*a*b^2*
c - 10*a^2*c^2)*e + (5*a*b^3 - 19*a^2*b*c)*f)/(c*x^4 + b*x^2 + a), x)/(b^2
*c^3 - 4*a*c^4) + 1/3*(c*f*x^3 + 3*(c*e - 2*b*f)*x)/c^3
```

3.68.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8946 vs. 2(506) = 1012.

Time = 2.03 (sec) , antiderivative size = 8946, normalized size of antiderivative = 16.27

$$\int \frac{x^6(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^6*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(b^2*c^2*d*x^3 - 2*a*c^3*d*x^3 - b^3*c*e*x^3 + 3*a*b*c^2*e*x^3 + b^4*
f*x^3 - 4*a*b^2*c*f*x^3 + 2*a^2*c^2*f*x^3 + a*b*c^2*d*x - a*b^2*c*e*x + 2*
a^2*c^2*e*x + a*b^3*f*x - 3*a^2*b*c*f*x)/((b^2*c^3 - 4*a*c^4)*(c*x^4 + b*x
^2 + a)) + 1/16*((2*b^4*c^4 - 20*a*b^2*c^5 + 48*a^2*c^6 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 10*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 24*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*c^5 - 2*(b^2 - 4*a*c)*b^2*c^4 + 12*(b^2 - 4*a*c)*a*
c^5)*(b^2*c^3 - 4*a*c^4)^2*d - (6*b^5*c^3 - 50*a*b^3*c^4 + 104*a^2*b*c^5 -
 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 25*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 6*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 52*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 26*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 3*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 13*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 6*(b^2 - 4*a*c)*b^3
*c^3 + 26*(b^2 - 4*a*c)*a*b*c^4)*(b^2*c^3 - 4*a*c^4)^2*e + (10*b^6*c^2 ...
```

3.68.9 Mupad [B] (verification not implemented)

Time = 10.44 (sec) , antiderivative size = 33799, normalized size of antiderivative = 61.45

$$\int \frac{x^6(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^6*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
x*(e/c^2 - (2*b*f)/c^3) + ((x^3*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3
*d - b^3*c*e + 3*a*b*c^2*e - 4*a*b^2*c*f))/(2*(4*a*c - b^2)) + (x*(2*a^2*c
^2*e + a*b^3*f + a*b*c^2*d - a*b^2*c*e - 3*a^2*b*c*f))/(2*(4*a*c - b^2)))/
(a*c^3 + c^4*x^4 + b*c^3*x^2) - atan(((((10240*a^5*c^9*e + 192*a^2*b^5*c^7
*d - 768*a^3*b^3*c^8*d - 736*a^2*b^6*c^6*e + 4224*a^3*b^4*c^7*e - 10752*a^
4*b^2*c^8*e + 1264*a^2*b^7*c^5*f - 7488*a^3*b^5*c^6*f + 19712*a^4*b^3*c^7*
f - 16*a*b^7*c^6*d + 1024*a^4*b*c^9*d + 48*a*b^8*c^5*e - 80*a*b^9*c^4*f -
19456*a^5*b*c^8*f)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^
7)) - (x*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 + 25*b^6*f^2*(-(4*
a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 - 9*a*c^5*d^2*
(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 8064
0*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d
^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 +
 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 + 25*a^2*c^4*e^2*(-(4*a*c -
 b^2)^9)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*
f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*
f^2 + 215040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 9
*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d
*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4
*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f - ...
```

3.68. $\int \frac{x^6(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

3.69 $\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2} \, dx$

3.69.1 Optimal result

Integrand size = 30, antiderivative size = 436

$$\begin{aligned}
&\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2} \, dx \\
&= \frac{fx}{c^2} + \frac{x(a(2c^2d-bce+b^2f-2acf)-(b^2ce-2ac^2e-b^3f-bc(cd-3af))\,x^2)}{2c^2\,(b^2-4ac)\,(a+bx^2+cx^4)} \\
&+ \frac{\left(b^2ce-6ac^2e-3b^3f+bc(cd+13af)-\frac{b^3ce-8abc^2e-3b^4f+4ac^2(cd-5af)+b^2c(cd+19af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}} \\
&+ \frac{\left(b^2ce-6ac^2e-3b^3f+bc(cd+13af)+\frac{b^3ce-8abc^2e-3b^4f+4ac^2(cd-5af)+b^2c(cd+19af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}\,(b^2-4ac)\,\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

output
```
f*x/c^2+1/2*x*(a*(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)-(b^2*c*e-2*a*c^2*e-b^3*f-b
*c*(-3*a*f+c*d))*x^2)/c^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2
)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*c*e-6*a*c^2*e-3*b^3*f+b*c*(13
*a*f+c*d)+(-b^3*c*e+8*a*b*c^2*e+3*b^4*f-4*a*c^2*(-5*a*f+c*d)-b^2*c*(19*a*f
+c*d))/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1
/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2
*c*e-6*a*c^2*e-3*b^3*f+b*c*(13*a*f+c*d)+(b^3*c*e-8*a*b*c^2*e-3*b^4*f+4*a*c
^2*(-5*a*f+c*d)+b^2*c*(19*a*f+c*d))/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^
2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.69.2 Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.17

$$\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{4\sqrt{c}fx + \frac{2\sqrt{c}x\left(-2a^2cf+b\left(c^2d-bce+b^2f\right)x^2+a\left(b^2f+2c^2\left(d+ex^2\right)-bc\left(e+3fx^2\right)\right)\right)}{(b^2-4ac)(a+bx^2+cx^4)} - \frac{\sqrt{2}\left(-3b^4f+2ac^2\left(2cd+3\sqrt{b^2-4ac}e-10af\right)+b^2c\left(cd\right.\right.}{}}{}$$

input
```
Integrate[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(4*Sqrt[c]*f*x + (2*Sqrt[c]*x*(-2*a^2*c*f + b*(c^2*d - b*c*e + b^2*f)*x^2
+ a*(b^2*f + 2*c^2*(d + e*x^2) - b*c*(e + 3*f*x^2))))/((b^2 - 4*a*c)*(a +
b*x^2 + c*x^4)) - (Sqrt[2]*(-3*b^4*f + 2*a*c^2*(2*c*d + 3*Sqrt[b^2 - 4*a*c
]*e - 10*a*f) + b^2*c*(c*d - Sqrt[b^2 - 4*a*c]*e + 19*a*f) + b^3*(c*e + 3*
Sqrt[b^2 - 4*a*c]*f) - b*c*(c*Sqrt[b^2 - 4*a*c]*d + 8*a*c*e + 13*a*Sqrt[b^
2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((
b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(3*b^4*f + 2*a*
c^2*(-2*c*d + 3*Sqrt[b^2 - 4*a*c]*e + 10*a*f) - b^2*c*(c*d + Sqrt[b^2 - 4*
a*c]*e + 19*a*f) + b^3*(-(c*e) + 3*Sqrt[b^2 - 4*a*c]*f) - b*c*(c*Sqrt[b^2
- 4*a*c]*d - 8*a*c*e + 13*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 -
4*a*c]]))/(4*c^(5/2))
```

3.69.3 Rubi [A] (verified)

Time = 3.15 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2197, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \; 2197$$

$$\frac{x\left(a\left(-2acf+b^2f-bce+2c^2d\right)-x^2\left(-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int\frac{2a\left(4a-\frac{b^2}{c}\right)fx^4-\frac{a\left(-fb^3+ceb^2+c(cd+5af)b-6ac^2e\right)x^2}{c^2}+\frac{a^2\left(fb^2+2c^2d-c(be+2af)\right)}{c^2}}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{2205}$$

$$\frac{x\left(a\left(-2acf+b^2f-bce+2c^2d\right)-x^2\left(-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int\left(\frac{a^2\left(3fb^2-ceb+2c^2d-10acf\right)-a\left(-3fb^3+ceb^2+c(cd+13af)b-6ac^2e\right)x^2}{c^2\left(cx^4+bx^2+a\right)}-\frac{2a\left(b^2-4ac\right)f}{c^2}\right)dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(a\left(-2acf+b^2f-bce+2c^2d\right)-x^2\left(-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-\frac{a\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{b^2c(19af+cd)-8abc^2e+4ac^2(cd-5af)-3b^4f+b^3ce}{\sqrt{b^2-4ac}}+bc(13af+cd)-6ac^2e-3b^3f+b^2ce\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{a\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\cdots\right.}{\cdots}}{2a\left(b^2-4ac\right)}$$

input
```
Int[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(x*(a*(2*c^2*d - b*c*e + b^2*f - 2*a*c*f) - (b^2*c*e - 2*a*c^2*e - b^3*f -
 b*c*(c*d - 3*a*f))*x^2))/(2*c^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((-2
*a*(b^2 - 4*a*c)*f*x)/c^2 - (a*(b^2*c*e - 6*a*c^2*e - 3*b^3*f + b*c*(c*d +
 13*a*f) - (b^3*c*e - 8*a*b*c^2*e - 3*b^4*f + 4*a*c^2*(c*d - 5*a*f) + b^2*
c*(c*d + 19*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - S
qrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (a*(b^
2*c*e - 6*a*c^2*e - 3*b^3*f + b*c*(c*d + 13*a*f) + (b^3*c*e - 8*a*b*c^2*e
- 3*b^4*f + 4*a*c^2*(c*d - 5*a*f) + b^2*c*(c*d + 19*a*f))/Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/
2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*(b^2 - 4*a*c))
```

3.69. $\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

3.69.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

3.69.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.56

method	result
risch	$\frac{fx}{c^2}+\frac{\frac{\left(3abcf-2a\,c^2e-b^3f+b^2ce-b\,c^2d\right)x^3}{8ac-2b^2}+\frac{a\left(2acf-b^2f+ebc-2c^2d\right)x}{8ac-2b^2}}{c^2(c\,x^4+b\,x^2+a)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(-\frac{\left(13abcf-6a\,c^2e-3b^3f+b^2ce\right.}{4ac-b^2}\right.}{}}{4c^2}$
default	$\frac{fx}{c^2}-\frac{-\frac{\left(3abcf-2a\,c^2e-b^3f+b^2ce-b\,c^2d\right)x^3}{2\left(4ac-b^2\right)}-\frac{a\left(2acf-b^2f+ebc-2c^2d\right)x}{2\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{\left(13\sqrt{-4ac+b^2}\,abcf-6\sqrt{-4ac+b^2}\,a\,c^2e-3b^3f\sqrt{-4ac+b^2}+b^2c\right.}{}\right.}{}$

input
```
int(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
f*x/c^2+(1/2*(3*a*b*c*f-2*a*c^2*e-b^3*f+b^2*c*e-b*c^2*d)/(4*a*c-b^2)*x^3+1
/2*a*(2*a*c*f-b^2*f+b*c*e-2*c^2*d)/(4*a*c-b^2)*x)/c^2/(c*x^4+b*x^2+a)+1/4/
c^2*sum((-(13*a*b*c*f-6*a*c^2*e-3*b^3*f+b^2*c*e+b*c^2*d)/(4*a*c-b^2)*_R^2-
a*(10*a*c*f-3*b^2*f+b*c*e-2*c^2*d)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_
R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.69.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12597 vs. 2(394) = 788.

Time = 23.67 (sec) , antiderivative size = 12597, normalized size of antiderivative = 28.89

$$\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.69.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.69.7 Maxima [F]

$$\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(fx^4+ex^2+d)x^4}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*((b*c^2*d - (b^2*c - 2*a*c^2)*e + (b^3 - 3*a*b*c)*f)*x^3 + (2*a*c^2*d
- a*b*c*e + (a*b^2 - 2*a^2*c)*f)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*
a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) + f*x/c^2 + 1/2*integrate(-(2*a*c^
2*d - a*b*c*e - (b*c^2*d + (b^2*c - 6*a*c^2)*e - (3*b^3 - 13*a*b*c)*f)*x^2
 + (3*a*b^2 - 10*a^2*c)*f)/(c*x^4 + b*x^2 + a), x)/(b^2*c^2 - 4*a*c^3)
```

3.69.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7479 vs. 2(394) = 788.

Time = 1.78 (sec) , antiderivative size = 7479, normalized size of antiderivative = 17.15

$$\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
f*x/c^2 + 1/2*(b*c^2*d*x^3 - b^2*c*e*x^3 + 2*a*c^2*e*x^3 + b^3*f*x^3 - 3*a
*b*c*f*x^3 + 2*a*c^2*d*x - a*b*c*e*x + a*b^2*f*x - 2*a^2*c*f*x)/((c*x^4 +
b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) - 1/16*((2*b^3*c^4 - 8*a*b*c^5 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^4 - 2*(b^2 - 4*a*c)*b*c^4)*(b^2*c^2
 - 4*a*c^3)^2*d + (2*b^4*c^3 - 20*a*b^2*c^4 + 48*a^2*c^5 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 10*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 12*(b^2 - 4*a*c)*a*c
^4)*(b^2*c^2 - 4*a*c^3)^2*e - (6*b^5*c^2 - 50*a*b^3*c^3 + 104*a^2*b*c^4 -
3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 25*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 6*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 52*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 26*sqrt(2)*sqrt...
```

3.69.9 Mupad [B] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 25862, normalized size of antiderivative = 59.32

$$\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
(f*x)/c^2 - atan(((((10240*a^5*c^7*f - 2048*a^4*c^8*d - 384*a^2*b^4*c^6*d
+ 1536*a^3*b^2*c^7*d + 192*a^2*b^5*c^5*e - 768*a^3*b^3*c^6*e - 736*a^2*b^6
*c^4*f + 4224*a^3*b^4*c^5*f - 10752*a^4*b^2*c^6*f + 32*a*b^6*c^5*d - 16*a*
b^7*c^4*e + 1024*a^4*b*c^7*e + 48*a*b^8*c^3*f)/(8*(64*a^3*c^6 - b^6*c^3 +
12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*((768*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c
^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c^2*e^2 - 9*b^4*f^2*(-(4*a*c - b^2)
^9)^(1/2) - 9*b^13*f^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 + 9*a*c^3*e
^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*
b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5
*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2
 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 - 25*a^2*c^2*f^2*(-(4*a*c
 - b^2)^9)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2
 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f
+ 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f + 10*a*c^3*d*f*
(-(4*a*c - b^2)^9)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^
10*c^2*e*f + 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c*f^2*(-(4*a*
c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b
^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*
d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f
- 30720*a^5*b^2*c^6*e*f + 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a...
```

3.69. $\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

3.70 $\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

3.70.1 Optimal result

Integrand size = 30, antiderivative size = 362

$$\begin{aligned}\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx &= -\frac{x(bcd-2ace+abf+(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} \\ &\quad - \frac{\left(2cd-be+6af-\frac{b^2f}{c}+\frac{b^2ce+4ac^2e+b^3f-4bc(cd+2af)}{c\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}} \\ &\quad - \frac{\left(2cd-be+6af-\frac{b^2f}{c}-\frac{b^2ce+4ac^2e+b^3f-4bc(cd+2af)}{c\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\,\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

output
```
-1/2*x*(b*c*d-2*a*c*e+a*b*f+(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x^2)/c/(-4*a*c+
b^2)/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(
1/2))*(2*c*d-b*e+6*a*f-b^2*f/c+(b^2*c*e+4*a*c^2*e+b^3*f-4*b*c*(2*a*f+c*d))
/c/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))
^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*c*d-b
*e+6*a*f-b^2*f/c+(-b^2*c*e-4*a*c^2*e-b^3*f+4*b*c*(2*a*f+c*d))/c/(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.70.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.14

$$\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{-\frac{2\sqrt{c}x\left(abf+2c^2dx^2+b^2fx^2+bc(d-ex^2)-2ac(e+fx^2)\right)}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(-b^3f+bc\left(4cd+\sqrt{b^2-4ac}e+8af\right)+b^2\left(-ce+\sqrt{b^2-4ac}f\right)-2c\left(c\sqrt{b^2-4ac}d+2a\right.\right.}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{}$$

input
```
Integrate[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((-2*Sqrt[c]*x*(a*b*f + 2*c^2*d*x^2 + b^2*f*x^2 + b*c*(d - e*x^2) - 2*a*c*
(e + f*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-(b^3*f) + b
*c*(4*c*d + Sqrt[b^2 - 4*a*c]*e + 8*a*f) + b^2*(-(c*e) + Sqrt[b^2 - 4*a*c]
*f) - 2*c*(c*Sqrt[b^2 - 4*a*c]*d + 2*a*c*e + 3*a*Sqrt[b^2 - 4*a*c]*f))*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)
*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^3*f + b*c*(-4*c*d + Sqrt[b^2 -
 4*a*c]*e - 8*a*f) + b^2*(c*e + Sqrt[b^2 - 4*a*c]*f) - 2*c*(c*Sqrt[b^2 - 4
*a*c]*d - 2*a*c*e + 3*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*
c]]))/(4*c^(3/2))
```

3.70.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2197, 25, 27, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \; 2197$$

$$-\frac{\int -\frac{a\left(-c\left(-\frac{fb^2}{c}-eb+2cd+6af\right)x^2+bcd-2ace+abf\right)}{c(cx^4+bx^2+a)}dx}{2a\left(b^2-4ac\right)}-\frac{x\left(x^2\left(-2acf+b^2f-bce+2c^2d\right)+abf-2ace+bcd\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{a(-((-fb^2-ceb+2c^2d+6acf)x^2)+bcd-2ace+abf)}{c(cx^4+bx^2+a)}dx}{2a\left(b^2-4ac\right)}-\frac{x\left(x^2\left(-2acf+b^2f-bce+2c^2d\right)+abf-2ace+bcd\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{-((-fb^2-ceb+2c^2d+6acf)x^2)+bcd-2ace+abf}{cx^4+bx^2+a}dx}{2c\left(b^2-4ac\right)}-\frac{x\left(x^2\left(-2acf+b^2f-bce+2c^2d\right)+abf-2ace+bcd\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$\Big\downarrow$ 1480

$$\frac{-\frac{1}{2}\left(\frac{-4bc(2af+cd)+4ac^2e+b^3f+b^2ce}{\sqrt{b^2-4ac}}+6acf+b^2(-f)-bce+2c^2d\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}\left(-\frac{-4bc(2af+cd)+4ac^2e+b}{\sqrt{b^2-4ac}}\right.}{2c\left(b^2-4ac\right)}$$
$$\frac{x\left(x^2\left(-2acf+b^2f-bce+2c^2d\right)+abf-2ace+bcd\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$\Big\downarrow$ 218

$$\frac{-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-4bc(2af+cd)+4ac^2e+b^3f+b^2ce}{\sqrt{b^2-4ac}}+6acf+b^2(-f)-bce+2c^2d\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-4bc(2af+cd)+4ac^2e+b^3f+}{\sqrt{b^2-4ac}}\right.}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2c\left(b^2-4ac\right)}$$
$$\frac{x\left(x^2\left(-2acf+b^2f-bce+2c^2d\right)+abf-2ace+bcd\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input
```
Int[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

3.70. $\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

output
```
-1/2*(x*(b*c*d - 2*a*c*e + a*b*f + (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^2
))/(c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (-(((2*c^2*d - b*c*e - b^2*f +
6*a*c*f + (b^2*c*e + 4*a*c^2*e + b^3*f - 4*b*c*(c*d + 2*a*f))/Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*S
qrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])) - ((2*c^2*d - b*c*e - b^2*f + 6*a*c*f
 - (b^2*c*e + 4*a*c^2*e + b^3*f - 4*b*c*(c*d + 2*a*f))/Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*
Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*c*(b^2 - 4*a*c))
```

3.70.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

3.70.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.55

method	result
risch	$\frac{-\frac{\left(2acf-b^2f+ebc-2c^2d\right)x^3}{2c\left(4ac-b^2\right)}+\frac{(abf-2ace+bcd)x}{2c\left(4ac-b^2\right)}}{c x^4+b x^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\frac{\left(6acf-b^2f-ebc+2c^2d\right)\_R^2}{4ac-b^2}-\frac{abf-2ace+bcd}{4ac-b^2}\right)\ln\left(x-\right.}{2c\_R^3+\_Rb}}{4c}$
default	$\frac{-\frac{\left(2acf-b^2f+ebc-2c^2d\right)x^3}{2c\left(4ac-b^2\right)}+\frac{(abf-2ace+bcd)x}{2c\left(4ac-b^2\right)}}{c x^4+b x^2+a}+\frac{\frac{\left(6acf\sqrt{-4ac+b^2}-b^2f\sqrt{-4ac+b^2}-ebc\sqrt{-4ac+b^2}+2c^2d\sqrt{-4ac+b^2}+8abcf-4a c^2e-b^3f-\right.}{4c\sqrt{-4ac+b^2}\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}}{}$

input
```
int(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*(2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c/(4*a*c-b^2)*x^3+1/2/c*(a*b*f-2*a*c*e
+b*c*d)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/4/c*sum(((6*a*c*f-b^2*f-b*c*e+2*c
^2*d)/(4*a*c-b^2)*_R^2-(a*b*f-2*a*c*e+b*c*d)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*
ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.70.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8951 vs. 2(320) = 640.

Time = 13.28 (sec) , antiderivative size = 8951, normalized size of antiderivative = 24.73

$$\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.70.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.70.7 Maxima [F]

$$\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(fx^4+ex^2+d)x^2}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((2*c^2*d - b*c*e + (b^2 - 2*a*c)*f)*x^3 + (b*c*d - 2*a*c*e + a*b*f)*
x)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^
2) - 1/2*integrate(-(b*c*d - 2*a*c*e + a*b*f - (2*c^2*d - b*c*e - (b^2 - 6
*a*c)*f)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)
```

3.70.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6200 vs. 2(320) = 640.

Time = 1.52 (sec) , antiderivative size = 6200, normalized size of antiderivative = 17.13

$$\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(2*c^2*d*x^3 - b*c*e*x^3 + b^2*f*x^3 - 2*a*c*f*x^3 + b*c*d*x - 2*a*c*
e*x + a*b*f*x)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2)) - 1/16*(2*(2*b^2*c^
4 - 8*a*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^3 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^4 - 2*(b^2 - 4*a*c
)*c^4)*(b^2*c - 4*a*c^2)^2*d - (2*b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3)*(b^2*c - 4*a*c^2)^2
*e - (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4*a*c)*a*c^3)*(b^2*c - 4*a*c^
2)^2*f - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 8*sqrt(2)...
```

3.70.9 Mupad [B] (verification not implemented)

Time = 12.24 (sec) , antiderivative size = 19494, normalized size of antiderivative = 53.85

$$\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
((x^3*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e))/(2*c*(4*a*c - b^2)) + (x*(a*b*f
 - 2*a*c*e + b*c*d))/(2*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) - atan(((((2
048*a^4*c^6*e + 16*b^7*c^3*d + 768*a^2*b^3*c^5*d + 384*a^2*b^4*c^4*e - 153
6*a^3*b^2*c^5*e - 192*a^2*b^5*c^3*f + 768*a^3*b^3*c^4*f - 192*a*b^5*c^4*d
- 1024*a^3*b*c^6*d - 32*a*b^6*c^3*e + 16*a*b^7*c^2*f - 1024*a^4*b*c^5*f)/(
8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*((768*a^4*b*c
^7*d^2 - b^9*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*f^2 - a*b
^9*c^2*e^2 + 768*a^5*b*c^6*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + a*c^
2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9
*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6
*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 15
04*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^
6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*
d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e -
 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2
*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^
5*e*f - 2*a*b^10*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a
^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6
 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^
4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2...
```

3.70. $\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

3.71 $\int \frac{d+ex^2+fx^4}{(a+bx^2+cx^4)^2} dx$

3.71.1 Optimal result

Integrand size = 27, antiderivative size = 346

$$\begin{aligned}
&\int \frac{d+ex^2+fx^4}{(a+bx^2+cx^4)^2} dx \\
&= \frac{x(b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} \\
&\quad+ \frac{\left(bcd-2ace+abf+\frac{4abce+b^2(cd-af)-4ac(3cd+af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad+ \frac{\left(bcd-2ace+abf-\frac{4abce+b^2(cd-af)-4ac(3cd+af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

output
```
1/2*x*(b^2*d-a*b*e-2*a*(-a*f+c*d)+(a*b*f-2*a*c*e+b*c*d)*x^2)/a/(-4*a*c+b^2
)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2
))*(b*c*d-2*a*c*e+a*b*f+(4*a*b*c*e+b^2*(-a*f+c*d)-4*a*c*(a*f+3*c*d))/(-4*a
*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)
+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*c*d-2*a*c*e
+a*b*f+(-4*a*b*c*e-b^2*(-a*f+c*d)+4*a*c*(a*f+3*c*d))/(-4*a*c+b^2)^(1/2))/a
/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.71.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.10

$$\int \frac{d+ex^2+fx^4}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{\frac{2x(b^2d+b(-ae+cdx^2+afx^2)+2a(af-c(d+ex^2)))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(b^2(cd-af)-2ac\left(6cd+\sqrt{b^2-4ac}e+2af\right)+b\left(c\sqrt{b^2-4ac}d+4ace+a\sqrt{b^2-4ac}f\right)\right)\text{arcta}}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{4a}$$

input
```
Integrate[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((2*x*(b^2*d + b*(-(a*e) + c*d*x^2 + a*f*x^2) + 2*a*(a*f - c*(d + e*x^2)))
)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(b^2*(c*d - a*f) - 2*a*c*
(6*c*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f) + b*(c*Sqrt[b^2 - 4*a*c]*d + 4*a*c*e
 + a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqr
t[2]*(b^2*(-(c*d) + a*f) + 2*a*c*(6*c*d - Sqrt[b^2 - 4*a*c]*e + 2*a*f) + b
*(c*Sqrt[b^2 - 4*a*c]*d - 4*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sq
rt[b + Sqrt[b^2 - 4*a*c]]))/(4*a)
```

3.71.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2206, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \; 2206$$

$$\frac{x(x^2(abf-2ace+bcd)-abe-2a(cd-af)+b^2d)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)}-\frac{\int -\frac{db^2+aeb+(bcd-2ace+abf)x^2-2a(3cd+af)}{cx^4+bx^2+a}dx}{2a\,(b^2-4ac)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{db^2+aeb+(bcd-2ace+abf)x^2-2a(3cd+af)}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}+\frac{x(x^2(abf-2ace+bcd)-abe-2a(cd-af)+b^2d)}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

$\downarrow$ 1480

$$\frac{\frac{1}{2}\left(\frac{b^2(cd-af)+4abce-4ac(af+3cd)}{\sqrt{b^2-4ac}}+abf-2ace+bcd\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(-\frac{b^2(cd-af)+4abce-4ac(af+3cd)}{\sqrt{b^2-4ac}}+abf\right.}{2a(b^2-4ac)}$$
$$\frac{x(x^2(abf-2ace+bcd)-abe-2a(cd-af)+b^2d)}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

$\downarrow$ 218

$$\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2(cd-af)+4abce-4ac(af+3cd)}{\sqrt{b^2-4ac}}+abf-2ace+bcd\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{b^2(cd-af)+4abce-4ac(af+3cd)}{\sqrt{b^2-4ac}}+abf-2a\right.}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2a(b^2-4ac)}$$
$$\frac{x(x^2(abf-2ace+bcd)-abe-2a(cd-af)+b^2d)}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

input
```
Int[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(x*(b^2*d - a*b*e - 2*a*(c*d - a*f) + (b*c*d - 2*a*c*e + a*b*f)*x^2))/(2*a
*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (((b*c*d - 2*a*c*e + a*b*f + (4*a*b*
c*e + b^2*(c*d - a*f) - 4*a*c*(3*c*d + a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - S
qrt[b^2 - 4*a*c]]) + ((b*c*d - 2*a*c*e + a*b*f - (4*a*b*c*e + b^2*(c*d - a
*f) - 4*a*c*(3*c*d + a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
)/(2*a*(b^2 - 4*a*c))
```

3.71.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

3.71.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.58

method	result
risch	$\frac{-\frac{(abf-2ace+bcd)x^3}{2a\left(4ac-b^2\right)}-\frac{\left(2f\,a^2-abe-2acd+b^2d\right)x}{2a\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(-\frac{(abf-2ace+bcd)\_R^2}{4ac-b^2}+\frac{2f\,a^2-abe+6acd-b^2d}{4ac-b^2}\right)\ln(}{2c\_R^3+\_Rb}}{4a}$
default	$\frac{-\frac{(abf-2ace+bcd)x^3}{2a\left(4ac-b^2\right)}-\frac{\left(2f\,a^2-abe-2acd+b^2d\right)x}{2a\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{\left(-\sqrt{-4ac+b^2}\,abf+2ace\sqrt{-4ac+b^2}-bcd\sqrt{-4ac+b^2}-4a^2cf-a\,b^2f+4abce-12a\,c^2d+b\right.}{8\sqrt{-4ac+b^2}\,c\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right.}{}$

input
```
int((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2/a*(a*b*f-2*a*c*e+b*c*d)/(4*a*c-b^2)*x^3-1/2*(2*a^2*f-a*b*e-2*a*c*d+b
^2*d)/a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/4/a*sum((-(a*b*f-2*a*c*e+b*c*d)/(
4*a*c-b^2)*_R^2+(2*a^2*f-a*b*e+6*a*c*d-b^2*d)/(4*a*c-b^2))/(2*_R^3*c+_R*b)
*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.71.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8991 vs. $2(304) = 608$.

Time = 11.61 (sec) , antiderivative size = 8991, normalized size of antiderivative = 25.99

$$\int \frac{d+ex^2+fx^4}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.71.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.71.7 Maxima [F]

$$\int \frac{d+ex^2+fx^4}{(a+bx^2+cx^4)^2}\,dx = \int \frac{fx^4+ex^2+d}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*((b*c*d - 2*a*c*e + a*b*f)*x^3 - (a*b*e - 2*a^2*f - (b^2 - 2*a*c)*d)*x
)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2
) + 1/2*integrate((a*b*e - 2*a^2*f + (b*c*d - 2*a*c*e + a*b*f)*x^2 + (b^2
- 6*a*c)*d)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

3.71.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6348 vs. $2(304) = 608$.

Time = 1.29 (sec) , antiderivative size = 6348, normalized size of antiderivative = 18.35

$$\int \frac{d+ex^2+fx^4}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b*c*d*x^3 - 2*a*c*e*x^3 + a*b*f*x^3 + b^2*d*x - 2*a*c*d*x - a*b*e*x +
 2*a^2*f*x)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) - 1/16*((2*b^3*c^3 - 8
*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*
c)*b*c^3)*(a*b^2 - 4*a^2*c)^2*d - 2*(2*a*b^2*c^3 - 8*a^2*c^4 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 4*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*a*c^3)*(a*b^2 - 4
*a^2*c)^2*e + (2*a*b^3*c^2 - 8*a^2*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*(a*b^2 - 4*a^2*c)^2*f - 2*(s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 14*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*b^4*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*b^5*c^2 - 2*a*b^6*c^2 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b
^2*c^3 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 + sqrt(...
```

3.71.9 Mupad [B] (verification not implemented)

Time = 12.16 (sec) , antiderivative size = 19589, normalized size of antiderivative = 56.62

$$\int \frac{d+ex^2+fx^4}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4)^2,x)
```

output
```
atan(((((6144*a^5*c^6*d + 2048*a^6*c^5*f - 288*a^2*b^6*c^3*d + 1920*a^3*b^
4*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*
a^4*b^3*c^4*e - 32*a^3*b^6*c^2*f + 384*a^4*b^4*c^3*f - 1536*a^5*b^2*c^4*f
+ 16*a*b^8*c^2*d - 1024*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4
*c + 48*a^4*b^2*c^2)) - (x*((27*a*b^9*c^2*d^2 - a^3*b^9*f^2 - a^3*f^2*(-(4
*a*c - b^2)^9)^(1/2) - b^11*c*d^2 + 3840*a^5*b*c^6*d^2 - 9*a*c^2*d^2*(-(4*
a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 + a^2*c*e^2*(-(4*a
*c - b^2)^9)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^
2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96
*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*
c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6
*b*c^5*d*f - 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*
a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2
*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*
f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e + 2*a*b*c*d
*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*
c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2
*c^6)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4
*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((27*a*b^9*c^2*d^2 -
a^3*b^9*f^2 - a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c*d^2 + 3840*a^5*...
```

3.71. $\int \frac{d+ex^2+fx^4}{(a+bx^2+cx^4)^2}\,dx$

3.72 $\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)^2}\,dx$

3.72.1 Optimal result

Integrand size = 30, antiderivative size = 399

$$\begin{aligned}&\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)^2}\,dx\\&=-\frac{d}{a^2x}-\frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)\right)+c(b^2d-abe-2a(cd-af))\,x^2\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\\&\quad-\frac{\sqrt{c}\left(3b^2d-abe-2a(5cd-af)+\frac{3b^3d-ab^2e+12a^2ce-4ab(4cd+af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}\\&\quad-\frac{\sqrt{c}\left(3b^2d-abe-2a(5cd-af)-\frac{3b^3d-ab^2e+12a^2ce-4ab(4cd+af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

output
```
-d/a^2/x-1/2*x*(a*(b^3*d/a-b*(b*e+3*c*d)+a*(b*f+2*c*e))+c*(b^2*d-a*b*e-2*a
*(-a*f+c*d))*x^2)/a^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^
(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^2*d-a*b*e-2*a*(-a*f+5*c*d
)+(3*b^3*d-a*b^2*e+12*a^2*c*e-4*a*b*(a*f+4*c*d))/(-4*a*c+b^2)^(1/2))/a^2/(
-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1
/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^2*d-a*b*e-2*a*(-a*f+5*c*d)+
(-3*b^3*d+a*b^2*e-12*a^2*c*e+4*a*b*(a*f+4*c*d))/(-4*a*c+b^2)^(1/2))/a^2/(-
4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.72.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.11

$$\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \frac{-\frac{4d}{x}-\frac{2x\left(b^3d+b^2\left(-ae+cdx^2\right)+ab\left(af-c\left(3d+ex^2\right)\right)+2ac\left(-cdx^2+a\left(e+fx^2\right)\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(-3b^3d+b^2\left(-3\sqrt{b^2-4ac}d+ae\right)+ab\left(16cd+\sqrt{b^2-4\cdots}\right)\cdots\right)}{\left(b^2\cdots\right)}}{\cdots}$$

input
```
Integrate[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

output
```
((-4*d)/x - (2*x*(b^3*d + b^2*(-(a*e) + c*d*x^2) + a*b*(a*f - c*(3*d + e*x
^2)) + 2*a*c*(-(c*d*x^2) + a*(e + f*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*
x^4)) + (Sqrt[2]*Sqrt[c]*(-3*b^3*d + b^2*(-3*Sqrt[b^2 - 4*a*c]*d + a*e) +
a*b*(16*c*d + Sqrt[b^2 - 4*a*c]*e + 4*a*f) - 2*a*(-5*c*Sqrt[b^2 - 4*a*c]*d
 + 6*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - S
qrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[2]*Sqrt[c]*(3*b^3*d - b^2*(3*Sqrt[b^2 - 4*a*c]*d + a*e) + a*b*(-16*c*d
 + Sqrt[b^2 - 4*a*c]*e - 4*a*f) + 2*a*(5*c*Sqrt[b^2 - 4*a*c]*d + 6*a*c*e -
 a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*
a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2)
```

3.72.3 Rubi [A] (verified)

Time = 1.90 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2198, 25, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$$

$$\Big\downarrow \text{ 2198}$$

$$-\frac{\int -\frac{-\frac{c\left(db^2-aeb-2a(cd-af)\right)x^4}{a}-\frac{\left(db^3-aeb^2-a(5cd+af)b+6a^2ce\right)x^2}{a}+2(b^2-4ac)d}{x^2(cx^4+bx^2+a)}dx}{2a\left(b^2-4ac\right)}-\frac{x\left(a\left(\frac{b^3d}{a}+a(bf+2ce)-b(be+3cd)\right)+cx^2\left(-abe-2a(cd-af)+b^2d\right)\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{-\frac{c\left(db^2-aeb-2a(cd-af)\right)x^4}{a}-\frac{\left(db^3-aeb^2-a(5cd+af)b+6a^2ce\right)x^2}{a}+2(b^2-4ac)d}{x^2(cx^4+bx^2+a)}dx}{2a\left(b^2-4ac\right)}-\frac{x\left(a\left(\frac{b^3d}{a}+a(bf+2ce)-b(be+3cd)\right)+cx^2\left(-abe-2a(cd-af)+b^2d\right)\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{ 2195}$$

$$\frac{\int \left(\frac{-3db^3+aeb^2+a(13cd+af)b-c(3db^2-aeb-2a(5cd-af))x^2-6a^2ce}{a(cx^4+bx^2+a)}-\frac{2(4ac-b^2)d}{ax^2}\right)dx}{2a\left(b^2-4ac\right)}-\frac{x\left(a\left(\frac{b^3d}{a}+a(bf+2ce)-b(be+3cd)\right)+cx^2\left(-abe-2a(cd-af)+b^2d\right)\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{12a^2ce-ab^2e-4ab(af+4cd)+3b^3d}{\sqrt{b^2-4ac}}-abe-2a(5cd-af)+3b^2d\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{12a^2ce-ab^2e-4ab(af\cdots}{\sqrt{b^2-4ac}}\cdots\right.}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}\cdots}}$$

$$\overline{\quad 2a\left(b^2-4ac\right)\quad}$$

$$\frac{x\left(a\left(\frac{b^3d}{a}+a(bf+2ce)-b(be+3cd)\right)+cx^2\left(-abe-2a(cd-af)+b^2d\right)\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input
```
Int[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

output
```
-1/2*(x*(a*((b^3*d)/a - b*(3*c*d + b*e) + a*(2*c*e + b*f)) + c*(b^2*d - a*
b*e - 2*a*(c*d - a*f))*x^2))/(a^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((-
2*(b^2 - 4*a*c)*d)/(a*x) - (Sqrt[c]*(3*b^2*d - a*b*e - 2*a*(5*c*d - a*f) +
 (3*b^3*d - a*b^2*e + 12*a^2*c*e - 4*a*b*(4*c*d + a*f))/Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(3*b^2*d - a*b*e - 2*a*(5*c*d - a*f) -
(3*b^3*d - a*b^2*e + 12*a^2*c*e - 4*a*b*(4*c*d + a*f))/Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b
 + Sqrt[b^2 - 4*a*c]]))/(2*a*(b^2 - 4*a*c))
```

3.72. $\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)^2}\,dx$

3.72.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

3.72.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.10

method	result
default	$-\frac{d}{a^2 x}+\frac{\frac{c\left(2f\,a^2-abe-2acd+b^2 d\right)x^3}{8ac-2b^2}+\frac{\left(a^2 bf+2a^2 ce-a\,b^2 e-3abcd+b^3 d\right)x}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{\left(2f\,a^2\sqrt{-4ac+b^2}-abe\sqrt{-4ac+b^2}-10acd\sqrt{-4ac+b^2}+3b^2 d\right.}{8\sqrt{-}}\right.}{}$
risch	Expression too large to display

input
```
int((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-d/a^2/x+1/a^2*((1/2*c*(2*a^2*f-a*b*e-2*a*c*d+b^2*d)/(4*a*c-b^2)*x^3+1/2*(
a^2*b*f+2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+
2/(4*a*c-b^2)*c*(1/8*(2*f*a^2*(-4*a*c+b^2)^(1/2)-a*b*e*(-4*a*c+b^2)^(1/2)-
10*a*c*d*(-4*a*c+b^2)^(1/2)+3*b^2*d*(-4*a*c+b^2)^(1/2)+4*a^2*b*f-12*a^2*c*
e+a*b^2*e+16*a*b*c*d-3*b^3*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(
2*f*a^2*(-4*a*c+b^2)^(1/2)-a*b*e*(-4*a*c+b^2)^(1/2)-10*a*c*d*(-4*a*c+b^2)^
(1/2)+3*b^2*d*(-4*a*c+b^2)^(1/2)-4*a^2*b*f+12*a^2*c*e-a*b^2*e-16*a*b*c*d+3
*b^3*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
nh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

3.72.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13111 vs. $2(357) = 714$.

Time = 28.00 (sec) , antiderivative size = 13111, normalized size of antiderivative = 32.86

$$\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.72.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x**4+e*x**2+d)/x**2/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.72.7 Maxima [F]

$$\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)^2}\,dx = \int \frac{fx^4+ex^2+d}{(cx^4+bx^2+a)^2x^2}\,dx$$

input
```
integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*((a*b*c*e - 2*a^2*c*f - (3*b^2*c - 10*a*c^2)*d)*x^4 - (a^2*b*f + (3*b^
3 - 11*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*x^2 - 2*(a*b^2 - 4*a^2*c)*d)/((a^2*
b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x
) - 1/2*integrate(-(a^2*b*f + (a*b*c*e - 2*a^2*c*f - (3*b^2*c - 10*a*c^2)*
d)*x^2 - (3*b^3 - 13*a*b*c)*d + (a*b^2 - 6*a^2*c)*e)/(c*x^4 + b*x^2 + a),
x)/(a^2*b^2 - 4*a^3*c)
```

3.72.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7173 vs. $2(357) = 714$.

Time = 1.44 (sec) , antiderivative size = 7173, normalized size of antiderivative = 17.98

$$\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(3*b^2*c*d*x^4 - 10*a*c^2*d*x^4 - a*b*c*e*x^4 + 2*a^2*c*f*x^4 + 3*b^3
*d*x^2 - 11*a*b*c*d*x^2 - a*b^2*e*x^2 + 2*a^2*c*e*x^2 + a^2*b*f*x^2 + 2*a*
b^2*d - 8*a^2*c*d)/((c*x^5 + b*x^3 + a*x)*(a^2*b^2 - 4*a^3*c)) - 1/16*((6*
b^4*c^2 - 44*a*b^2*c^3 + 80*a^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
c^3 - 6*(b^2 - 4*a*c)*b^2*c^2 + 20*(b^2 - 4*a*c)*a*c^3)*(a^2*b^2 - 4*a^3*c
)^2*d - (2*a*b^3*c^2 - 8*a^2*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*(a^2*b^2 - 4*a^3*c)^2*e + 2*(2*a^2
*b^2*c^2 - 8*a^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^...
```

3.72.9 Mupad [B] (verification not implemented)

Time = 12.19 (sec) , antiderivative size = 28164, normalized size of antiderivative = 70.59

$$\int \frac{d + ex^2 + fx^4}{x^2 (a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)^2),x)
```

output
```
((x^2*(3*b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 11*a*b*c*d))/(2*a^2*(4*a*
c - b^2)) - d/a + (c*x^4*(3*b^2*d + 2*a^2*f - a*b*e - 10*a*c*d))/(2*a^2*(4
*a*c - b^2)))/(a*x + b*x^3 + c*x^5) - atan(((x*(204800*a^12*c^9*d^2 - 7372
8*a^13*c^8*e^2 + 8192*a^14*c^7*f^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7*b^10*
c^4*d^2 + 30112*a^8*b^8*c^5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10*b^4
*c^7*d^2 - 458752*a^11*b^2*c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*c^4
*e^2 + 4608*a^10*b^6*c^5*e^2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c^7
*e^2 + 160*a^10*b^8*c^3*f^2 - 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f^
2 - 16384*a^13*b^2*c^6*f^2 - 81920*a^13*c^8*d*f + 237568*a^12*b*c^8*d*e +
40960*a^13*b*c^7*e*f - 96*a^7*b^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e - 22528*
a^9*b^7*c^5*d*e + 107520*a^10*b^5*c^6*d*e - 253952*a^11*b^3*c^7*d*e - 96*a
^8*b^10*c^3*d*f + 1472*a^9*b^8*c^4*d*f - 7168*a^10*b^6*c^5*d*f + 6144*a^11
*b^4*c^6*d*f + 40960*a^12*b^2*c^7*d*f + 32*a^9*b^9*c^3*e*f - 1024*a^10*b^7
*c^4*e*f + 9216*a^11*b^5*c^5*e*f - 32768*a^12*b^3*c^6*e*f) + ((27*a^3*b^9*
c*e^2 - a^2*b^11*e^2 - 9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*f^2 -
a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 - 9*b^13*d^2 + 3840
*a^7*b*c^5*e^2 + 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2
+ 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*
b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2)
 - 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504...
```

3.72. $\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)^2}\,dx$

3.73 $\int \frac{d+ex^2+fx^4}{x^4(a+bx^2+cx^4)^2}\,dx$

3.73.1 Optimal result

Integrand size = 30, antiderivative size = 575

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{d}{3a^2x^3}+\frac{2bd-ae}{a^3x}$$
$$+\frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-2acf\right)+c(b^3d-ab^2e+2a^2ce-ab(3cd-af))\,x^2\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$
$$+\frac{\sqrt{c}\left(5b^4d+b^3\left(5\sqrt{b^2-4ac}d-3ae\right)+2a^2c\left(14cd+5\sqrt{b^2-4ac}e-6af\right)-ab^2\left(29cd+3\sqrt{b^2-4ac}e-a\right.}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4}}}$$
$$-\frac{\sqrt{c}\left(5b^4d-b^3\left(5\sqrt{b^2-4ac}d+3ae\right)+2a^2c\left(14cd-5\sqrt{b^2-4ac}e-6af\right)-ab^2\left(29cd-3\sqrt{b^2-4ac}e-a\right.}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4}}}$$

output
```
-1/3*d/a^2/x^3+(-a*e+2*b*d)/a^3/x+1/2*x*(a^2*(b^4*d/a^2+2*c^2*d+3*b*c*e-b^
2*(b*e+4*c*d)/a+b^2*f-2*a*c*f)+c*(b^3*d-a*b^2*e+2*a^2*c*e-a*b*(-a*f+3*c*d)
)*x^2)/a^3/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-
4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(5*b^4*d+b^3*(-3*a*e+5*d*(-4*a*c+b^2)^(1/
2))-a*b^2*(29*c*d-a*f+3*e*(-4*a*c+b^2)^(1/2))+2*a^2*c*(14*c*d-6*a*f+5*e*(-
4*a*c+b^2)^(1/2))-a*b*(-16*a*c*e+19*c*d*(-4*a*c+b^2)^(1/2)-a*f*(-4*a*c+b^2
)^(1/2)))/a^3/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*
arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(5*b^4*d-b^
3*(3*a*e+5*d*(-4*a*c+b^2)^(1/2))+2*a^2*c*(14*c*d-6*a*f-5*e*(-4*a*c+b^2)^(1
/2))-a*b^2*(29*c*d-a*f-3*e*(-4*a*c+b^2)^(1/2))+a*b*(16*a*c*e+19*c*d*(-4*a*
c+b^2)^(1/2)-a*f*(-4*a*c+b^2)^(1/2)))/a^3/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-
4*a*c+b^2)^(1/2))^(1/2)
```

3.73.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 548, normalized size of antiderivative = 0.95

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{-\frac{4ad}{x^3}+\frac{24bd-12ae}{x}+\frac{6x\left(b^4d+b^3\left(-ae+cdx^2\right)+abc\left(3ae-3cdx^2+afx^2\right)+2a^2c\left(-af+c\left(d+ex^2\right)\right)+ab^2\left(af-c\left(4d+ex^2\right)\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{2}\sqrt{c}\left(5b^4d+\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

output
```
((-4*a*d)/x^3 + (24*b*d - 12*a*e)/x + (6*x*(b^4*d + b^3*(-(a*e) + c*d*x^2)
 + a*b*c*(3*a*e - 3*c*d*x^2 + a*f*x^2) + 2*a^2*c*(-(a*f) + c*(d + e*x^2))
+ a*b^2*(a*f - c*(4*d + e*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (3
*Sqrt[2]*Sqrt[c]*(5*b^4*d + b^3*(5*Sqrt[b^2 - 4*a*c]*d - 3*a*e) + 2*a^2*c*
(14*c*d + 5*Sqrt[b^2 - 4*a*c]*e - 6*a*f) + a*b^2*(-29*c*d - 3*Sqrt[b^2 - 4
*a*c]*e + a*f) + a*b*(-19*c*Sqrt[b^2 - 4*a*c]*d + 16*a*c*e + a*Sqrt[b^2 -
4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2
- 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^4*d
 + b^3*(5*Sqrt[b^2 - 4*a*c]*d + 3*a*e) - a*b^2*(-29*c*d + 3*Sqrt[b^2 - 4*a
*c]*e + a*f) + 2*a^2*c*(-14*c*d + 5*Sqrt[b^2 - 4*a*c]*e + 6*a*f) + a*b*(-1
9*c*Sqrt[b^2 - 4*a*c]*d - 16*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b +
Sqrt[b^2 - 4*a*c]]))/(12*a^3)
```

3.73.3 Rubi [A] (verified)

Time = 4.75 (sec) , antiderivative size = 605, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2198, 25, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)^2}\,dx$$

$$\Big\downarrow \; 2198$$

$$\frac{x\left(a^2\left(\frac{b^4d}{a^2}-\frac{b^2(be+4cd)}{a}-2acf+b^2f+3bce+2c^2d\right)+cx^2\left(2a^2ce-ab^2e-ab(3cd-af)+b^3d\right)\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{\frac{c\left(db^3-aeb^2-a(3cd-af)b+2a^2ce\right)x^6}{a^2}+\left(\frac{db^4}{a^2}-\frac{(6cd+be)b^2}{a}+fb^2+5ceb+6c^2d-6acf\right)x^4-\frac{2\left(b^2-4ac\right)(bd-ae)x^2}{a}+2\left(b^2-4ac\right)d}{x^4\left(cx^4+bx^2+a\right)}dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\frac{c\left(db^3-aeb^2-a(3cd-af)b+2a^2ce\right)x^6}{a^2}+\left(\frac{db^4}{a^2}-\frac{(6cd+be)b^2}{a}+fb^2+5ceb+6c^2d-6acf\right)x^4-\frac{2\left(b^2-4ac\right)(bd-ae)x^2}{a}+2\left(b^2-4ac\right)d}{x^4\left(cx^4+bx^2+a\right)}dx}{2a\left(b^2-4ac\right)}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}-\frac{b^2(be+4cd)}{a}-2acf+b^2f+3bce+2c^2d\right)+cx^2\left(2a^2ce-ab^2e-ab(3cd-af)+b^3d\right)\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 2195$$

$$\frac{\int \left(-\frac{2\left(4ac-b^2\right)d}{ax^4}+\frac{5db^4-3aeb^3-a(24cd-af)b^2+13a^2ceb+c\left(5db^3-3aeb^2-a(19cd-af)b+10a^2ce\right)x^2+2a^2c(7cd-3af)}{a^2\left(cx^4+bx^2+a\right)}-\frac{2\left(4ac-b^2\right)(ae-2bd)}{a^2x^2}\right.}{2a\left(b^2-4ac\right)}$$
$$\frac{x\left(a^2\left(\frac{b^4d}{a^2}-\frac{b^2(be+4cd)}{a}-2acf+b^2f+3bce+2c^2d\right)+cx^2\left(2a^2ce-ab^2e-ab(3cd-af)+b^3d\right)\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(2a^2c\left(5e\sqrt{b^2-4ac}-6af+14cd\right)-ab^2\left(3e\sqrt{b^2-4ac}-af+29cd\right)-ab\left(19cd\sqrt{b^2-4ac}-af\sqrt{b^2-4ac}-16ace\right)+b^3\left(5d\sqrt{b^2-\right.\right.}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}}{}$$
$$\frac{x\left(a^2\left(\frac{b^4d}{a^2}-\frac{b^2(be+4cd)}{a}-2acf+b^2f+3bce+2c^2d\right)+cx^2\left(2a^2ce-ab^2e-ab(3cd-af)+b^3d\right)\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input
```
Int[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

3.73. $\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)^2}\,dx$

output
```
(x*(a^2*((b^4*d)/a^2 + 2*c^2*d + 3*b*c*e - (b^2*(4*c*d + b*e))/a + b^2*f -
 2*a*c*f) + c*(b^3*d - a*b^2*e + 2*a^2*c*e - a*b*(3*c*d - a*f))*x^2))/(2*a
^3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((-2*(b^2 - 4*a*c)*d)/(3*a*x^3) +
(2*(b^2 - 4*a*c)*(2*b*d - a*e))/(a^2*x) + (Sqrt[c]*(5*b^4*d + b^3*(5*Sqrt[
b^2 - 4*a*c]*d - 3*a*e) + 2*a^2*c*(14*c*d + 5*Sqrt[b^2 - 4*a*c]*e - 6*a*f)
 - a*b^2*(29*c*d + 3*Sqrt[b^2 - 4*a*c]*e - a*f) - a*b*(19*c*Sqrt[b^2 - 4*a
*c]*d - 16*a*c*e - a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) - (Sqrt[c]*(5*b^4*d - b^3*(5*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + 2*
a^2*c*(14*c*d - 5*Sqrt[b^2 - 4*a*c]*e - 6*a*f) - a*b^2*(29*c*d - 3*Sqrt[b^
2 - 4*a*c]*e - a*f) + a*b*(19*c*Sqrt[b^2 - 4*a*c]*d + 16*a*c*e - a*Sqrt[b^
2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(S
qrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*(b^2 - 4*a
*c))
```

3.73.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

3.73.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 570, normalized size of antiderivative = 0.99

method	result
default	$-\frac{d}{3a^2x^3}-\frac{ae-2bd}{a^3x}+\frac{-\frac{c\left(a^2bf+2a^2ce-a\,b^2e-3abcd+b^3d\right)x^3}{2\left(4ac-b^2\right)}+\frac{\left(2a^3cf-a^2b^2f-3a^2bce-2a^2c^2d+a\,b^3e+4a\,b^2cd-d\,b^4\right)x}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{\left(-a^2bf\sqrt{-}\right.}{}\right.}{}$
risch	Expression too large to display

input
```
int((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*d/a^2/x^3-(a*e-2*b*d)/a^3/x+1/a^3*((-1/2*c*(a^2*b*f+2*a^2*c*e-a*b^2*e
-3*a*b*c*d+b^3*d)/(4*a*c-b^2)*x^3+1/2*(2*a^3*c*f-a^2*b^2*f-3*a^2*b*c*e-2*a
^2*c^2*d+a*b^3*e+4*a*b^2*c*d-b^4*d)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+2/(4*a*
c-b^2)*c*(1/8*(-a^2*b*f*(-4*a*c+b^2)^(1/2)-10*a^2*c*e*(-4*a*c+b^2)^(1/2)+3
*a*b^2*e*(-4*a*c+b^2)^(1/2)+19*a*b*c*d*(-4*a*c+b^2)^(1/2)-5*b^3*d*(-4*a*c+
b^2)^(1/2)-12*a^3*c*f+a^2*b^2*f+16*a^2*b*c*e+28*a^2*c^2*d-3*a*b^3*e-29*a*b
^2*c*d+5*d*b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(-a^2*b*f*(-4*a
*c+b^2)^(1/2)-10*a^2*c*e*(-4*a*c+b^2)^(1/2)+3*a*b^2*e*(-4*a*c+b^2)^(1/2)+1
9*a*b*c*d*(-4*a*c+b^2)^(1/2)-5*b^3*d*(-4*a*c+b^2)^(1/2)+12*a^3*c*f-a^2*b^2
*f-16*a^2*b*c*e-28*a^2*c^2*d+3*a*b^3*e+29*a*b^2*c*d-5*d*b^4)/(-4*a*c+b^2)^
(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

3.73.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19333 vs. 2(510) = 1020.

Time = 88.78 (sec) , antiderivative size = 19333, normalized size of antiderivative = 33.62

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.73.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{x^4(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x**4+e*x**2+d)/x**4/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.73.7 Maxima [F]

$$\int \frac{d+ex^2+fx^4}{x^4(a+bx^2+cx^4)^2}\,dx = \int \frac{fx^4+ex^2+d}{(cx^4+bx^2+a)^2x^4}\,dx$$

input
```
integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/6*(3*(a^2*b*c*f + (5*b^3*c - 19*a*b*c^2)*d - (3*a*b^2*c - 10*a^2*c^2)*e)
*x^6 + ((15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d - 3*(3*a*b^3 - 11*a^2*b*c)*e
+ 3*(a^2*b^2 - 2*a^3*c)*f)*x^4 + 2*(5*(a*b^3 - 4*a^2*b*c)*d - 3*(a^2*b^2 -
 4*a^3*c)*e)*x^2 - 2*(a^2*b^2 - 4*a^3*c)*d)/((a^3*b^2*c - 4*a^4*c^2)*x^7 +
 (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3) + 1/2*integrate(((a^
2*b*c*f + (5*b^3*c - 19*a*b*c^2)*d - (3*a*b^2*c - 10*a^2*c^2)*e)*x^2 + (5*
b^4 - 24*a*b^2*c + 14*a^2*c^2)*d - (3*a*b^3 - 13*a^2*b*c)*e + (a^2*b^2 - 6
*a^3*c)*f)/(c*x^4 + b*x^2 + a), x)/(a^3*b^2 - 4*a^4*c)
```

3.73.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8649 vs. $2(510) = 1020$.

Time = 1.65 (sec) , antiderivative size = 8649, normalized size of antiderivative = 15.04

$$\int \frac{d+ex^2+fx^4}{x^4(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b^3*c*d*x^3 - 3*a*b*c^2*d*x^3 - a*b^2*c*e*x^3 + 2*a^2*c^2*e*x^3 + a^2
*b*c*f*x^3 + b^4*d*x - 4*a*b^2*c*d*x + 2*a^2*c^2*d*x - a*b^3*e*x + 3*a^2*b
*c*e*x + a^2*b^2*f*x - 2*a^3*c*f*x)/((a^3*b^2 - 4*a^4*c)*(c*x^4 + b*x^2 +
a)) + 1/16*((10*b^5*c^2 - 78*a*b^3*c^3 + 152*a^2*b*c^4 - 5*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 39*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 76*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 38*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 10*(b^2 - 4*a*c)*b^3*c^2 + 38*(b^2 - 4*a*c
)*a*b*c^3)*(a^3*b^2 - 4*a^4*c)^2*d - (6*a*b^4*c^2 - 44*a^2*b^2*c^3 + 80*a^
3*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4
+ 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c +
 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 40*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 20*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 3*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 10*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 6*(b^2 - 4*a
*c)*a*b^2*c^2 + 20*(b^2 - 4*a*c)*a^2*c^3)*(a^3*b^2 - 4*a^4*c)^2*e + (2*...
```

3.73.9 Mupad [B] (verification not implemented)

Time = 12.50 (sec) , antiderivative size = 36097, normalized size of antiderivative = 62.78

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)^2),x)
```

output
```
atan(((x*(204800*a^17*c^9*e^2 - 401408*a^16*c^10*d^2 - 73728*a^18*c^8*f^2
+ 400*a^9*b^14*c^3*d^2 - 9440*a^10*b^12*c^4*d^2 + 92816*a^11*b^10*c^5*d^2
- 488096*a^12*b^8*c^6*d^2 + 1458688*a^13*b^6*c^7*d^2 - 2401280*a^14*b^4*c^
8*d^2 + 1871872*a^15*b^2*c^9*d^2 + 144*a^11*b^12*c^3*e^2 - 3264*a^12*b^10*
c^4*e^2 + 30112*a^13*b^8*c^5*e^2 - 143360*a^14*b^6*c^6*e^2 + 365568*a^15*b
^4*c^7*e^2 - 458752*a^16*b^2*c^8*e^2 + 16*a^13*b^10*c^3*f^2 - 416*a^14*b^8
*c^4*f^2 + 4608*a^15*b^6*c^5*f^2 - 25600*a^16*b^4*c^6*f^2 + 69632*a^17*b^2
*c^7*f^2 + 344064*a^17*c^9*d*f - 1236992*a^16*b*c^9*d*e + 237568*a^17*b*c^
8*e*f - 480*a^10*b^13*c^3*d*e + 11104*a^11*b^11*c^4*d*e - 105824*a^12*b^9*
c^5*d*e + 530432*a^13*b^7*c^6*d*e - 1469440*a^14*b^5*c^7*d*e + 2121728*a^1
5*b^3*c^8*d*e + 160*a^11*b^12*c^3*d*f - 3968*a^12*b^10*c^4*d*f + 39488*a^1
3*b^8*c^5*d*f - 200704*a^14*b^6*c^6*d*f + 542720*a^15*b^4*c^7*d*f - 720896
*a^16*b^2*c^8*d*f - 96*a^12*b^11*c^3*e*f + 2336*a^13*b^9*c^4*e*f - 22528*a
^14*b^7*c^5*e*f + 107520*a^15*b^5*c^6*e*f - 253952*a^16*b^3*c^7*e*f) + (-(
25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b
^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 -
 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1
/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 1169
28*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a
^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)...
```

3.73. $\int \frac{d+ex^2+fx^4}{x^4(a+bx^2+cx^4)^2}\,dx$

3.74 $\int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$

3.74.1 Optimal result

Integrand size = 31, antiderivative size = 68

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx = -\frac{293x^2}{2}+\frac{49x^4}{2}-\frac{9x^6}{2}+\frac{5x^8}{8}+\frac{414+415x^2}{2(2+3x^2+x^4)} + 2\log(1+x^2)+392\log(2+x^2)$$

output
```
-293/2*x^2+49/2*x^4-9/2*x^6+5/8*x^8+1/2*(415*x^2+414)/(x^4+3*x^2+2)+2*ln(x
^2+1)+392*ln(x^2+2)
```

3.74.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx = \frac{1}{8}\left(-1172x^2+196x^4-36x^6+5x^8+\frac{4(414+415x^2)}{2+3x^2+x^4} + 16\log(1+x^2)+3136\log(2+x^2)\right)$$

input
```
Integrate[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
(-1172*x^2 + 196*x^4 - 36*x^6 + 5*x^8 + (4*(414 + 415*x^2))/(2 + 3*x^2 + x
^4) + 16*Log[1 + x^2] + 3136*Log[2 + x^2])/8
```

3.74.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2194, 2191, 25, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9(5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^2}dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{x^8(5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^2}dx^2$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{2}\left(\frac{415x^2+414}{x^4+3x^2+2}-\int -\frac{5x^{10}-12x^8+27x^6-53x^4+105x^2+206}{x^4+3x^2+2}dx^2\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\int \frac{5x^{10}-12x^8+27x^6-53x^4+105x^2+206}{x^4+3x^2+2}dx^2+\frac{415x^2+414}{x^4+3x^2+2}\right)$$

$$\Big\downarrow \text{2188}$$

$$\frac{1}{2}\left(\int \left(5x^6-27x^4+98x^2+\frac{4(197x^2+198)}{x^4+3x^2+2}-293\right)dx^2+\frac{415x^2+414}{x^4+3x^2+2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{5x^8}{4}-9x^6+49x^4-293x^2+4\log\left(x^2+1\right)+784\log\left(x^2+2\right)+\frac{415x^2+414}{x^4+3x^2+2}\right)$$

input
```
Int[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
(-293*x^2 + 49*x^4 - 9*x^6 + (5*x^8)/4 + (414 + 415*x^2)/(2 + 3*x^2 + x^4)
 + 4*Log[1 + x^2] + 784*Log[2 + x^2])/2
```

3.74.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.74.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

method	result
default	$392\ln\left(x^2+2\right)+\frac{208}{x^2+2}+\frac{5x^8}{8}-\frac{9x^6}{2}+\frac{49x^4}{2}-\frac{293x^2}{2}+2\ln\left(x^2+1\right)-\frac{1}{2(x^2+1)}$
norman	$\frac{1086x^2-82x^6+\frac{49}{4}x^8-\frac{21}{8}x^{10}+\frac{5}{8}x^{12}+988}{x^4+3x^2+2}+2\ln\left(x^2+1\right)+392\ln\left(x^2+2\right)$
risch	$\frac{5x^8}{8}-\frac{9x^6}{2}+\frac{49x^4}{2}-\frac{293x^2}{2}+\frac{\frac{415x^2}{2}+207}{x^4+3x^2+2}+2\ln\left(x^2+1\right)+392\ln\left(x^2+2\right)$
parallelrisch	$\frac{5x^{12}-21x^{10}+98x^8-656x^6+16\ln(x^2+1)x^4+3136\ln(x^2+2)x^4+7904+48\ln(x^2+1)x^2+9408\ln(x^2+2)x^2+8688x^2+32\ln(x^2+}{8x^4+24x^2+16}$

input
```
int(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
392*ln(x^2+2)+208/(x^2+2)+5/8*x^8-9/2*x^6+49/2*x^4-293/2*x^2+2*ln(x^2+1)-1
/2/(x^2+1)
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.21

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$$
$$= \frac{5\,x^{12} - 21\,x^{10} + 98\,x^8 - 656\,x^6 - 3124\,x^4 - 684\,x^2 + 3136\,(x^4 + 3\,x^2 + 2)\log\left(x^2 + 2\right) + 16\,(x^4 + 3\,x^2 + 2}{8\,(x^4 + 3\,x^2 + 2)}$$

input
```
integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
1/8*(5*x^12 - 21*x^10 + 98*x^8 - 656*x^6 - 3124*x^4 - 684*x^2 + 3136*(x^4
+ 3*x^2 + 2)*log(x^2 + 2) + 16*(x^4 + 3*x^2 + 2)*log(x^2 + 1) + 1656)/(x^4
 + 3*x^2 + 2)
```

3.74.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5x^8}{8} - \frac{9x^6}{2} + \frac{49x^4}{2} - \frac{293x^2}{2} + \frac{415x^2+414}{2x^4+6x^2+4} + 2\log\left(x^2+1\right) + 392\log\left(x^2+2\right)$$

input
```
integrate(x**9*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

output
```
5*x**8/8 - 9*x**6/2 + 49*x**4/2 - 293*x**2/2 + (415*x**2 + 414)/(2*x**4 +
6*x**2 + 4) + 2*log(x**2 + 1) + 392*log(x**2 + 2)
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.85

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{8}\,x^8 - \frac{9}{2}\,x^6 + \frac{49}{2}\,x^4 - \frac{293}{2}\,x^2 + \frac{415\,x^2+414}{2\,(x^4+3\,x^2+2)} + 392\,\log\left(x^2+2\right) + 2\,\log\left(x^2+1\right)$$

input
```
integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
5/8*x^8 - 9/2*x^6 + 49/2*x^4 - 293/2*x^2 + 1/2*(415*x^2 + 414)/(x^4 + 3*x^
2 + 2) + 392*log(x^2 + 2) + 2*log(x^2 + 1)
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{8}\,x^8 - \frac{9}{2}\,x^6 + \frac{49}{2}\,x^4 - \frac{293}{2}\,x^2 - \frac{394\,x^4+767\,x^2+374}{2\,(x^4+3\,x^2+2)} + 392\,\log\left(x^2+2\right) + 2\,\log\left(x^2+1\right)$$

input
```
integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
5/8*x^8 - 9/2*x^6 + 49/2*x^4 - 293/2*x^2 - 1/2*(394*x^4 + 767*x^2 + 374)/(
x^4 + 3*x^2 + 2) + 392*log(x^2 + 2) + 2*log(x^2 + 1)
```

3.74.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = 2\,\ln\left(x^2+1\right) + 392\,\ln\left(x^2+2\right) + \frac{\frac{415\,x^2}{2}+207}{x^4+3\,x^2+2} - \frac{293\,x^2}{2} + \frac{49\,x^4}{2} - \frac{9\,x^6}{2} + \frac{5\,x^8}{8}$$

input
```
int((x^9*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

output
```
2*log(x^2 + 1) + 392*log(x^2 + 2) + ((415*x^2)/2 + 207)/(3*x^2 + x^4 + 2)
- (293*x^2)/2 + (49*x^4)/2 - (9*x^6)/2 + (5*x^8)/8
```

3.75 $\int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

3.75.1 Optimal result

Integrand size = 31, antiderivative size = 61

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = 49x^2-\frac{27x^4}{4}+\frac{5x^6}{6}-\frac{206+207x^2}{2(2+3x^2+x^4)} - \frac{5}{2}\log\left(1+x^2\right)-144\log\left(2+x^2\right)$$

output
```
49*x^2-27/4*x^4+5/6*x^6+1/2*(-207*x^2-206)/(x^4+3*x^2+2)-5/2*ln(x^2+1)-144
*ln(x^2+2)
```

3.75.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = 49x^2-\frac{27x^4}{4}+\frac{5x^6}{6}+\frac{-206-207x^2}{2(2+3x^2+x^4)} - \frac{5}{2}\log\left(1+x^2\right)-144\log\left(2+x^2\right)$$

input
```
Integrate[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
49*x^2 - (27*x^4)/4 + (5*x^6)/6 + (-206 - 207*x^2)/(2*(2 + 3*x^2 + x^4)) -
 (5*Log[1 + x^2])/2 - 144*Log[2 + x^2]
```

3.75.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2194, 2191, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 (5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^2} \, dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2} \int \frac{x^6 (5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^2} dx^2$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{2} \left(-\int \frac{-5x^8+12x^6-27x^4+53x^2+102}{x^4+3x^2+2} dx^2 - \frac{207x^2+206}{x^4+3x^2+2}\right)$$

$$\Big\downarrow \text{2188}$$

$$\frac{1}{2} \left(-\int \left(-5x^4+27x^2+\frac{293x^2+298}{x^4+3x^2+2}-98\right) dx^2 - \frac{207x^2+206}{x^4+3x^2+2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{5x^6}{3} - \frac{27x^4}{2} + 98x^2 - 5\log\left(x^2+1\right) - 288\log\left(x^2+2\right) - \frac{207x^2+206}{x^4+3x^2+2}\right)$$

input
```
Int[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
(98*x^2 - (27*x^4)/2 + (5*x^6)/3 - (206 + 207*x^2)/(2 + 3*x^2 + x^4) - 5*L
og[1 + x^2] - 288*Log[2 + x^2])/2
```

3.75.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.75.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

method	result
default	$-144\ln\left(x^{2}+2\right)-\frac{104}{x^{2}+2}+\frac{5x^{6}}{6}-\frac{27x^{4}}{4}+49x^{2}-\frac{5\ln\left(x^{2}+1\right)}{2}+\frac{1}{2x^{2}+2}$
norman	$\frac{-406x^{2}+\frac{365}{12}x^{6}-\frac{17}{4}x^{8}+\frac{5}{6}x^{10}-370}{x^{4}+3x^{2}+2}-\frac{5\ln\left(x^{2}+1\right)}{2}-144\ln\left(x^{2}+2\right)$
risch	$\frac{5x^{6}}{6}-\frac{27x^{4}}{4}+49x^{2}+\frac{-\frac{207x^{2}}{2}-103}{x^{4}+3x^{2}+2}-\frac{5\ln\left(x^{2}+1\right)}{2}-144\ln\left(x^{2}+2\right)$
parallelrisch	$-\frac{-10x^{10}+51x^{8}-365x^{6}+30\ln\left(x^{2}+1\right)x^{4}+1728\ln\left(x^{2}+2\right)x^{4}+4440+90\ln\left(x^{2}+1\right)x^{2}+5184\ln\left(x^{2}+2\right)x^{2}+4872x^{2}+60\ln\left(x^{2}+1\right)}{12\left(x^{4}+3x^{2}+2\right)}$

input
```
int(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
-144*ln(x^2+2)-104/(x^2+2)+5/6*x^6-27/4*x^4+49*x^2-5/2*ln(x^2+1)+1/2/(x^2+
1)
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.26

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$$
$$= \frac{10\,x^{10}-51\,x^8+365\,x^6+1602\,x^4-66\,x^2-1728\,(x^4+3\,x^2+2)\log(x^2+2)-30\,(x^4+3\,x^2+2)\log(x^2}{12\,(x^4+3\,x^2+2)}$$

input
```
integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
1/12*(10*x^10 - 51*x^8 + 365*x^6 + 1602*x^4 - 66*x^2 - 1728*(x^4 + 3*x^2 +
 2)*log(x^2 + 2) - 30*(x^4 + 3*x^2 + 2)*log(x^2 + 1) - 1236)/(x^4 + 3*x^2
+ 2)
```

3.75.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5x^6}{6}-\frac{27x^4}{4}+49x^2+\frac{-207x^2-206}{2x^4+6x^2+4} - \frac{5\log(x^2+1)}{2}-144\log\left(x^2+2\right)$$

input
```
integrate(x**7*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

output
```
5*x**6/6 - 27*x**4/4 + 49*x**2 + (-207*x**2 - 206)/(2*x**4 + 6*x**2 + 4) -
 5*log(x**2 + 1)/2 - 144*log(x**2 + 2)
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{6}\,x^6 - \frac{27}{4}\,x^4 + 49\,x^2 - \frac{207\,x^2+206}{2\,(x^4+3\,x^2+2)} - 144\,\log\left(x^2+2\right) - \frac{5}{2}\,\log\left(x^2+1\right)$$

input
```
integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
5/6*x^6 - 27/4*x^4 + 49*x^2 - 1/2*(207*x^2 + 206)/(x^4 + 3*x^2 + 2) - 144*
log(x^2 + 2) - 5/2*log(x^2 + 1)
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.95

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{6}\,x^6 - \frac{27}{4}\,x^4 + 49\,x^2 + \frac{293\,x^4+465\,x^2+174}{4\,(x^4+3\,x^2+2)} - 144\,\log\left(x^2+2\right) - \frac{5}{2}\,\log\left(x^2+1\right)$$

input
```
integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
5/6*x^6 - 27/4*x^4 + 49*x^2 + 1/4*(293*x^4 + 465*x^2 + 174)/(x^4 + 3*x^2 +
 2) - 144*log(x^2 + 2) - 5/2*log(x^2 + 1)
```

3.75.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = 49\,x^2 - 144\,\ln\left(x^2+2\right) - \frac{\frac{207\,x^2}{2}+103}{x^4+3\,x^2+2} - \frac{5\,\ln\left(x^2+1\right)}{2} - \frac{27\,x^4}{4} + \frac{5\,x^6}{6}$$

input
```
int((x^7*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

output
```
49*x^2 - 144*log(x^2 + 2) - ((207*x^2)/2 + 103)/(3*x^2 + x^4 + 2) - (5*log
(x^2 + 1))/2 - (27*x^4)/4 + (5*x^6)/6
```

3.75. $\int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

3.76 $\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

3.76.1 Optimal result

Integrand size = 31, antiderivative size = 54

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = -\frac{27x^2}{2}+\frac{5x^4}{4}+\frac{102+103x^2}{2(2+3x^2+x^4)}+3\log(1+x^2)+46\log(2+x^2)$$

output
```
-27/2*x^2+5/4*x^4+1/2*(103*x^2+102)/(x^4+3*x^2+2)+3*ln(x^2+1)+46*ln(x^2+2)
```

3.76.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = -\frac{27x^2}{2}+\frac{5x^4}{4}+\frac{102+103x^2}{2(2+3x^2+x^4)}+3\log(1+x^2)+46\log(2+x^2)$$

input
```
Integrate[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
(-27*x^2)/2 + (5*x^4)/4 + (102 + 103*x^2)/(2*(2 + 3*x^2 + x^4)) + 3*Log[1
+ x^2] + 46*Log[2 + x^2]
```

3.76.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2194, 2191, 25, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^2} \, dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2} \int \frac{x^4 (5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^2} dx^2$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{2} \left(\frac{103x^2+102}{x^4+3x^2+2} - \int -\frac{5x^6-12x^4+27x^2+50}{x^4+3x^2+2} dx^2\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2} \left(\int \frac{5x^6-12x^4+27x^2+50}{x^4+3x^2+2} dx^2 + \frac{103x^2+102}{x^4+3x^2+2}\right)$$

$$\Big\downarrow \text{2188}$$

$$\frac{1}{2} \left(\int \left(5x^2 + \frac{2(49x^2+52)}{x^4+3x^2+2} - 27\right) dx^2 + \frac{103x^2+102}{x^4+3x^2+2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{5x^4}{2} - 27x^2 + 6\log(x^2+1) + 92\log(x^2+2) + \frac{103x^2+102}{x^4+3x^2+2}\right)$$

input
```
Int[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
(-27*x^2 + (5*x^4)/2 + (102 + 103*x^2)/(2 + 3*x^2 + x^4) + 6*Log[1 + x^2]
+ 92*Log[2 + x^2])/2
```

3.76.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.76.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

method	result
default	$46\ln\left(x^2+2\right)+\frac{52}{x^2+2}+\frac{5x^4}{4}-\frac{27x^2}{2}+3\ln\left(x^2+1\right)-\frac{1}{2(x^2+1)}$
norman	$\frac{\frac{277}{2}x^2-\frac{39}{4}x^6+\frac{5}{4}x^8+127}{x^4+3x^2+2}+3\ln\left(x^2+1\right)+46\ln\left(x^2+2\right)$
risch	$\frac{5x^4}{4}-\frac{27x^2}{2}+\frac{729}{20}+\frac{\frac{103x^2}{2}+51}{x^4+3x^2+2}+3\ln\left(x^2+1\right)+46\ln\left(x^2+2\right)$
parallelrisch	$\frac{5x^8-39x^6+12\ln(x^2+1)x^4+184\ln(x^2+2)x^4+508+36\ln(x^2+1)x^2+552\ln(x^2+2)x^2+554x^2+24\ln(x^2+1)+368\ln(x^2+2)}{4x^4+12x^2+8}$

input
```
int(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
46*ln(x^2+2)+52/(x^2+2)+5/4*x^4-27/2*x^2+3*ln(x^2+1)-1/2/(x^2+1)
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$$
$$= \frac{5\,x^8 - 39\,x^6 - 152\,x^4 + 98\,x^2 + 184\,(x^4+3\,x^2+2)\log\left(x^2+2\right) + 12\,(x^4+3\,x^2+2)\log\left(x^2+1\right) + 204}{4\,(x^4+3\,x^2+2)}$$

input
```
integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
1/4*(5*x^8 - 39*x^6 - 152*x^4 + 98*x^2 + 184*(x^4 + 3*x^2 + 2)*log(x^2 + 2
) + 12*(x^4 + 3*x^2 + 2)*log(x^2 + 1) + 204)/(x^4 + 3*x^2 + 2)
```

3.76.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5x^4}{4} - \frac{27x^2}{2} + \frac{103x^2+102}{2x^4+6x^2+4} + 3\log\left(x^2+1\right) + 46\log\left(x^2+2\right)$$

input
```
integrate(x**5*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

output
```
5*x**4/4 - 27*x**2/2 + (103*x**2 + 102)/(2*x**4 + 6*x**2 + 4) + 3*log(x**2
 + 1) + 46*log(x**2 + 2)
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{4}\,x^4 - \frac{27}{2}\,x^2 + \frac{103\,x^2+102}{2\,(x^4+3\,x^2+2)} + 46\,\log\left(x^2+2\right) + 3\,\log\left(x^2+1\right)$$

input
```
integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
5/4*x^4 - 27/2*x^2 + 1/2*(103*x^2 + 102)/(x^4 + 3*x^2 + 2) + 46*log(x^2 +
2) + 3*log(x^2 + 1)
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{4}\,x^4 - \frac{27}{2}\,x^2 - \frac{49\,x^4+44\,x^2-4}{2\,(x^4+3\,x^2+2)} + 46\,\log\left(x^2+2\right) + 3\,\log\left(x^2+1\right)$$

input
```
integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
5/4*x^4 - 27/2*x^2 - 1/2*(49*x^4 + 44*x^2 - 4)/(x^4 + 3*x^2 + 2) + 46*log(
x^2 + 2) + 3*log(x^2 + 1)
```

3.76.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = 3\,\ln\left(x^2+1\right) + 46\,\ln\left(x^2+2\right) + \frac{\frac{103\,x^2}{2}+51}{x^4+3\,x^2+2} - \frac{27\,x^2}{2} + \frac{5\,x^4}{4}$$

input
```
int((x^5*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

output
```
3*log(x^2 + 1) + 46*log(x^2 + 2) + ((103*x^2)/2 + 51)/(3*x^2 + x^4 + 2) -
(27*x^2)/2 + (5*x^4)/4
```

3.77 $\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

3.77.1 Optimal result

Integrand size = 31, antiderivative size = 49

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5x^2}{2} - \frac{50+51x^2}{2(2+3x^2+x^4)} - \frac{7}{2}\log(1+x^2) - 10\log(2+x^2)$$

output
```
5/2*x^2+1/2*(-51*x^2-50)/(x^4+3*x^2+2)-7/2*ln(x^2+1)-10*ln(x^2+2)
```

3.77.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5x^2}{2} + \frac{-50-51x^2}{2(2+3x^2+x^4)} - \frac{7}{2}\log(1+x^2) - 10\log(2+x^2)$$

input
```
Integrate[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
(5*x^2)/2 + (-50 - 51*x^2)/(2*(2 + 3*x^2 + x^4)) - (7*Log[1 + x^2])/2 - 10
*Log[2 + x^2]
```

3.77.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2194, 2191, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^2} dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{x^2 (5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^2} dx^2$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{2}\left(-\int \frac{-5x^4+12x^2+24}{x^4+3x^2+2} dx^2-\frac{51x^2+50}{x^4+3x^2+2}\right)$$

$$\Big\downarrow \text{2188}$$

$$\frac{1}{2}\left(-\int \left(\frac{27x^2+34}{x^4+3x^2+2}-5\right) dx^2-\frac{51x^2+50}{x^4+3x^2+2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(5x^2-7\log\left(x^2+1\right)-20\log\left(x^2+2\right)-\frac{51x^2+50}{x^4+3x^2+2}\right)$$

input
```
Int[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
(5*x^2 - (50 + 51*x^2)/(2 + 3*x^2 + x^4) - 7*Log[1 + x^2] - 20*Log[2 + x^2
])/2
```

3.77.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.77.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

method	result	size
default	$-10\ln\left(x^{2}+2\right)-\frac{26}{x^{2}+2}+\frac{5x^{2}}{2}-\frac{7\ln\left(x^{2}+1\right)}{2}+\frac{1}{2x^{2}+2}$	41
norman	$\frac{-43x^{2}+\frac{5}{2}x^{6}-40}{x^{4}+3x^{2}+2}-\frac{7\ln\left(x^{2}+1\right)}{2}-10\ln\left(x^{2}+2\right)$	43
risch	$\frac{5x^{2}}{2}+\frac{-\frac{51x^{2}}{2}-25}{x^{4}+3x^{2}+2}-\frac{7\ln\left(x^{2}+1\right)}{2}-10\ln\left(x^{2}+2\right)$	43
parallelrisch	$-\frac{-5x^{6}+7\ln\left(x^{2}+1\right)x^{4}+20\ln\left(x^{2}+2\right)x^{4}+80+21\ln\left(x^{2}+1\right)x^{2}+60\ln\left(x^{2}+2\right)x^{2}+86x^{2}+14\ln\left(x^{2}+1\right)+40\ln\left(x^{2}+2\right)}{2\left(x^{4}+3x^{2}+2\right)}$	87

input
```
int(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
-10*ln(x^2+2)-26/(x^2+2)+5/2*x^2-7/2*ln(x^2+1)+1/2/(x^2+1)
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.37

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5\,x^6+15\,x^4-41\,x^2-20\,(x^4+3\,x^2+2)\log(x^2+2)-7\,(x^4+3\,x^2+2)\log(x^2+1)-50}{2\,(x^4+3\,x^2+2)}$$

input
```
integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
1/2*(5*x^6 + 15*x^4 - 41*x^2 - 20*(x^4 + 3*x^2 + 2)*log(x^2 + 2) - 7*(x^4
+ 3*x^2 + 2)*log(x^2 + 1) - 50)/(x^4 + 3*x^2 + 2)
```

3.77.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5x^2}{2}+\frac{-51x^2-50}{2x^4+6x^2+4}-\frac{7\log(x^2+1)}{2}-10\log(x^2+2)$$

input
```
integrate(x**3*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

output
```
5*x**2/2 + (-51*x**2 - 50)/(2*x**4 + 6*x**2 + 4) - 7*log(x**2 + 1)/2 - 10*
log(x**2 + 2)
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{2}\,x^2-\frac{51\,x^2+50}{2\,(x^4+3\,x^2+2)}-10\,\log(x^2+2)-\frac{7}{2}\,\log(x^2+1)$$

input
```
integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
5/2*x^2 - 1/2*(51*x^2 + 50)/(x^4 + 3*x^2 + 2) - 10*log(x^2 + 2) - 7/2*log(
x^2 + 1)
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} \, dx = \frac{5}{2} \, x^2 - \frac{51 \, x^2 + 50}{2 \, (x^2+2)(x^2+1)} - 10 \, \log \left(x^2+2\right) - \frac{7}{2} \, \log \left(x^2+1\right)$$

input
```
integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
5/2*x^2 - 1/2*(51*x^2 + 50)/((x^2 + 2)*(x^2 + 1)) - 10*log(x^2 + 2) - 7/2*
log(x^2 + 1)
```

3.77.9 Mupad [B] (verification not implemented)

Time = 8.49 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} \, dx = \frac{5 \, x^2}{2} - 10 \, \ln \left(x^2+2\right) - \frac{\frac{51 \, x^2}{2}+25}{x^4+3 \, x^2+2} - \frac{7 \, \ln \left(x^2+1\right)}{2}$$

input
```
int((x^3*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

output
```
(5*x^2)/2 - 10*log(x^2 + 2) - ((51*x^2)/2 + 25)/(3*x^2 + x^4 + 2) - (7*log
(x^2 + 1))/2
```

3.78 $\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

3.78.1 Optimal result

Integrand size = 29, antiderivative size = 42

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{24+25x^2}{2(2+3x^2+x^4)} + 4\log(1+x^2) - \frac{3}{2}\log(2+x^2)$$

output
```
1/2*(25*x^2+24)/(x^4+3*x^2+2)+4*ln(x^2+1)-3/2*ln(x^2+2)
```

3.78.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{24+25x^2}{2(2+3x^2+x^4)} + 4\log(1+x^2) - \frac{3}{2}\log(2+x^2)$$

input
```
Integrate[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
(24 + 25*x^2)/(2*(2 + 3*x^2 + x^4)) + 4*Log[1 + x^2] - (3*Log[2 + x^2])/2
```

3.78.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2194, 2191, 25, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^2}dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{5x^6+3x^4+x^2+4}{(x^4+3x^2+2)^2}dx^2$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{2}\left(\frac{25x^2+24}{x^4+3x^2+2}-\int -\frac{5x^2+13}{x^4+3x^2+2}dx^2\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\int \frac{5x^2+13}{x^4+3x^2+2}dx^2+\frac{25x^2+24}{x^4+3x^2+2}\right)$$

$$\Big\downarrow \text{1141}$$

$$\frac{1}{2}\left(\int \left(\frac{8}{x^2+1}-\frac{3}{x^2+2}\right)dx^2+\frac{25x^2+24}{x^4+3x^2+2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(8\log(x^2+1)-3\log(x^2+2)+\frac{25x^2+24}{x^4+3x^2+2}\right)$$

input
```
Int[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
((24 + 25*x^2)/(2 + 3*x^2 + x^4) + 8*Log[1 + x^2] - 3*Log[2 + x^2])/2
```

3.78.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.78. $\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} \, dx$

3.78.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{3\ln(x^2+2)}{2}+\frac{13}{x^2+2}+4\ln(x^2+1)-\frac{1}{2(x^2+1)}$	36
norman	$\frac{\frac{25x^2}{2}+12}{x^4+3x^2+2}+4\ln(x^2+1)-\frac{3\ln(x^2+2)}{2}$	38
risch	$\frac{\frac{25x^2}{2}+12}{x^4+3x^2+2}+4\ln(x^2+1)-\frac{3\ln(x^2+2)}{2}$	38
parallelrisch	$\frac{8\ln(x^2+1)x^4-3\ln(x^2+2)x^4+24+24\ln(x^2+1)x^2-9\ln(x^2+2)x^2+25x^2+16\ln(x^2+1)-6\ln(x^2+2)}{2x^4+6x^2+4}$	82

input
```
int(x*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
-3/2*ln(x^2+2)+13/(x^2+2)+4*ln(x^2+1)-1/2/(x^2+1)
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.36

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{25\,x^2-3\,(x^4+3\,x^2+2)\log(x^2+2)+8\,(x^4+3\,x^2+2)\log(x^2+1)+24}{2\,(x^4+3\,x^2+2)}$$

input
```
integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
1/2*(25*x^2 - 3*(x^4 + 3*x^2 + 2)*log(x^2 + 2) + 8*(x^4 + 3*x^2 + 2)*log(x
^2 + 1) + 24)/(x^4 + 3*x^2 + 2)
```

3.78.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{25x^2+24}{2x^4+6x^2+4} + 4\log\left(x^2+1\right) - \frac{3\log\left(x^2+2\right)}{2}$$

input
```
integrate(x*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

output
```
(25*x**2 + 24)/(2*x**4 + 6*x**2 + 4) + 4*log(x**2 + 1) - 3*log(x**2 + 2)/2
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{25\,x^2+24}{2\,(x^4+3\,x^2+2)} - \frac{3}{2}\,\log\left(x^2+2\right) + 4\,\log\left(x^2+1\right)$$

input
```
integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
1/2*(25*x^2 + 24)/(x^4 + 3*x^2 + 2) - 3/2*log(x^2 + 2) + 4*log(x^2 + 1)
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{25\,x^2+24}{2\,(x^2+2)(x^2+1)} - \frac{3}{2}\,\log\left(x^2+2\right) + 4\,\log\left(x^2+1\right)$$

input
```
integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
1/2*(25*x^2 + 24)/((x^2 + 2)*(x^2 + 1)) - 3/2*log(x^2 + 2) + 4*log(x^2 + 1
)
```

3.78.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = 4\,\ln\left(x^2+1\right) - \frac{3\,\ln\left(x^2+2\right)}{2} + \frac{\frac{25\,x^2}{2}+12}{x^4+3\,x^2+2}$$

input
```
int((x*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

output
```
4*log(x^2 + 1) - (3*log(x^2 + 2))/2 + ((25*x^2)/2 + 12)/(3*x^2 + x^4 + 2)
```

3.79 $\int \frac{4+x^2+3x^4+5x^6}{x(2+3x^2+x^4)^2} dx$

3.79.1 Optimal result

Integrand size = 31, antiderivative size = 44

$$\int \frac{4+x^2+3x^4+5x^6}{x(2+3x^2+x^4)^2} dx = -\frac{11+12x^2}{2(2+3x^2+x^4)} + \log(x) - \frac{9}{2}\log(1+x^2) + 4\log(2+x^2)$$

output
```
1/2*(-12*x^2-11)/(x^4+3*x^2+2)+ln(x)-9/2*ln(x^2+1)+4*ln(x^2+2)
```

3.79.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{4+x^2+3x^4+5x^6}{x(2+3x^2+x^4)^2} dx = \frac{-11-12x^2}{2(2+3x^2+x^4)} + \log(x) - \frac{9}{2}\log(1+x^2) + 4\log(2+x^2)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(2 + 3*x^2 + x^4)^2),x]
```

output
```
(-11 - 12*x^2)/(2*(2 + 3*x^2 + x^4)) + Log[x] - (9*Log[1 + x^2])/2 + 4*Log
[2 + x^2]
```

3.79.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2194, 2177, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x\left(x^4+3x^2+2\right)^2}dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{5x^6+3x^4+x^2+4}{x^2\left(x^4+3x^2+2\right)^2}dx^2$$

$$\Big\downarrow \text{2177}$$

$$\frac{1}{2}\left(-\int -\frac{2-7x^2}{x^2\left(x^4+3x^2+2\right)}dx^2-\frac{12x^2+11}{x^4+3x^2+2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\int \frac{2-7x^2}{x^2\left(x^4+3x^2+2\right)}dx^2-\frac{12x^2+11}{x^4+3x^2+2}\right)$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{2}\left(\int \left(\frac{8}{x^2+2}+\frac{1}{x^2}-\frac{9}{x^2+1}\right)dx^2-\frac{12x^2+11}{x^4+3x^2+2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\log\left(x^2\right)-9\log\left(x^2+1\right)+8\log\left(x^2+2\right)-\frac{12x^2+11}{x^4+3x^2+2}\right)$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(2 + 3*x^2 + x^4)^2),x]
```

output
```
(-((11 + 12*x^2)/(2 + 3*x^2 + x^4)) + Log[x^2] - 9*Log[1 + x^2] + 8*Log[2
+ x^2])/2
```

3.79.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.79.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

method	result
default	$4\ln\left(x^{2}+2\right)-\frac{13}{2\left(x^{2}+2\right)}+\ln\left(x\right)-\frac{9\ln\left(x^{2}+1\right)}{2}+\frac{1}{2x^{2}+2}$
norman	$\frac{-6x^{2}-\frac{11}{2}}{x^{4}+3x^{2}+2}-\frac{9\ln\left(x^{2}+1\right)}{2}+4\ln\left(x^{2}+2\right)+\ln\left(x\right)$
risch	$\frac{-6x^{2}-\frac{11}{2}}{x^{4}+3x^{2}+2}-\frac{9\ln\left(x^{2}+1\right)}{2}+4\ln\left(x^{2}+2\right)+\ln\left(x\right)$
parallelrisch	$\frac{2\ln(x)x^{4}-9\ln\left(x^{2}+1\right)x^{4}+8\ln\left(x^{2}+2\right)x^{4}-11+6\ln(x)x^{2}-27\ln\left(x^{2}+1\right)x^{2}+24\ln\left(x^{2}+2\right)x^{2}-12x^{2}+4\ln(x)-18\ln\left(x^{2}+1\right)+16\ln(}{2x^{4}+6x^{2}+4}$

input
```
int((5*x^6+3*x^4+x^2+4)/x/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
4*ln(x^2+2)-13/2/(x^2+2)+ln(x)-9/2*ln(x^2+1)+1/2/(x^2+1)
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.61

$$\int \frac{4+x^2+3x^4+5x^6}{x\left(2+3x^2+x^4\right)^2}\,dx = -\frac{12\,x^{2}-8\,\left(x^{4}+3\,x^{2}+2\right)\log\left(x^{2}+2\right)+9\,\left(x^{4}+3\,x^{2}+2\right)\log\left(x^{2}+1\right)-2\,\left(x^{4}+3\,x^{2}+2\right)\log\left(x\right)+11}{2\,\left(x^{4}+3\,x^{2}+2\right)}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
-1/2*(12*x^2 - 8*(x^4 + 3*x^2 + 2)*log(x^2 + 2) + 9*(x^4 + 3*x^2 + 2)*log(
x^2 + 1) - 2*(x^4 + 3*x^2 + 2)*log(x) + 11)/(x^4 + 3*x^2 + 2)
```

3.79.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{4+x^2+3x^4+5x^6}{x(2+3x^2+x^4)^2}\,dx = \frac{-12x^2-11}{2x^4+6x^2+4} + \log(x) - \frac{9\log(x^2+1)}{2} + 4\log(x^2+2)$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x/(x**4+3*x**2+2)**2,x)
```

output
```
(-12*x**2 - 11)/(2*x**4 + 6*x**2 + 4) + log(x) - 9*log(x**2 + 1)/2 + 4*log
(x**2 + 2)
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{4+x^2+3x^4+5x^6}{x(2+3x^2+x^4)^2}\,dx = -\frac{12\,x^2+11}{2\,(x^4+3\,x^2+2)} + 4\,\log(x^2+2) - \frac{9}{2}\,\log(x^2+1) + \frac{1}{2}\,\log(x^2)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
-1/2*(12*x^2 + 11)/(x^4 + 3*x^2 + 2) + 4*log(x^2 + 2) - 9/2*log(x^2 + 1) +
 1/2*log(x^2)
```

3.79.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int \frac{4+x^2+3x^4+5x^6}{x(2+3x^2+x^4)^2}\,dx = \frac{x^4-21\,x^2-20}{4\,(x^4+3\,x^2+2)} + 4\,\log(x^2+2) - \frac{9}{2}\,\log(x^2+1) + \frac{1}{2}\,\log(x^2)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
1/4*(x^4 - 21*x^2 - 20)/(x^4 + 3*x^2 + 2) + 4*log(x^2 + 2) - 9/2*log(x^2 +
 1) + 1/2*log(x^2)
```

3.79.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int \frac{4+x^2+3x^4+5x^6}{x\left(2+3x^2+x^4\right)^2}\,dx = 4\,\ln\left(x^2+2\right)-\frac{9\,\ln\left(x^2+1\right)}{2}+\ln\left(x\right)-\frac{6\,x^2+\frac{11}{2}}{x^4+3\,x^2+2}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x*(3*x^2 + x^4 + 2)^2),x)
```

output
```
4*log(x^2 + 2) - (9*log(x^2 + 1))/2 + log(x) - (6*x^2 + 11/2)/(3*x^2 + x^4
 + 2)
```

3.80 $\int \frac{4+x^2+3x^4+5x^6}{x^3(2+3x^2+x^4)^2} dx$

3.80.1 Optimal result

Integrand size = 31, antiderivative size = 55

$$\int \frac{4+x^2+3x^4+5x^6}{x^3(2+3x^2+x^4)^2} dx = -\frac{1}{2x^2}+\frac{9+11x^2}{4(2+3x^2+x^4)}-\frac{11\log(x)}{4}+5\log(1+x^2)-\frac{29}{8}\log(2+x^2)$$

output
```
-1/2/x^2+1/4*(11*x^2+9)/(x^4+3*x^2+2)-11/4*ln(x)+5*ln(x^2+1)-29/8*ln(x^2+2
)
```

3.80.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{4+x^2+3x^4+5x^6}{x^3(2+3x^2+x^4)^2} dx = \frac{1}{8}\left(-\frac{4}{x^2}+\frac{18+22x^2}{2+3x^2+x^4}-22\log(x)+40\log(1+x^2)-29\log(2+x^2)\right)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(2 + 3*x^2 + x^4)^2),x]
```

output
```
(-4/x^2 + (18 + 22*x^2)/(2 + 3*x^2 + x^4) - 22*Log[x] + 40*Log[1 + x^2] -
29*Log[2 + x^2])/8
```

3.80.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2194, 2177, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^3\left(x^4+3x^2+2\right)^2}dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{5x^6+3x^4+x^2+4}{x^4\left(x^4+3x^2+2\right)^2}dx^2$$

$$\Big\downarrow \text{2177}$$

$$\frac{1}{2}\left(\frac{11x^2+9}{2\left(x^4+3x^2+2\right)}-\int -\frac{11x^4-5x^2+4}{2x^4\left(x^4+3x^2+2\right)}dx^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{11x^4-5x^2+4}{x^4\left(x^4+3x^2+2\right)}dx^2+\frac{11x^2+9}{2\left(x^4+3x^2+2\right)}\right)$$

$$\Big\downarrow \text{2159}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \left(-\frac{29}{2\left(x^2+2\right)}-\frac{11}{2x^2}+\frac{2}{x^4}+\frac{20}{x^2+1}\right)dx^2+\frac{11x^2+9}{2\left(x^4+3x^2+2\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{2}{x^2}-\frac{11}{2}\log\left(x^2\right)+20\log\left(x^2+1\right)-\frac{29}{2}\log\left(x^2+2\right)\right)+\frac{11x^2+9}{2\left(x^4+3x^2+2\right)}\right)$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(2 + 3*x^2 + x^4)^2),x]
```

output
```
((9 + 11*x^2)/(2*(2 + 3*x^2 + x^4)) + (-2/x^2 - (11*Log[x^2])/2 + 20*Log[1
 + x^2] - (29*Log[2 + x^2])/2)/2)/2
```

3.80.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.80. $\int \frac{4+x^2+3x^4+5x^6}{x^3(2+3x^2+x^4)^2}\,dx$

3.80.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

method	result
default	$-\frac{29\ln(x^2+2)}{8}+\frac{13}{4(x^2+2)}-\frac{1}{2x^2}-\frac{11\ln(x)}{4}+5\ln\left(x^2+1\right)-\frac{1}{2(x^2+1)}$
norman	$\frac{-1+\frac{3}{4}x^2+\frac{9}{4}x^4}{x^2(x^4+3x^2+2)}-\frac{11\ln(x)}{4}+5\ln\left(x^2+1\right)-\frac{29\ln(x^2+2)}{8}$
risch	$\frac{-1+\frac{3}{4}x^2+\frac{9}{4}x^4}{x^2(x^4+3x^2+2)}-\frac{11\ln(x)}{4}+5\ln\left(x^2+1\right)-\frac{29\ln(x^2+2)}{8}$
parallelrisch	$-\frac{22\ln(x)x^6-40\ln(x^2+1)x^6+29\ln(x^2+2)x^6+8+66\ln(x)x^4-120\ln(x^2+1)x^4+87\ln(x^2+2)x^4-18x^4+44\ln(x)x^2-80\ln(x^2}{8x^2(x^4+3x^2+2)}$

input
```
int((5*x^6+3*x^4+x^2+4)/x^3/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
-29/8*ln(x^2+2)+13/4/(x^2+2)-1/2/x^2-11/4*ln(x)+5*ln(x^2+1)-1/2/(x^2+1)
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.67

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(2+3x^2+x^4\right)^2}\,dx$$
$$=\frac{18\,x^4+6\,x^2-29\,(x^6+3\,x^4+2\,x^2)\log\left(x^2+2\right)+40\,(x^6+3\,x^4+2\,x^2)\log\left(x^2+1\right)-22\,(x^6+3\,x^4+2\,x^2}{8\,(x^6+3\,x^4+2\,x^2)}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
1/8*(18*x^4 + 6*x^2 - 29*(x^6 + 3*x^4 + 2*x^2)*log(x^2 + 2) + 40*(x^6 + 3*
x^4 + 2*x^2)*log(x^2 + 1) - 22*(x^6 + 3*x^4 + 2*x^2)*log(x) - 8)/(x^6 + 3*
x^4 + 2*x^2)
```

3.80.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(2+3x^2+x^4\right)^2}\,dx = \frac{9x^4+3x^2-4}{4x^6+12x^4+8x^2} - \frac{11\log\left(x\right)}{4} + 5\log\left(x^2+1\right) - \frac{29\log\left(x^2+2\right)}{8}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**3/(x**4+3*x**2+2)**2,x)
```

output
```
(9*x**4 + 3*x**2 - 4)/(4*x**6 + 12*x**4 + 8*x**2) - 11*log(x)/4 + 5*log(x*
*2 + 1) - 29*log(x**2 + 2)/8
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(2+3x^2+x^4\right)^2}\,dx = \frac{9\,x^4+3\,x^2-4}{4\left(x^6+3\,x^4+2\,x^2\right)} - \frac{29}{8}\log\left(x^2+2\right) + 5\log\left(x^2+1\right) - \frac{11}{8}\log\left(x^2\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
1/4*(9*x^4 + 3*x^2 - 4)/(x^6 + 3*x^4 + 2*x^2) - 29/8*log(x^2 + 2) + 5*log(
x^2 + 1) - 11/8*log(x^2)
```

3.80.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(2+3x^2+x^4\right)^2}\,dx = \frac{9\,x^4+3\,x^2-4}{4\left(x^6+3\,x^4+2\,x^2\right)} - \frac{29}{8}\log\left(x^2+2\right) + 5\log\left(x^2+1\right) - \frac{11}{8}\log\left(x^2\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
1/4*(9*x^4 + 3*x^2 - 4)/(x^6 + 3*x^4 + 2*x^2) - 29/8*log(x^2 + 2) + 5*log(
x^2 + 1) - 11/8*log(x^2)
```

3.80.9 Mupad [B] (verification not implemented)

Time = 8.52 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(2+3x^2+x^4\right)^2}\,dx = 5\,\ln\left(x^2+1\right) - \frac{29\,\ln\left(x^2+2\right)}{8} - \frac{11\,\ln\left(x\right)}{4} + \frac{\frac{9\,x^4}{4}+\frac{3\,x^2}{4}-1}{x^6+3\,x^4+2\,x^2}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^3*(3*x^2 + x^4 + 2)^2),x)
```

output
```
5*log(x^2 + 1) - (29*log(x^2 + 2))/8 - (11*log(x))/4 + ((3*x^2)/4 + (9*x^4
)/4 - 1)/(2*x^2 + 3*x^4 + x^6)
```

3.81 $\int \frac{4+x^2+3x^4+5x^6}{x^5(2+3x^2+x^4)^2}\,dx$

3.81.1 Optimal result

Integrand size = 31, antiderivative size = 64

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(2+3x^2+x^4\right)^2}\,dx = -\frac{1}{4x^4}+\frac{11}{8x^2}-\frac{5+9x^2}{8\left(2+3x^2+x^4\right)}+\frac{23\log(x)}{4} -\frac{11}{2}\log\left(1+x^2\right)+\frac{21}{8}\log\left(2+x^2\right)$$

output
```
-1/4/x^4+11/8/x^2+1/8*(-9*x^2-5)/(x^4+3*x^2+2)+23/4*ln(x)-11/2*ln(x^2+1)+2
1/8*ln(x^2+2)
```

3.81.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(2+3x^2+x^4\right)^2}\,dx = \frac{1}{8}\left(-\frac{2}{x^4}+\frac{11}{x^2}-\frac{5+9x^2}{2+3x^2+x^4}+46\log(x)-44\log\left(1+x^2\right) +21\log\left(2+x^2\right)\right)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(2 + 3*x^2 + x^4)^2),x]
```

output
```
(-2/x^4 + 11/x^2 - (5 + 9*x^2)/(2 + 3*x^2 + x^4) + 46*Log[x] - 44*Log[1 +
x^2] + 21*Log[2 + x^2])/8
```

3.81.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2194, 2177, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^5\left(x^4+3x^2+2\right)^2}dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{5x^6+3x^4+x^2+4}{x^6\left(x^4+3x^2+2\right)^2}dx^2$$

$$\Big\downarrow \text{2177}$$

$$\frac{1}{2}\left(-\int -\frac{-9x^6+17x^4-10x^2+8}{4x^6\left(x^4+3x^2+2\right)}dx^2-\frac{9x^2+5}{4\left(x^4+3x^2+2\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{4}\int \frac{-9x^6+17x^4-10x^2+8}{x^6\left(x^4+3x^2+2\right)}dx^2-\frac{9x^2+5}{4\left(x^4+3x^2+2\right)}\right)$$

$$\Big\downarrow \text{2159}$$

$$\frac{1}{2}\left(\frac{1}{4}\int \left(\frac{21}{x^2+2}+\frac{23}{x^2}-\frac{11}{x^4}+\frac{4}{x^6}-\frac{44}{x^2+1}\right)dx^2-\frac{9x^2+5}{4\left(x^4+3x^2+2\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(-\frac{2}{x^4}+\frac{11}{x^2}+23\log\left(x^2\right)-44\log\left(x^2+1\right)+21\log\left(x^2+2\right)\right)-\frac{9x^2+5}{4\left(x^4+3x^2+2\right)}\right)$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(2 + 3*x^2 + x^4)^2),x]
```

output
```
(-1/4*(5 + 9*x^2)/(2 + 3*x^2 + x^4) + (-2/x^4 + 11/x^2 + 23*Log[x^2] - 44*
Log[1 + x^2] + 21*Log[2 + x^2])/4)/2
```

3.81.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.81.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.78

method	result
default	$\frac{21\ln(x^2+2)}{8}-\frac{13}{8(x^2+2)}-\frac{1}{4x^4}+\frac{11}{8x^2}+\frac{23\ln(x)}{4}-\frac{11\ln(x^2+1)}{2}+\frac{1}{2x^2+2}$
norman	$\frac{-\frac{1}{2}+\frac{1}{4}x^6+\frac{13}{4}x^4+2x^2}{x^4(x^4+3x^2+2)}+\frac{23\ln(x)}{4}-\frac{11\ln(x^2+1)}{2}+\frac{21\ln(x^2+2)}{8}$
risch	$\frac{-\frac{1}{2}+\frac{1}{4}x^6+\frac{13}{4}x^4+2x^2}{x^4(x^4+3x^2+2)}+\frac{23\ln(x)}{4}-\frac{11\ln(x^2+1)}{2}+\frac{21\ln(x^2+2)}{8}$
parallelrisch	$\frac{46\ln(x)x^8-44\ln(x^2+1)x^8+21\ln(x^2+2)x^8-4+138\ln(x)x^6-132\ln(x^2+1)x^6+63\ln(x^2+2)x^6+2x^6+92\ln(x)x^4-88\ln(x^2+1}{8x^4(x^4+3x^2+2)}$

input
```
int((5*x^6+3*x^4+x^2+4)/x^5/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
21/8*ln(x^2+2)-13/8/(x^2+2)-1/4/x^4+11/8/x^2+23/4*ln(x)-11/2*ln(x^2+1)+1/2
/(x^2+1)
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.52

$$\int \frac{4+x^2+3x^4+5x^6}{x^5(2+3x^2+x^4)^2}\,dx$$
$$=\frac{2\,x^6+26\,x^4+16\,x^2+21\,(x^8+3\,x^6+2\,x^4)\log(x^2+2)-44\,(x^8+3\,x^6+2\,x^4)\log(x^2+1)+46\,(x^8+3\,x}{8\,(x^8+3\,x^6+2\,x^4)}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
1/8*(2*x^6 + 26*x^4 + 16*x^2 + 21*(x^8 + 3*x^6 + 2*x^4)*log(x^2 + 2) - 44*
(x^8 + 3*x^6 + 2*x^4)*log(x^2 + 1) + 46*(x^8 + 3*x^6 + 2*x^4)*log(x) - 4)/
(x^8 + 3*x^6 + 2*x^4)
```

3.81.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(2+3x^2+x^4\right)^2}\,dx=\frac{23\log{(x)}}{4}-\frac{11\log{(x^2+1)}}{2}+\frac{21\log{(x^2+2)}}{8}+\frac{x^6+13x^4+8x^2-2}{4x^8+12x^6+8x^4}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**5/(x**4+3*x**2+2)**2,x)
```

output
```
23*log(x)/4 - 11*log(x**2 + 1)/2 + 21*log(x**2 + 2)/8 + (x**6 + 13*x**4 +
8*x**2 - 2)/(4*x**8 + 12*x**6 + 8*x**4)
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(2+3x^2+x^4\right)^2}\,dx=\frac{x^6+13\,x^4+8\,x^2-2}{4\,(x^8+3\,x^6+2\,x^4)}+\frac{21}{8}\,\log\left(x^2+2\right)-\frac{11}{2}\,\log\left(x^2+1\right)+\frac{23}{8}\,\log\left(x^2\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
1/4*(x^6 + 13*x^4 + 8*x^2 - 2)/(x^8 + 3*x^6 + 2*x^4) + 21/8*log(x^2 + 2) -
 11/2*log(x^2 + 1) + 23/8*log(x^2)
```

3.81.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(2+3x^2+x^4\right)^2}\,dx=\frac{23\,x^4+51\,x^2+36}{16\,(x^4+3\,x^2+2)}-\frac{69\,x^4-22\,x^2+4}{16\,x^4}+\frac{21}{8}\,\log\left(x^2+2\right)-\frac{11}{2}\,\log\left(x^2+1\right)+\frac{23}{8}\,\log\left(x^2\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
1/16*(23*x^4 + 51*x^2 + 36)/(x^4 + 3*x^2 + 2) - 1/16*(69*x^4 - 22*x^2 + 4)
/x^4 + 21/8*log(x^2 + 2) - 11/2*log(x^2 + 1) + 23/8*log(x^2)
```

3.81.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{4+x^2+3x^4+5x^6}{x^5(2+3x^2+x^4)^2}\,dx = \frac{21\ln(x^2+2)}{8} - \frac{11\ln(x^2+1)}{2} + \frac{23\ln(x)}{4} + \frac{\frac{x^6}{4}+\frac{13\,x^4}{4}+2\,x^2-\frac{1}{2}}{x^8+3\,x^6+2\,x^4}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^5*(3*x^2 + x^4 + 2)^2),x)
```

output
```
(21*log(x^2 + 2))/8 - (11*log(x^2 + 1))/2 + (23*log(x))/4 + (2*x^2 + (13*x
^4)/4 + x^6/4 - 1/2)/(2*x^4 + 3*x^6 + x^8)
```

3.82 $\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} \, dx$

3.82.1 Optimal result

Integrand size = 31, antiderivative size = 70

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} \, dx = -293x+\frac{98x^3}{3}-\frac{27x^5}{5}+\frac{5x^7}{7}-\frac{x(206+207x^2)}{2(2+3x^2+x^4)} +\frac{9\arctan(x)}{2}+340\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

output
```
-293*x+98/3*x^3-27/5*x^5+5/7*x^7-1/2*x*(207*x^2+206)/(x^4+3*x^2+2)+9/2*arc
tan(x)+340*arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.82.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} \, dx = -293x+\frac{98x^3}{3}-\frac{27x^5}{5}+\frac{5x^7}{7}+\frac{-206x-207x^3}{2(2+3x^2+x^4)} +\frac{9\arctan(x)}{2}+340\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

input
```
Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
-293*x + (98*x^3)/3 - (27*x^5)/5 + (5*x^7)/7 + (-206*x - 207*x^3)/(2*(2 +
3*x^2 + x^4)) + (9*ArcTan[x])/2 + 340*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

3.82.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2197, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 (5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^2} \, dx$$

$$\Big\downarrow \; 2197$$

$$-\frac{1}{4}\int -\frac{2(10x^{10}-24x^8+54x^6-106x^4+3x^2+206)}{x^4+3x^2+2}dx-\frac{x(207x^2+206)}{2(x^4+3x^2+2)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\int \frac{10x^{10}-24x^8+54x^6-106x^4+3x^2+206}{x^4+3x^2+2}dx-\frac{x(207x^2+206)}{2(x^4+3x^2+2)}$$

$$\Big\downarrow \; 2205$$

$$\frac{1}{2}\int \left(10x^6-54x^4+196x^2+\frac{1369x^2+1378}{x^4+3x^2+2}-586\right)dx-\frac{x(207x^2+206)}{2(x^4+3x^2+2)}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(9\arctan(x)+680\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+\frac{10x^7}{7}-\frac{54x^5}{5}+\frac{196x^3}{3}-586x\right)-\frac{x(207x^2+206)}{2(x^4+3x^2+2)}$$

input
```
Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
-1/2*(x*(206 + 207*x^2))/(2 + 3*x^2 + x^4) + (-586*x + (196*x^3)/3 - (54*x
^5)/5 + (10*x^7)/7 + 9*ArcTan[x] + 680*Sqrt[2]*ArcTan[x/Sqrt[2]])/2
```

3.82.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

3.82.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{104x}{x^2+2}+340\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}+\frac{5x^7}{7}-\frac{27x^5}{5}+\frac{98x^3}{3}-293x+\frac{x}{2x^2+2}+\frac{9\arctan(x)}{2}$	56
risch	$\frac{5x^7}{7}-\frac{27x^5}{5}+\frac{98x^3}{3}-293x+\frac{-\frac{207}{2}x^3-103x}{x^4+3x^2+2}+\frac{9\arctan(x)}{2}+340\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$	58

input
```
int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
-104*x/(x^2+2)+340*arctan(1/2*x*2^(1/2))*2^(1/2)+5/7*x^7-27/5*x^5+98/3*x^3
-293*x+1/2*x/(x^2+1)+9/2*arctan(x)
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.13

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$$
$$= \frac{150\,x^{11} - 684\,x^9 + 3758\,x^7 - 43218\,x^5 - 192605\,x^3 + 71400\,\sqrt{2}(x^4+3\,x^2+2)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + 945\,(x^4 +}{210\,(x^4+3\,x^2+2)}$$

input
```
integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
1/210*(150*x^11 - 684*x^9 + 3758*x^7 - 43218*x^5 - 192605*x^3 + 71400*sqrt
(2)*(x^4 + 3*x^2 + 2)*arctan(1/2*sqrt(2)*x) + 945*(x^4 + 3*x^2 + 2)*arctan
(x) - 144690*x)/(x^4 + 3*x^2 + 2)
```

3.82.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5x^7}{7} - \frac{27x^5}{5} + \frac{98x^3}{3} - 293x + \frac{-207x^3-206x}{2x^4+6x^2+4} + \frac{9\,\mathrm{atan}\,(x)}{2} + 340\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

input
```
integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

output
```
5*x**7/7 - 27*x**5/5 + 98*x**3/3 - 293*x + (-207*x**3 - 206*x)/(2*x**4 + 6
*x**2 + 4) + 9*atan(x)/2 + 340*sqrt(2)*atan(sqrt(2)*x/2)
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{7}\,x^7 - \frac{27}{5}\,x^5 + \frac{98}{3}\,x^3 + 340\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - 293\,x - \frac{207\,x^3+206\,x}{2\,(x^4+3\,x^2+2)} + \frac{9}{2}\,\arctan\left(x\right)$$

input
```
integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
5/7*x^7 - 27/5*x^5 + 98/3*x^3 + 340*sqrt(2)*arctan(1/2*sqrt(2)*x) - 293*x
- 1/2*(207*x^3 + 206*x)/(x^4 + 3*x^2 + 2) + 9/2*arctan(x)
```

3.82.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{7}\,x^7 - \frac{27}{5}\,x^5 + \frac{98}{3}\,x^3 + 340\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - 293\,x - \frac{207\,x^3+206\,x}{2\,(x^4+3\,x^2+2)} + \frac{9}{2}\,\arctan\left(x\right)$$

input
```
integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
5/7*x^7 - 27/5*x^5 + 98/3*x^3 + 340*sqrt(2)*arctan(1/2*sqrt(2)*x) - 293*x
- 1/2*(207*x^3 + 206*x)/(x^4 + 3*x^2 + 2) + 9/2*arctan(x)
```

3.82.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{9\,\mathrm{atan}(x)}{2} - 293\,x + 340\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right) - \frac{\frac{207\,x^3}{2}+103\,x}{x^4+3\,x^2+2} + \frac{98\,x^3}{3} - \frac{27\,x^5}{5} + \frac{5\,x^7}{7}$$

input
```
int((x^8*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

output
```
(9*atan(x))/2 - 293*x + 340*2^(1/2)*atan((2^(1/2)*x)/2) - (103*x + (207*x^
3)/2)/(3*x^2 + x^4 + 2) + (98*x^3)/3 - (27*x^5)/5 + (5*x^7)/7
```

3.83 $\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$

3.83.1 Optimal result

Integrand size = 31, antiderivative size = 57

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx = 98x-9x^3+x^5+\frac{x(102+103x^2)}{2(2+3x^2+x^4)} - \frac{11\arctan(x)}{2} - 118\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

output
```
98*x-9*x^3+x^5+1/2*x*(103*x^2+102)/(x^4+3*x^2+2)-11/2*arctan(x)-118*arctan
(1/2*x*2^(1/2))*2^(1/2)
```

3.83.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx = 98x-9x^3+x^5+\frac{102x+103x^3}{2(2+3x^2+x^4)} - \frac{11\arctan(x)}{2} - 118\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

input
```
Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
98*x - 9*x^3 + x^5 + (102*x + 103*x^3)/(2*(2 + 3*x^2 + x^4)) - (11*ArcTan[
x])/2 - 118*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

3.83.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2197, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^2}dx$$

$$\Big\downarrow \text{2197}$$

$$\frac{x(103x^2+102)}{2(x^4+3x^2+2)}-\frac{1}{4}\int \frac{2(-10x^8+24x^6-54x^4+3x^2+102)}{x^4+3x^2+2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x(103x^2+102)}{2(x^4+3x^2+2)}-\frac{1}{2}\int \frac{-10x^8+24x^6-54x^4+3x^2+102}{x^4+3x^2+2}dx$$

$$\Big\downarrow \text{2205}$$

$$\frac{x(103x^2+102)}{2(x^4+3x^2+2)}-\frac{1}{2}\int \left(-10x^4+54x^2+\frac{483x^2+494}{x^4+3x^2+2}-196\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-11\arctan(x)-236\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+2x^5-18x^3+196x\right)+\frac{x(103x^2+102)}{2(x^4+3x^2+2)}$$

input
```
Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
(x*(102 + 103*x^2))/(2*(2 + 3*x^2 + x^4)) + (196*x - 18*x^3 + 2*x^5 - 11*A
rcTan[x] - 236*Sqrt[2]*ArcTan[x/Sqrt[2]])/2
```

3.83.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

3.83.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{52x}{x^2+2}-118\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}+x^5-9x^3+98x-\frac{x}{2(x^2+1)}-\frac{11\arctan(x)}{2}$	49
risch	$x^5-9x^3+98x+\frac{\frac{103}{2}x^3+51x}{x^4+3x^2+2}-\frac{11\arctan(x)}{2}-118\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$	51

input
```
int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
52*x/(x^2+2)-118*arctan(1/2*x*2^(1/2))*2^(1/2)+x^5-9*x^3+98*x-1/2*x/(x^2+1
)-11/2*arctan(x)
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.30

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{2\,x^9-12\,x^7+146\,x^5+655\,x^3-236\,\sqrt{2}(x^4+3\,x^2+2)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-11\,(x^4+3\,x^2+2)\arctan\left(x\right)+}{2\,(x^4+3\,x^2+2)}$$

input
```
integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
1/2*(2*x^9 - 12*x^7 + 146*x^5 + 655*x^3 - 236*sqrt(2)*(x^4 + 3*x^2 + 2)*ar
ctan(1/2*sqrt(2)*x) - 11*(x^4 + 3*x^2 + 2)*arctan(x) + 494*x)/(x^4 + 3*x^2
 + 2)
```

3.83.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = x^5-9x^3+98x+\frac{103x^3+102x}{2x^4+6x^2+4} - \frac{11\,\mathrm{atan}\,(x)}{2}-118\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

input
```
integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

output
```
x**5 - 9*x**3 + 98*x + (103*x**3 + 102*x)/(2*x**4 + 6*x**2 + 4) - 11*atan(
x)/2 - 118*sqrt(2)*atan(sqrt(2)*x/2)
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = x^5-9\,x^3-118\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + 98\,x+\frac{103\,x^3+102\,x}{2\,(x^4+3\,x^2+2)}-\frac{11}{2}\,\arctan\left(x\right)$$

input
```
integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
x^5 - 9*x^3 - 118*sqrt(2)*arctan(1/2*sqrt(2)*x) + 98*x + 1/2*(103*x^3 + 10
2*x)/(x^4 + 3*x^2 + 2) - 11/2*arctan(x)
```

3.83.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = x^5 - 9\,x^3 - 118\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + 98\,x + \frac{103\,x^3 + 102\,x}{2\,(x^4 + 3\,x^2 + 2)} - \frac{11}{2}\,\arctan\left(x\right)$$

input
```
integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
x^5 - 9*x^3 - 118*sqrt(2)*arctan(1/2*sqrt(2)*x) + 98*x + 1/2*(103*x^3 + 10
2*x)/(x^4 + 3*x^2 + 2) - 11/2*arctan(x)
```

3.83.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = 98\,x - \frac{11\,\mathrm{atan}(x)}{2} - 118\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right) + \frac{\frac{103\,x^3}{2} + 51\,x}{x^4 + 3\,x^2 + 2} - 9\,x^3 + x^5$$

input
```
int((x^6*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

output
```
98*x - (11*atan(x))/2 - 118*2^(1/2)*atan((2^(1/2)*x)/2) + (51*x + (103*x^3
)/2)/(3*x^2 + x^4 + 2) - 9*x^3 + x^5
```

3.84 $\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} \, dx$

3.84.1 Optimal result

Integrand size = 31, antiderivative size = 56

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} \, dx = -27x+\frac{5x^3}{3}-\frac{x(50+51x^2)}{2(2+3x^2+x^4)}+\frac{13\arctan(x)}{2}+33\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

output
```
-27*x+5/3*x^3-1/2*x*(51*x^2+50)/(x^4+3*x^2+2)+13/2*arctan(x)+33*arctan(1/2
*x*2^(1/2))*2^(1/2)
```

3.84.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} \, dx = -27x+\frac{5x^3}{3}+\frac{-50x-51x^3}{2(2+3x^2+x^4)}+\frac{13\arctan(x)}{2}+33\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

input
```
Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
-27*x + (5*x^3)/3 + (-50*x - 51*x^3)/(2*(2 + 3*x^2 + x^4)) + (13*ArcTan[x]
)/2 + 33*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

3.84.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2197, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(5x^6+3x^4+x^2+4\right)}{\left(x^4+3x^2+2\right)^2}dx$$

$$\Big\downarrow \text{2197}$$

$$-\frac{1}{4}\int -\frac{2\left(10x^6-24x^4+3x^2+50\right)}{x^4+3x^2+2}dx-\frac{x\left(51x^2+50\right)}{2\left(x^4+3x^2+2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{10x^6-24x^4+3x^2+50}{x^4+3x^2+2}dx-\frac{x\left(51x^2+50\right)}{2\left(x^4+3x^2+2\right)}$$

$$\Big\downarrow \text{2205}$$

$$\frac{1}{2}\int \left(10x^2+\frac{145x^2+158}{x^4+3x^2+2}-54\right)dx-\frac{x\left(51x^2+50\right)}{2\left(x^4+3x^2+2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(13\arctan(x)+66\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+\frac{10x^3}{3}-54x\right)-\frac{x\left(51x^2+50\right)}{2\left(x^4+3x^2+2\right)}$$

input
```
Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
-1/2*(x*(50 + 51*x^2))/(2 + 3*x^2 + x^4) + (-54*x + (10*x^3)/3 + 13*ArcTan
[x] + 66*Sqrt[2]*ArcTan[x/Sqrt[2]])/2
```

3.84.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

3.84.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{26x}{x^2+2}+33\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}+\frac{5x^3}{3}-27x+\frac{x}{2x^2+2}+\frac{13\arctan(x)}{2}$	46
risch	$\frac{5x^3}{3}-27x+\frac{-\frac{51}{2}x^3-25x}{x^4+3x^2+2}+\frac{13\arctan(x)}{2}+33\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$	48

input
```
int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
-26*x/(x^2+2)+33*arctan(1/2*x*2^(1/2))*2^(1/2)+5/3*x^3-27*x+1/2*x/(x^2+1)+
13/2*arctan(x)
```

3.84.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.23

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{10\,x^7 - 132\,x^5 - 619\,x^3 + 198\,\sqrt{2}(x^4+3\,x^2+2)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + 39\,(x^4+3\,x^2+2)\arctan(x) - 474\,x}{6\,(x^4+3\,x^2+2)}$$

input
```
integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
1/6*(10*x^7 - 132*x^5 - 619*x^3 + 198*sqrt(2)*(x^4 + 3*x^2 + 2)*arctan(1/2
*sqrt(2)*x) + 39*(x^4 + 3*x^2 + 2)*arctan(x) - 474*x)/(x^4 + 3*x^2 + 2)
```

3.84.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5x^3}{3} - 27x + \frac{-51x^3-50x}{2x^4+6x^2+4} + \frac{13\,\mathrm{atan}(x)}{2} + 33\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

input
```
integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

output
```
5*x**3/3 - 27*x + (-51*x**3 - 50*x)/(2*x**4 + 6*x**2 + 4) + 13*atan(x)/2 +
 33*sqrt(2)*atan(sqrt(2)*x/2)
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{3}\,x^3 + 33\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - 27\,x - \frac{51\,x^3+50\,x}{2\,(x^4+3\,x^2+2)} + \frac{13}{2}\,\arctan(x)$$

input
```
integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
5/3*x^3 + 33*sqrt(2)*arctan(1/2*sqrt(2)*x) - 27*x - 1/2*(51*x^3 + 50*x)/(x
^4 + 3*x^2 + 2) + 13/2*arctan(x)
```

3.84.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{3}\,x^3 + 33\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - 27\,x - \frac{51\,x^3+50\,x}{2\,(x^4+3\,x^2+2)} + \frac{13}{2}\,\arctan\left(x\right)$$

input
```
integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
5/3*x^3 + 33*sqrt(2)*arctan(1/2*sqrt(2)*x) - 27*x - 1/2*(51*x^3 + 50*x)/(x
^4 + 3*x^2 + 2) + 13/2*arctan(x)
```

3.84.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{13\,\mathrm{atan}(x)}{2} - 27\,x + 33\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right) - \frac{\frac{51\,x^3}{2}+25\,x}{x^4+3\,x^2+2} + \frac{5\,x^3}{3}$$

input
```
int((x^4*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

output
```
(13*atan(x))/2 - 27*x + 33*2^(1/2)*atan((2^(1/2)*x)/2) - (25*x + (51*x^3)/
2)/(3*x^2 + x^4 + 2) + (5*x^3)/3
```

3.85 $\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$

3.85.1 Optimal result

Integrand size = 31, antiderivative size = 49

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx = 5x + \frac{x(24+25x^2)}{2(2+3x^2+x^4)} - \frac{15\arctan(x)}{2} - \frac{7\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

output
```
5*x+1/2*x*(25*x^2+24)/(x^4+3*x^2+2)-15/2*arctan(x)-7/2*arctan(1/2*x*2^(1/2
))*2^(1/2)
```

3.85.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx = 5x + \frac{24x+25x^3}{2(2+3x^2+x^4)} - \frac{15\arctan(x)}{2} - \frac{7\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

input
```
Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
5*x + (24*x + 25*x^3)/(2*(2 + 3*x^2 + x^4)) - (15*ArcTan[x])/2 - (7*ArcTan
[x/Sqrt[2]])/Sqrt[2]
```

3.85.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2197, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^2}dx$$

$$\Big\downarrow \text{2197}$$

$$\frac{x(25x^2+24)}{2(x^4+3x^2+2)}-\frac{1}{4}\int \frac{2(-10x^4-x^2+24)}{x^4+3x^2+2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x(25x^2+24)}{2(x^4+3x^2+2)}-\frac{1}{2}\int \frac{-10x^4-x^2+24}{x^4+3x^2+2}dx$$

$$\Big\downarrow \text{2205}$$

$$\frac{x(25x^2+24)}{2(x^4+3x^2+2)}-\frac{1}{2}\int \left(\frac{29x^2+44}{x^4+3x^2+2}-10\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-15\arctan(x)-7\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+10x\right)+\frac{x(25x^2+24)}{2(x^4+3x^2+2)}$$

input
```
Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
(x*(24 + 25*x^2))/(2*(2 + 3*x^2 + x^4)) + (10*x - 15*ArcTan[x] - 7*Sqrt[2]
*ArcTan[x/Sqrt[2]])/2
```

3.85.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

3.85.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

method	result	size
default	$5x+\frac{13x}{x^{2}+2}-\frac{7\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{2}-\frac{x}{2(x^{2}+1)}-\frac{15\arctan(x)}{2}$	41
risch	$5x+\frac{\frac{25}{2}x^{3}+12x}{x^{4}+3x^{2}+2}-\frac{7\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{2}-\frac{15\arctan(x)}{2}$	43

input
```
int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
5*x+13*x/(x^2+2)-7/2*arctan(1/2*x*2^(1/2))*2^(1/2)-1/2*x/(x^2+1)-15/2*arct
an(x)
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.31

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{10\,x^5+55\,x^3-7\,\sqrt{2}(x^4+3\,x^2+2)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-15\,(x^4+3\,x^2+2)\arctan\left(x\right)+44\,x}{2\,(x^4+3\,x^2+2)}$$

input
```
integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
1/2*(10*x^5 + 55*x^3 - 7*sqrt(2)*(x^4 + 3*x^2 + 2)*arctan(1/2*sqrt(2)*x) -
 15*(x^4 + 3*x^2 + 2)*arctan(x) + 44*x)/(x^4 + 3*x^2 + 2)
```

3.85.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = 5x+\frac{25x^3+24x}{2x^4+6x^2+4}-\frac{15\,\mathrm{atan}\,(x)}{2}-\frac{7\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

input
```
integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

output
```
5*x + (25*x**3 + 24*x)/(2*x**4 + 6*x**2 + 4) - 15*atan(x)/2 - 7*sqrt(2)*at
an(sqrt(2)*x/2)/2
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = -\frac{7}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+5\,x+\frac{25\,x^3+24\,x}{2\,(x^4+3\,x^2+2)}-\frac{15}{2}\,\arctan\left(x\right)$$

input
```
integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
-7/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x + 1/2*(25*x^3 + 24*x)/(x^4 + 3*x^
2 + 2) - 15/2*arctan(x)
```

3.85.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = -\frac{7}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+5\,x + \frac{25\,x^3+24\,x}{2\,(x^4+3\,x^2+2)}-\frac{15}{2}\arctan\left(x\right)$$

input
```
integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
-7/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x + 1/2*(25*x^3 + 24*x)/(x^4 + 3*x^
2 + 2) - 15/2*arctan(x)
```

3.85.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = 5\,x-\frac{15\,\mathrm{atan}(x)}{2}-\frac{7\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{2}+\frac{\frac{25\,x^3}{2}+12\,x}{x^4+3\,x^2+2}$$

input
```
int((x^2*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

output
```
5*x - (15*atan(x))/2 - (7*2^(1/2)*atan((2^(1/2)*x)/2))/2 + (12*x + (25*x^3
)/2)/(3*x^2 + x^4 + 2)
```

3.86 $\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^2} \, dx$

3.86.1 Optimal result

Integrand size = 28, antiderivative size = 48

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^2} \, dx = -\frac{x(11+12x^2)}{2(2+3x^2+x^4)} + \frac{17\arctan(x)}{2} - \frac{19\arctan\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

output
```
-1/2*x*(12*x^2+11)/(x^4+3*x^2+2)+17/2*arctan(x)-19/4*arctan(1/2*x*2^(1/2))
*2^(1/2)
```

3.86.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^2} \, dx = \frac{1}{4}\left(-\frac{2x(11+12x^2)}{2+3x^2+x^4} + 34\arctan(x) - 19\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^2,x]
```

output
```
((-2*x*(11 + 12*x^2))/(2 + 3*x^2 + x^4) + 34*ArcTan[x] - 19*Sqrt[2]*ArcTan
[x/Sqrt[2]])/4
```

3.86.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2206, 27, 1480, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{(x^4+3x^2+2)^2}dx$$

$$\Big\downarrow \text{2206}$$

$$-\frac{1}{4}\int -\frac{2(15-2x^2)}{x^4+3x^2+2}dx-\frac{x(12x^2+11)}{2(x^4+3x^2+2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{15-2x^2}{x^4+3x^2+2}dx-\frac{x(12x^2+11)}{2(x^4+3x^2+2)}$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{2}\left(17\int \frac{1}{x^2+1}dx-19\int \frac{1}{x^2+2}dx\right)-\frac{x(12x^2+11)}{2(x^4+3x^2+2)}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(17\arctan(x)-\frac{19\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}\right)-\frac{x(12x^2+11)}{2(x^4+3x^2+2)}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^2,x]
```

output
```
-1/2*(x*(11 + 12*x^2))/(2 + 3*x^2 + x^4) + (17*ArcTan[x] - (19*ArcTan[x/Sq
rt[2]])/Sqrt[2])/2
```

3.86.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

3.86.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{13x}{2(x^2+2)}-\frac{19\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{4}+\frac{x}{2x^2+2}+\frac{17\arctan(x)}{2}$	38
risch	$\frac{-6x^3-\frac{11}{2}x}{x^4+3x^2+2}+\frac{17\arctan(x)}{2}-\frac{19\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{4}$	40

input
```
int((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
-13/2*x/(x^2+2)-19/4*arctan(1/2*x*2^(1/2))*2^(1/2)+1/2*x/(x^2+1)+17/2*arct
an(x)
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^2}\,dx$$
$$= -\frac{24\,x^3+19\,\sqrt{2}(x^4+3\,x^2+2)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-34\,(x^4+3\,x^2+2)\arctan\left(x\right)+22\,x}{4\,(x^4+3\,x^2+2)}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
-1/4*(24*x^3 + 19*sqrt(2)*(x^4 + 3*x^2 + 2)*arctan(1/2*sqrt(2)*x) - 34*(x^
4 + 3*x^2 + 2)*arctan(x) + 22*x)/(x^4 + 3*x^2 + 2)
```

3.86.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^2}\,dx = \frac{-12x^3-11x}{2x^4+6x^2+4}+\frac{17\,\mathrm{atan}\,(x)}{2}-\frac{19\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{4}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

output
```
(-12*x**3 - 11*x)/(2*x**4 + 6*x**2 + 4) + 17*atan(x)/2 - 19*sqrt(2)*atan(s
qrt(2)*x/2)/4
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^2}\,dx = -\frac{19}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{12\,x^3+11\,x}{2\,(x^4+3\,x^2+2)} + \frac{17}{2}\arctan(x)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
-19/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/2*(12*x^3 + 11*x)/(x^4 + 3*x^2 + 2
) + 17/2*arctan(x)
```

3.86.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^2}\,dx = -\frac{19}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{12\,x^3+11\,x}{2\,(x^4+3\,x^2+2)} + \frac{17}{2}\arctan(x)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
-19/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/2*(12*x^3 + 11*x)/(x^4 + 3*x^2 + 2
) + 17/2*arctan(x)
```

3.86.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^2}\,dx = \frac{17\,\mathrm{atan}(x)}{2} - \frac{19\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{4} - \frac{6\,x^3+\frac{11\,x}{2}}{x^4+3\,x^2+2}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(3*x^2 + x^4 + 2)^2,x)
```

output
```
(17*atan(x))/2 - (19*2^(1/2)*atan((2^(1/2)*x)/2))/4 - ((11*x)/2 + 6*x^3)/(
3*x^2 + x^4 + 2)
```

3.87 $\int \frac{4+x^2+3x^4+5x^6}{x^2(2+3x^2+x^4)^2}\,dx$

3.87.1 Optimal result

Integrand size = 31, antiderivative size = 53

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^2}\,dx=-\frac{1}{x}+\frac{x(9+11x^2)}{4\left(2+3x^2+x^4\right)}-\frac{19\arctan(x)}{2}+\frac{45\arctan\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

output
```
-1/x+1/4*x*(11*x^2+9)/(x^4+3*x^2+2)-19/2*arctan(x)+45/8*arctan(1/2*x*2^(1/
2))*2^(1/2)
```

3.87.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^2}\,dx=\frac{1}{8}\left(-\frac{8}{x}+\frac{2x(9+11x^2)}{2+3x^2+x^4}-76\arctan(x)+45\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^2),x]
```

output
```
(-8/x + (2*x*(9 + 11*x^2))/(2 + 3*x^2 + x^4) - 76*ArcTan[x] + 45*Sqrt[2]*A
rcTan[x/Sqrt[2]])/8
```

3.87.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2198, 25, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^2\left(x^4+3x^2+2\right)^2}dx$$

$$\Big\downarrow \text{2198}$$

$$\frac{x\left(11x^2+9\right)}{4\left(x^4+3x^2+2\right)}-\frac{1}{4}\int -\frac{11x^4-19x^2+8}{x^2\left(x^4+3x^2+2\right)}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\int \frac{11x^4-19x^2+8}{x^2\left(x^4+3x^2+2\right)}dx+\frac{x\left(11x^2+9\right)}{4\left(x^4+3x^2+2\right)}$$

$$\Big\downarrow \text{2195}$$

$$\frac{1}{4}\int \left(\frac{45}{x^2+2}+\frac{4}{x^2}-\frac{38}{x^2+1}\right)dx+\frac{x\left(11x^2+9\right)}{4\left(x^4+3x^2+2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-38\arctan(x)+\frac{45\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{4}{x}\right)+\frac{x\left(11x^2+9\right)}{4\left(x^4+3x^2+2\right)}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^2),x]
```

output
```
(x*(9 + 11*x^2))/(4*(2 + 3*x^2 + x^4)) + (-4/x - 38*ArcTan[x] + (45*ArcTan
[x/Sqrt[2]])/Sqrt[2])/4
```

3.87.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

3.87.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{13x}{4(x^2+2)}+\frac{45\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{8}-\frac{1}{x}-\frac{x}{2(x^2+1)}-\frac{19\arctan(x)}{2}$	43
risch	$\frac{\frac{7}{4}x^4-\frac{3}{4}x^2-2}{x(x^4+3x^2+2)}-\frac{19\arctan(x)}{2}+\frac{45\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{8}$	46

input
```
int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
13/4*x/(x^2+2)+45/8*arctan(1/2*x*2^(1/2))*2^(1/2)-1/x-1/2*x/(x^2+1)-19/2*a
rctan(x)
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.28

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^2}\,dx = \frac{14\,x^4+45\,\sqrt{2}(x^5+3\,x^3+2\,x)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-6\,x^2-76\,(x^5+3\,x^3+2\,x)\arctan\left(x\right)-16}{8\,(x^5+3\,x^3+2\,x)}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
1/8*(14*x^4 + 45*sqrt(2)*(x^5 + 3*x^3 + 2*x)*arctan(1/2*sqrt(2)*x) - 6*x^2
 - 76*(x^5 + 3*x^3 + 2*x)*arctan(x) - 16)/(x^5 + 3*x^3 + 2*x)
```

3.87.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^2}\,dx = \frac{7x^4-3x^2-8}{4x^5+12x^3+8x} - \frac{19\,\mathrm{atan}\left(x\right)}{2} + \frac{45\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{8}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+3*x**2+2)**2,x)
```

output
```
(7*x**4 - 3*x**2 - 8)/(4*x**5 + 12*x**3 + 8*x) - 19*atan(x)/2 + 45*sqrt(2)
*atan(sqrt(2)*x/2)/8
```

3.87.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^2}\,dx = \frac{45}{8}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + \frac{7\,x^4-3\,x^2-8}{4\,(x^5+3\,x^3+2\,x)} - \frac{19}{2}\,\arctan\left(x\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
45/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(7*x^4 - 3*x^2 - 8)/(x^5 + 3*x^3
+ 2*x) - 19/2*arctan(x)
```

3.87.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^2}\,dx = \frac{45}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{7\,x^4-3\,x^2-8}{4\,(x^5+3\,x^3+2\,x)}-\frac{19}{2}\arctan\left(x\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
45/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(7*x^4 - 3*x^2 - 8)/(x^5 + 3*x^3
+ 2*x) - 19/2*arctan(x)
```

3.87.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^2}\,dx = \frac{45\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{8}-\frac{19\,\mathrm{atan}(x)}{2}-\frac{-\frac{7\,x^4}{4}+\frac{3\,x^2}{4}+2}{x^5+3\,x^3+2\,x}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^2*(3*x^2 + x^4 + 2)^2),x)
```

output
```
(45*2^(1/2)*atan((2^(1/2)*x)/2))/8 - (19*atan(x))/2 - ((3*x^2)/4 - (7*x^4)
/4 + 2)/(2*x + 3*x^3 + x^5)
```

3.88 $\int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^2}\,dx$

3.88.1 Optimal result

Integrand size = 31, antiderivative size = 62

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^2}\,dx=-\frac{1}{3x^3}+\frac{11}{4x}-\frac{x(5+9x^2)}{8\left(2+3x^2+x^4\right)}+\frac{21\arctan(x)}{2}-\frac{71\arctan\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}$$

output
```
-1/3/x^3+11/4/x-1/8*x*(9*x^2+5)/(x^4+3*x^2+2)+21/2*arctan(x)-71/16*arctan(
1/2*x*2^(1/2))*2^(1/2)
```

3.88.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^2}\,dx=\frac{1}{48}\left(-\frac{16}{x^3}+\frac{132}{x}-\frac{6x(5+9x^2)}{2+3x^2+x^4}+504\arctan(x)-213\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^2),x]
```

output
```
(-16/x^3 + 132/x - (6*x*(5 + 9*x^2))/(2 + 3*x^2 + x^4) + 504*ArcTan[x] - 2
13*Sqrt[2]*ArcTan[x/Sqrt[2]])/48
```

3.88.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2198, 27, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^4\left(x^4+3x^2+2\right)^2}dx$$

$$\Big\downarrow \text{ 2198}$$

$$-\frac{1}{4}\int -\frac{-9x^6+39x^4-20x^2+16}{2x^4\left(x^4+3x^2+2\right)}dx-\frac{x\left(9x^2+5\right)}{8\left(x^4+3x^2+2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{8}\int \frac{-9x^6+39x^4-20x^2+16}{x^4\left(x^4+3x^2+2\right)}dx-\frac{x\left(9x^2+5\right)}{8\left(x^4+3x^2+2\right)}$$

$$\Big\downarrow \text{ 2195}$$

$$\frac{1}{8}\int \left(-\frac{71}{x^2+2}-\frac{22}{x^2}+\frac{8}{x^4}+\frac{84}{x^2+1}\right)dx-\frac{x\left(9x^2+5\right)}{8\left(x^4+3x^2+2\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{8}\left(84\arctan(x)-\frac{71\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{8}{3x^3}+\frac{22}{x}\right)-\frac{x\left(9x^2+5\right)}{8\left(x^4+3x^2+2\right)}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^2),x]
```

output
```
-1/8*(x*(5 + 9*x^2))/(2 + 3*x^2 + x^4) + (-8/(3*x^3) + 22/x + 84*ArcTan[x]
 - (71*ArcTan[x/Sqrt[2]])/Sqrt[2])/8
```

3.88.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

3.88.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{13x}{8(x^2+2)}-\frac{71\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{16}-\frac{1}{3x^3}+\frac{11}{4x}+\frac{x}{2x^2+2}+\frac{21\arctan(x)}{2}$	48
risch	$\frac{\frac{13}{8}x^6+\frac{175}{24}x^4+\frac{9}{2}x^2-\frac{2}{3}}{x^3(x^4+3x^2+2)}-\frac{71\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{16}+\frac{21\arctan(x)}{2}$	51

input
```
int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
-13/8*x/(x^2+2)-71/16*arctan(1/2*x*2^(1/2))*2^(1/2)-1/3/x^3+11/4/x+1/2*x/(
x^2+1)+21/2*arctan(x)
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.27

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^2}\,dx = \frac{78\,x^6+350\,x^4-213\,\sqrt{2}(x^7+3\,x^5+2\,x^3)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+216\,x^2+504\,(x^7+3\,x^5+2\,x^3)\arctan\left(x\right)-}{48\,(x^7+3\,x^5+2\,x^3)}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
1/48*(78*x^6 + 350*x^4 - 213*sqrt(2)*(x^7 + 3*x^5 + 2*x^3)*arctan(1/2*sqrt
(2)*x) + 216*x^2 + 504*(x^7 + 3*x^5 + 2*x^3)*arctan(x) - 32)/(x^7 + 3*x^5
+ 2*x^3)
```

3.88.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^2}\,dx = \frac{21\,\mathrm{atan}\,(x)}{2}-\frac{71\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{16}+\frac{39x^6+175x^4+108x^2-16}{24x^7+72x^5+48x^3}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+3*x**2+2)**2,x)
```

output
```
21*atan(x)/2 - 71*sqrt(2)*atan(sqrt(2)*x/2)/16 + (39*x**6 + 175*x**4 + 108
*x**2 - 16)/(24*x**7 + 72*x**5 + 48*x**3)
```

3.88.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^2}\,dx = -\frac{71}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + \frac{39\,x^6+175\,x^4+108\,x^2-16}{24\,(x^7+3\,x^5+2\,x^3)}+\frac{21}{2}\,\arctan\left(x\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
-71/16*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/24*(39*x^6 + 175*x^4 + 108*x^2 -
16)/(x^7 + 3*x^5 + 2*x^3) + 21/2*arctan(x)
```

3.88.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^2}\,dx = -\frac{71}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{9\,x^3+5\,x}{8\,(x^4+3\,x^2+2)} + \frac{33\,x^2-4}{12\,x^3} + \frac{21}{2}\arctan\left(x\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
-71/16*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/8*(9*x^3 + 5*x)/(x^4 + 3*x^2 + 2)
 + 1/12*(33*x^2 - 4)/x^3 + 21/2*arctan(x)
```

3.88.9 Mupad [B] (verification not implemented)

Time = 8.51 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^2}\,dx = \frac{21\,\mathrm{atan}(x)}{2} - \frac{71\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{16} + \frac{\frac{13\,x^6}{8}+\frac{175\,x^4}{24}+\frac{9\,x^2}{2}-\frac{2}{3}}{x^7+3\,x^5+2\,x^3}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^4*(3*x^2 + x^4 + 2)^2),x)
```

output
```
(21*atan(x))/2 - (71*2^(1/2)*atan((2^(1/2)*x)/2))/16 + ((9*x^2)/2 + (175*x
^4)/24 + (13*x^6)/8 - 2/3)/(2*x^3 + 3*x^5 + x^7)
```

3.89 $\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^2}\,dx$

3.89.1 Optimal result

Integrand size = 31, antiderivative size = 69

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^2}\,dx=-\frac{1}{5x^5}+\frac{11}{12x^3}-\frac{23}{4x}-\frac{x(3-5x^2)}{16\left(2+3x^2+x^4\right)}-\frac{23\arctan(x)}{2}+\frac{97\arctan\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}$$

output
```
-1/5/x^5+11/12/x^3-23/4/x-1/16*x*(-5*x^2+3)/(x^4+3*x^2+2)-23/2*arctan(x)+9
7/32*arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.89.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^2}\,dx=\frac{1}{480}\left(-\frac{96}{x^5}+\frac{440}{x^3}-\frac{2760}{x}+\frac{30x(-3+5x^2)}{2+3x^2+x^4}-5520\arctan(x)+1455\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^2),x]
```

output
```
(-96/x^5 + 440/x^3 - 2760/x + (30*x*(-3 + 5*x^2))/(2 + 3*x^2 + x^4) - 5520
*ArcTan[x] + 1455*Sqrt[2]*ArcTan[x/Sqrt[2]])/480
```

3.89.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2198, 27, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^6\left(x^4+3x^2+2\right)^2}dx$$

$$\Big\downarrow \; 2198$$

$$-\frac{1}{4}\int -\frac{5x^8-39x^6+68x^4-40x^2+32}{4x^6\left(x^4+3x^2+2\right)}dx-\frac{x\left(3-5x^2\right)}{16\left(x^4+3x^2+2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{16}\int \frac{5x^8-39x^6+68x^4-40x^2+32}{x^6\left(x^4+3x^2+2\right)}dx-\frac{x\left(3-5x^2\right)}{16\left(x^4+3x^2+2\right)}$$

$$\Big\downarrow \; 2195$$

$$\frac{1}{16}\int \left(\frac{97}{x^2+2}+\frac{92}{x^2}-\frac{44}{x^4}+\frac{16}{x^6}-\frac{184}{x^2+1}\right)dx-\frac{x\left(3-5x^2\right)}{16\left(x^4+3x^2+2\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{16}\left(-184\arctan(x)+\frac{97\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{16}{5x^5}+\frac{44}{3x^3}-\frac{92}{x}\right)-\frac{x\left(3-5x^2\right)}{16\left(x^4+3x^2+2\right)}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^2),x]
```

output
```
-1/16*(x*(3 - 5*x^2))/(2 + 3*x^2 + x^4) + (-16/(5*x^5) + 44/(3*x^3) - 92/x
 - 184*ArcTan[x] + (97*ArcTan[x/Sqrt[2]])/Sqrt[2])/16
```

3.89.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

3.89.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{13x}{16(x^2+2)}+\frac{97\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{32}-\frac{1}{5x^5}+\frac{11}{12x^3}-\frac{23}{4x}-\frac{x}{2(x^2+1)}-\frac{23\arctan(x)}{2}$	53
risch	$\frac{-\frac{87}{16}x^8-\frac{793}{48}x^6-\frac{179}{20}x^4+\frac{37}{30}x^2-\frac{2}{5}}{x^5(x^4+3x^2+2)}+\frac{97\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{32}-\frac{23\arctan(x)}{2}$	56

input
```
int((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
13/16*x/(x^2+2)+97/32*arctan(1/2*x*2^(1/2))*2^(1/2)-1/5/x^5+11/12/x^3-23/4
/x-1/2*x/(x^2+1)-23/2*arctan(x)
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.22

$$\int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^2}\,dx = -\frac{2610\,x^8+7930\,x^6+4296\,x^4-1455\,\sqrt{2}(x^9+3\,x^7+2\,x^5)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-592\,x^2+5520\,(x^9+3\,x^7+}{480\,(x^9+3\,x^7+2\,x^5)}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
-1/480*(2610*x^8 + 7930*x^6 + 4296*x^4 - 1455*sqrt(2)*(x^9 + 3*x^7 + 2*x^5
)*arctan(1/2*sqrt(2)*x) - 592*x^2 + 5520*(x^9 + 3*x^7 + 2*x^5)*arctan(x) +
 192)/(x^9 + 3*x^7 + 2*x^5)
```

3.89.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^2}\,dx = -\frac{23\,\mathrm{atan}\,(x)}{2} + \frac{97\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{32} + \frac{-1305x^8-3965x^6-2148x^4+296x^2-96}{240x^9+720x^7+480x^5}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**6/(x**4+3*x**2+2)**2,x)
```

output
```
-23*atan(x)/2 + 97*sqrt(2)*atan(sqrt(2)*x/2)/32 + (-1305*x**8 - 3965*x**6
- 2148*x**4 + 296*x**2 - 96)/(240*x**9 + 720*x**7 + 480*x**5)
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^2}\,dx = \frac{97}{32}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - \frac{1305\,x^8+3965\,x^6+2148\,x^4-296\,x^2+96}{240\,(x^9+3\,x^7+2\,x^5)} - \frac{23}{2}\,\arctan\left(x\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
97/32*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/240*(1305*x^8 + 3965*x^6 + 2148*x^
4 - 296*x^2 + 96)/(x^9 + 3*x^7 + 2*x^5) - 23/2*arctan(x)
```

3.89.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^2}\,dx = \frac{97}{32}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + \frac{5\,x^3-3\,x}{16\,(x^4+3\,x^2+2)} - \frac{345\,x^4-55\,x^2+12}{60\,x^5} - \frac{23}{2}\arctan\left(x\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
97/32*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/16*(5*x^3 - 3*x)/(x^4 + 3*x^2 + 2)
 - 1/60*(345*x^4 - 55*x^2 + 12)/x^5 - 23/2*arctan(x)
```

3.89.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^2}\,dx = \frac{97\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{32} - \frac{23\,\mathrm{atan}(x)}{2} - \frac{\frac{87\,x^8}{16}+\frac{793\,x^6}{48}+\frac{179\,x^4}{20}-\frac{37\,x^2}{30}+\frac{2}{5}}{x^9+3\,x^7+2\,x^5}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^6*(3*x^2 + x^4 + 2)^2),x)
```

output
```
(97*2^(1/2)*atan((2^(1/2)*x)/2))/32 - (23*atan(x))/2 - ((179*x^4)/20 - (37
*x^2)/30 + (793*x^6)/48 + (87*x^8)/16 + 2/5)/(2*x^5 + 3*x^7 + x^9)
```

3.90 $\int \frac{4+x^2+3x^4+5x^6}{x^8(2+3x^2+x^4)^2}\,dx$

3.90.1 Optimal result

Integrand size = 31, antiderivative size = 76

$$\int \frac{4+x^2+3x^4+5x^6}{x^8\left(2+3x^2+x^4\right)^2}\,dx=-\frac{1}{7x^7}+\frac{11}{20x^5}-\frac{23}{12x^3}+\frac{137}{16x}+\frac{x(19+3x^2)}{32\left(2+3x^2+x^4\right)}+\frac{25\arctan(x)}{2}-\frac{123\arctan\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

output
```
-1/7/x^7+11/20/x^5-23/12/x^3+137/16/x+1/32*x*(3*x^2+19)/(x^4+3*x^2+2)+25/2
*arctan(x)-123/64*arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.90.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01

$$\int \frac{4+x^2+3x^4+5x^6}{x^8\left(2+3x^2+x^4\right)^2}\,dx=-\frac{1}{7x^7}+\frac{11}{20x^5}-\frac{23}{12x^3}+\frac{137}{16x}+\frac{19x+3x^3}{32\left(2+3x^2+x^4\right)}+\frac{25\arctan(x)}{2}-\frac{123\arctan\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^8*(2 + 3*x^2 + x^4)^2),x]
```

output
```
-1/7*1/x^7 + 11/(20*x^5) - 23/(12*x^3) + 137/(16*x) + (19*x + 3*x^3)/(32*(
2 + 3*x^2 + x^4)) + (25*ArcTan[x])/2 - (123*ArcTan[x/Sqrt[2]])/(32*Sqrt[2]
)
```

3.90.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2198, 27, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^8\left(x^4+3x^2+2\right)^2}dx$$

$$\Big\downarrow \text{2198}$$

$$\frac{x\left(3x^2+19\right)}{32\left(x^4+3x^2+2\right)}-\frac{1}{4}\int -\frac{3x^{10}+39x^8-84x^6+136x^4-80x^2+64}{8x^8\left(x^4+3x^2+2\right)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\int \frac{3x^{10}+39x^8-84x^6+136x^4-80x^2+64}{x^8\left(x^4+3x^2+2\right)}dx+\frac{x\left(3x^2+19\right)}{32\left(x^4+3x^2+2\right)}$$

$$\Big\downarrow \text{2195}$$

$$\frac{1}{32}\int \left(-\frac{123}{x^2+2}-\frac{274}{x^2}+\frac{184}{x^4}-\frac{88}{x^6}+\frac{32}{x^8}+\frac{400}{x^2+1}\right)dx+\frac{x\left(3x^2+19\right)}{32\left(x^4+3x^2+2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{32}\left(400\arctan(x)-\frac{123\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{32}{7x^7}+\frac{88}{5x^5}-\frac{184}{3x^3}+\frac{274}{x}\right)+\frac{x\left(3x^2+19\right)}{32\left(x^4+3x^2+2\right)}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^8*(2 + 3*x^2 + x^4)^2),x]
```

output
```
(x*(19 + 3*x^2))/(32*(2 + 3*x^2 + x^4)) + (-32/(7*x^7) + 88/(5*x^5) - 184/
(3*x^3) + 274/x + 400*ArcTan[x] - (123*ArcTan[x/Sqrt[2]])/Sqrt[2])/32
```

3.90.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

3.90.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{13x}{32(x^2+2)}-\frac{123\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{64}-\frac{1}{7x^7}+\frac{11}{20x^5}-\frac{23}{12x^3}+\frac{137}{16x}+\frac{x}{2x^2+2}+\frac{25\arctan(x)}{2}$	58
risch	$\frac{\frac{277}{32}x^{10}+\frac{2339}{96}x^8+\frac{477}{40}x^6-\frac{977}{420}x^4+\frac{47}{70}x^2-\frac{2}{7}}{x^7(x^4+3x^2+2)}-\frac{123\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{64}+\frac{25\arctan(x)}{2}$	61

input
```
int((5*x^6+3*x^4+x^2+4)/x^8/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
-13/32*x/(x^2+2)-123/64*arctan(1/2*x*2^(1/2))*2^(1/2)-1/7/x^7+11/20/x^5-23
/12/x^3+137/16/x+1/2*x/(x^2+1)+25/2*arctan(x)
```

3.90.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.17

$$\int \frac{4+x^2+3x^4+5x^6}{x^8\left(2+3x^2+x^4\right)^2}\,dx$$
$$= \frac{58170\,x^{10}+163730\,x^{8}+80136\,x^{6}-15632\,x^{4}-12915\,\sqrt{2}(x^{11}+3\,x^{9}+2\,x^{7})\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+4512\,x^{2}+}{6720\,(x^{11}+3\,x^{9}+2\,x^{7})}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^8/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
1/6720*(58170*x^10 + 163730*x^8 + 80136*x^6 - 15632*x^4 - 12915*sqrt(2)*(x
^11 + 3*x^9 + 2*x^7)*arctan(1/2*sqrt(2)*x) + 4512*x^2 + 84000*(x^11 + 3*x^
9 + 2*x^7)*arctan(x) - 1920)/(x^11 + 3*x^9 + 2*x^7)
```

3.90.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \frac{4+x^2+3x^4+5x^6}{x^8\left(2+3x^2+x^4\right)^2}\,dx = \frac{25\,\mathrm{atan}\,(x)}{2}-\frac{123\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{64} + \frac{29085x^{10}+81865x^{8}+40068x^{6}-7816x^{4}+2256x^{2}-960}{3360x^{11}+10080x^{9}+6720x^{7}}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**8/(x**4+3*x**2+2)**2,x)
```

output
```
25*atan(x)/2 - 123*sqrt(2)*atan(sqrt(2)*x/2)/64 + (29085*x**10 + 81865*x**
8 + 40068*x**6 - 7816*x**4 + 2256*x**2 - 960)/(3360*x**11 + 10080*x**9 + 6
720*x**7)
```

3.90.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.82

$$\int \frac{4+x^2+3x^4+5x^6}{x^8\left(2+3x^2+x^4\right)^2}\,dx = -\frac{123}{64}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + \frac{29085\,x^{10}+81865\,x^8+40068\,x^6-7816\,x^4+2256\,x^2-960}{3360\,(x^{11}+3\,x^9+2\,x^7)} + \frac{25}{2}\,\arctan\left(x\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^8/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
-123/64*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/3360*(29085*x^10 + 81865*x^8 + 4
0068*x^6 - 7816*x^4 + 2256*x^2 - 960)/(x^11 + 3*x^9 + 2*x^7) + 25/2*arctan
(x)
```

3.90.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.82

$$\int \frac{4+x^2+3x^4+5x^6}{x^8\left(2+3x^2+x^4\right)^2}\,dx = -\frac{123}{64}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + \frac{3\,x^3+19\,x}{32\,(x^4+3\,x^2+2)} + \frac{14385\,x^6-3220\,x^4+924\,x^2-240}{1680\,x^7} + \frac{25}{2}\,\arctan\left(x\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^8/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
-123/64*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/32*(3*x^3 + 19*x)/(x^4 + 3*x^2 +
 2) + 1/1680*(14385*x^6 - 3220*x^4 + 924*x^2 - 240)/x^7 + 25/2*arctan(x)
```

3.90.9 Mupad [B] (verification not implemented)

Time = 8.74 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

$$\int \frac{4+x^2+3x^4+5x^6}{x^8 (2+3x^2+x^4)^2} \, dx = \frac{25 \operatorname{atan}(x)}{2} - \frac{123 \sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{64} + \frac{\frac{277\,x^{10}}{32} + \frac{2339\,x^8}{96} + \frac{477\,x^6}{40} - \frac{977\,x^4}{420} + \frac{47\,x^2}{70} - \frac{2}{7}}{x^{11} + 3\,x^9 + 2\,x^7}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^8*(3*x^2 + x^4 + 2)^2),x)
```

output
```
(25*atan(x))/2 - (123*2^(1/2)*atan((2^(1/2)*x)/2))/64 + ((47*x^2)/70 - (97
7*x^4)/420 + (477*x^6)/40 + (2339*x^8)/96 + (277*x^10)/32 - 2/7)/(2*x^7 +
3*x^9 + x^11)
```

3.91 $\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$

3.91.1 Optimal result

Integrand size = 31, antiderivative size = 81

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = 214x-14x^3+x^5+\frac{x(414+415x^2)}{4(2+3x^2+x^4)^2}+\frac{x(824+1669x^2)}{8(2+3x^2+x^4)} + \frac{477\arctan(x)}{8}-351\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

output
```
214*x-14*x^3+x^5+1/4*x*(415*x^2+414)/(x^4+3*x^2+2)^2+1/8*x*(1669*x^2+824)/
(x^4+3*x^2+2)+477/8*arctan(x)-351*arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.91.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{x(9324+26736x^2+26775x^4+10581x^6+1144x^8-64x^{10}+8x^{12})}{8(2+3x^2+x^4)^2} + \frac{477\arctan(x)}{8}-351\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

input
```
Integrate[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

output
```
(x*(9324 + 26736*x^2 + 26775*x^4 + 10581*x^6 + 1144*x^8 - 64*x^10 + 8*x^12
))/(8*(2 + 3*x^2 + x^4)^2) + (477*ArcTan[x])/8 - 351*Sqrt[2]*ArcTan[x/Sqrt
[2]]
```

3.91.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2197, 27, 2206, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}(5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^3}dx$$

$$\Big\downarrow \text{2197}$$

$$\frac{x(415x^2+414)}{4(x^4+3x^2+2)^2}-\frac{1}{8}\int \frac{2(-20x^{12}+48x^{10}-108x^8+212x^6-420x^4-1239x^2+414)}{(x^4+3x^2+2)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x(415x^2+414)}{4(x^4+3x^2+2)^2}-\frac{1}{4}\int \frac{-20x^{12}+48x^{10}-108x^8+212x^6-420x^4-1239x^2+414}{(x^4+3x^2+2)^2}dx$$

$$\Big\downarrow \text{2206}$$

$$\frac{1}{4}\left(\frac{1}{4}\int -\frac{2(-40x^8+216x^6-784x^4+675x^2+1238)}{x^4+3x^2+2}dx+\frac{x(1669x^2+824)}{2(x^4+3x^2+2)}\right)+\frac{x(415x^2+414)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{x(1669x^2+824)}{2(x^4+3x^2+2)}-\frac{1}{2}\int \frac{-40x^8+216x^6-784x^4+675x^2+1238}{x^4+3x^2+2}dx\right)+\frac{x(415x^2+414)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{2205}$$

$$\frac{1}{4}\left(\frac{x(1669x^2+824)}{2(x^4+3x^2+2)}-\frac{1}{2}\int \left(-40x^4+336x^2+\frac{9(571x^2+518)}{x^4+3x^2+2}-1712\right)dx\right)+\frac{x(415x^2+414)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(477\arctan(x)-2808\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+8x^5-112x^3+1712x\right)+\frac{x(1669x^2+824)}{2(x^4+3x^2+2)}\right)+\frac{x(415x^2+414)}{4(x^4+3x^2+2)^2}$$

input
```
Int[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

output
```
(x*(414 + 415*x^2))/(4*(2 + 3*x^2 + x^4)^2) + ((x*(824 + 1669*x^2))/(2*(2
+ 3*x^2 + x^4)) + (1712*x - 112*x^3 + 8*x^5 + 477*ArcTan[x] - 2808*Sqrt[2]
*ArcTan[x/Sqrt[2]])/2)/4
```

3.91.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

3.91.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.75

method	result	size
risch	$x^5-14x^3+214x+\frac{\frac{1669}{8}x^7+\frac{5831}{8}x^5+830x^3+\frac{619}{2}x}{(x^4+3x^2+2)^2}+\frac{477\arctan(x)}{8}-351\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$	61
default	$-\frac{16\left(-\frac{105}{8}x^3-\frac{79}{4}x\right)}{(x^2+2)^2}-351\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}+x^5-14x^3+214x+\frac{-\frac{11}{8}x^3-\frac{13}{8}x}{(x^2+1)^2}+\frac{477\arctan(x)}{8}$	64

input
```
int(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
```

output
```
x^5-14*x^3+214*x+(1669/8*x^7+5831/8*x^5+830*x^3+619/2*x)/(x^4+3*x^2+2)^2+4
77/8*arctan(x)-351*arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.41

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{8\,x^{13}-64\,x^{11}+1144\,x^9+10581\,x^7+26775\,x^5+26736\,x^3-2808\,\sqrt{2}(x^8+6\,x^6+13\,x^4+12\,x^2+4)\,\mathrm{arct}}{8\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}$$

input
```
integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

output
```
1/8*(8*x^13 - 64*x^11 + 1144*x^9 + 10581*x^7 + 26775*x^5 + 26736*x^3 - 280
8*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(1/2*sqrt(2)*x) + 477*
(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(x) + 9324*x)/(x^8 + 6*x^6 + 13*
x^4 + 12*x^2 + 4)
```

3.91.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = x^5-14x^3+214x+\frac{1669x^7+5831x^5+6640x^3+2476x}{8x^8+48x^6+104x^4+96x^2+32} + \frac{477\operatorname{atan}(x)}{8}-351\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

input
```
integrate(x**10*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

output
```
x**5 - 14*x**3 + 214*x + (1669*x**7 + 5831*x**5 + 6640*x**3 + 2476*x)/(8*x
**8 + 48*x**6 + 104*x**4 + 96*x**2 + 32) + 477*atan(x)/8 - 351*sqrt(2)*ata
n(sqrt(2)*x/2)
```

3.91.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = x^5-14\,x^3-351\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+214\,x + \frac{1669\,x^7+5831\,x^5+6640\,x^3+2476\,x}{8\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}+\frac{477}{8}\,\arctan(x)$$

input
```
integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

output
```
x^5 - 14*x^3 - 351*sqrt(2)*arctan(1/2*sqrt(2)*x) + 214*x + 1/8*(1669*x^7 +
 5831*x^5 + 6640*x^3 + 2476*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) + 477/8
*arctan(x)
```

3.91.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.75

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = x^5 - 14\,x^3 - 351\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + 214\,x + \frac{1669\,x^7 + 5831\,x^5 + 6640\,x^3 + 2476\,x}{8\,(x^4 + 3\,x^2 + 2)^2} + \frac{477}{8}\,\arctan(x)$$

input
```
integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

output
```
x^5 - 14*x^3 - 351*sqrt(2)*arctan(1/2*sqrt(2)*x) + 214*x + 1/8*(1669*x^7 +
 5831*x^5 + 6640*x^3 + 2476*x)/(x^4 + 3*x^2 + 2)^2 + 477/8*arctan(x)
```

3.91.9 Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = 214\,x + \frac{477\,\mathrm{atan}(x)}{8} - 351\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right) + \frac{\frac{1669\,x^7}{8} + \frac{5831\,x^5}{8} + 830\,x^3 + \frac{619\,x}{2}}{x^8 + 6\,x^6 + 13\,x^4 + 12\,x^2 + 4} - 14\,x^3 + x^5$$

input
```
int((x^10*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^3,x)
```

output
```
214*x + (477*atan(x))/8 - 351*2^(1/2)*atan((2^(1/2)*x)/2) + ((619*x)/2 + 8
30*x^3 + (5831*x^5)/8 + (1669*x^7)/8)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4)
- 14*x^3 + x^5
```

3.92 $\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$

3.92.1 Optimal result

Integrand size = 31, antiderivative size = 80

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = -42x+\frac{5x^3}{3}-\frac{x(206+207x^2)}{4(2+3x^2+x^4)^2}+\frac{x(24-409x^2)}{8(2+3x^2+x^4)} - \frac{449\arctan(x)}{8}+\frac{219\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

output
```
-42*x+5/3*x^3-1/4*x*(207*x^2+206)/(x^4+3*x^2+2)^2+1/8*x*(-409*x^2+24)/(x^4
+3*x^2+2)-449/8*arctan(x)+219/2*arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.92.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.82

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{x(-5124-15416x^2-16233x^4-6755x^6-768x^8+40x^{10})}{24(2+3x^2+x^4)^2} - \frac{449\arctan(x)}{8}+\frac{219\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

input
```
Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

output
```
(x*(-5124 - 15416*x^2 - 16233*x^4 - 6755*x^6 - 768*x^8 + 40*x^10))/(24*(2
+ 3*x^2 + x^4)^2) - (449*ArcTan[x])/8 + (219*ArcTan[x/Sqrt[2]])/Sqrt[2]
```

3.92.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2197, 27, 2206, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 (5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^3} \, dx$$

$$\Big\downarrow \text{2197}$$

$$-\frac{1}{8} \int -\frac{2 (20x^{10}-48x^8+108x^6-212x^4-615x^2+206)}{(x^4+3x^2+2)^2} dx - \frac{x (207x^2+206)}{4 (x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4} \int \frac{20x^{10}-48x^8+108x^6-212x^4-615x^2+206}{(x^4+3x^2+2)^2} dx - \frac{x (207x^2+206)}{4 (x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{2206}$$

$$\frac{1}{4} \left(\frac{x (24-409x^2)}{2 (x^4+3x^2+2)} - \frac{1}{4} \int -\frac{2 (40x^6-216x^4+375x^2+182)}{x^4+3x^2+2} dx \right) - \frac{x (207x^2+206)}{4 (x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4} \left(\frac{1}{2} \int \frac{40x^6-216x^4+375x^2+182}{x^4+3x^2+2} dx + \frac{x (24-409x^2)}{2 (x^4+3x^2+2)} \right) - \frac{x (207x^2+206)}{4 (x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{2205}$$

$$\frac{1}{4} \left(\frac{1}{2} \int \left(40x^2 + \frac{1303x^2+854}{x^4+3x^2+2} - 336 \right) dx + \frac{x (24-409x^2)}{2 (x^4+3x^2+2)} \right) - \frac{x (207x^2+206)}{4 (x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4} \left(\frac{1}{2} \left(-449 \arctan(x) + 876\sqrt{2} \arctan\left(\frac{x}{\sqrt{2}}\right) + \frac{40x^3}{3} - 336x \right) + \frac{x (24-409x^2)}{2 (x^4+3x^2+2)} \right) - \frac{x (207x^2+206)}{4 (x^4+3x^2+2)^2}$$

input
```
Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

output
```
-1/4*(x*(206 + 207*x^2))/(2 + 3*x^2 + x^4)^2 + ((x*(24 - 409*x^2))/(2*(2 +
 3*x^2 + x^4)) + (-336*x + (40*x^3)/3 - 449*ArcTan[x] + 876*Sqrt[2]*ArcTan
[x/Sqrt[2]])/2)/4
```

3.92.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

3.92. $\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3} \, dx$

3.92.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.72

method	result	size
risch	$\frac{5x^3}{3} - 42x + \frac{-\frac{409}{8}x^7 - \frac{1203}{8}x^5 - 145x^3 - \frac{91}{2}x}{(x^4+3x^2+2)^2} - \frac{449\arctan(x)}{8} + \frac{219\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{2}$	58
default	$\frac{-53x^3-54x}{(x^2+2)^2} + \frac{219\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{2} + \frac{5x^3}{3} - 42x - \frac{-\frac{15}{8}x^3 - \frac{17}{8}x}{(x^2+1)^2} - \frac{449\arctan(x)}{8}$	62

input
```
int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
```

output
```
5/3*x^3-42*x+(-409/8*x^7-1203/8*x^5-145*x^3-91/2*x)/(x^4+3*x^2+2)^2-449/8*
arctan(x)+219/2*arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.36

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{40\,x^{11} - 768\,x^9 - 6755\,x^7 - 16233\,x^5 - 15416\,x^3 + 2628\,\sqrt{2}(x^8+6\,x^6+13\,x^4+12\,x^2+4)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right.}{24\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}$$

input
```
integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

output
```
1/24*(40*x^11 - 768*x^9 - 6755*x^7 - 16233*x^5 - 15416*x^3 + 2628*sqrt(2)*
(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(1/2*sqrt(2)*x) - 1347*(x^8 + 6*
x^6 + 13*x^4 + 12*x^2 + 4)*arctan(x) - 5124*x)/(x^8 + 6*x^6 + 13*x^4 + 12*
x^2 + 4)
```

3.92.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.95

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{5x^3}{3} - 42x + \frac{-409x^7 - 1203x^5 - 1160x^3 - 364x}{8x^8 + 48x^6 + 104x^4 + 96x^2 + 32} - \frac{449\,\mathrm{atan}\,(x)}{8} + \frac{219\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

input
```
integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

output
```
5*x**3/3 - 42*x + (-409*x**7 - 1203*x**5 - 1160*x**3 - 364*x)/(8*x**8 + 48
*x**6 + 104*x**4 + 96*x**2 + 32) - 449*atan(x)/8 + 219*sqrt(2)*atan(sqrt(2
)*x/2)/2
```

3.92.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{5}{3}\,x^3 + \frac{219}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - 42\,x - \frac{409\,x^7 + 1203\,x^5 + 1160\,x^3 + 364\,x}{8\,(x^8 + 6\,x^6 + 13\,x^4 + 12\,x^2 + 4)} - \frac{449}{8}\,\arctan\left(x\right)$$

input
```
integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

output
```
5/3*x^3 + 219/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 42*x - 1/8*(409*x^7 + 1203
*x^5 + 1160*x^3 + 364*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) - 449/8*arcta
n(x)
```

3.92.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.72

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{5}{3}\,x^3 + \frac{219}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - 42\,x - \frac{409\,x^7+1203\,x^5+1160\,x^3+364\,x}{8\,(x^4+3\,x^2+2)^2} - \frac{449}{8}\,\arctan\left(x\right)$$

input
```
integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

output
```
5/3*x^3 + 219/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 42*x - 1/8*(409*x^7 + 1203
*x^5 + 1160*x^3 + 364*x)/(x^4 + 3*x^2 + 2)^2 - 449/8*arctan(x)
```

3.92.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{219\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{2} - \frac{449\,\mathrm{atan}(x)}{8} - 42\,x - \frac{\frac{409\,x^7}{8}+\frac{1203\,x^5}{8}+145\,x^3+\frac{91\,x}{2}}{x^8+6\,x^6+13\,x^4+12\,x^2+4} + \frac{5\,x^3}{3}$$

input
```
int((x^8*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^3,x)
```

output
```
(219*2^(1/2)*atan((2^(1/2)*x)/2))/2 - (449*atan(x))/8 - 42*x - ((91*x)/2 +
 145*x^3 + (1203*x^5)/8 + (409*x^7)/8)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4)
 + (5*x^3)/3
```

3.93 $\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$

3.93.1 Optimal result

Integrand size = 31, antiderivative size = 75

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = 5x+\frac{x(102+103x^2)}{4(2+3x^2+x^4)^2}-\frac{x(244+15x^2)}{8(2+3x^2+x^4)}+\frac{413\arctan(x)}{8}-\frac{191\arctan\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

output
```
5*x+1/4*x*(103*x^2+102)/(x^4+3*x^2+2)^2-1/8*x*(15*x^2+244)/(x^4+3*x^2+2)+4
13/8*arctan(x)-191/4*arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.93.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{1}{8}\left(\frac{x(-124-76x^2+231x^4+225x^6+40x^8)}{(2+3x^2+x^4)^2}+413\arctan(x)-382\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)$$

input
```
Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

output
```
((x*(-124 - 76*x^2 + 231*x^4 + 225*x^6 + 40*x^8))/(2 + 3*x^2 + x^4)^2 + 41
3*ArcTan[x] - 382*Sqrt[2]*ArcTan[x/Sqrt[2]])/8
```

3.93.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2197, 27, 2206, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 (5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^3} dx$$

$$\Big\downarrow \text{2197}$$

$$\frac{x(103x^2+102)}{4(x^4+3x^2+2)^2}-\frac{1}{8}\int \frac{2(-20x^8+48x^6-108x^4-303x^2+102)}{(x^4+3x^2+2)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x(103x^2+102)}{4(x^4+3x^2+2)^2}-\frac{1}{4}\int \frac{-20x^8+48x^6-108x^4-303x^2+102}{(x^4+3x^2+2)^2}dx$$

$$\Big\downarrow \text{2206}$$

$$\frac{1}{4}\left(\frac{1}{4}\int \frac{2(40x^4-231x^2+142)}{x^4+3x^2+2}dx-\frac{x(15x^2+244)}{2(x^4+3x^2+2)}\right)+\frac{x(103x^2+102)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{40x^4-231x^2+142}{x^4+3x^2+2}dx-\frac{x(15x^2+244)}{2(x^4+3x^2+2)}\right)+\frac{x(103x^2+102)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{2205}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \left(\frac{62-351x^2}{x^4+3x^2+2}+40\right)dx-\frac{x(15x^2+244)}{2(x^4+3x^2+2)}\right)+\frac{x(103x^2+102)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(413\arctan(x)-382\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+40x\right)-\frac{x(15x^2+244)}{2(x^4+3x^2+2)}\right)+\frac{x(103x^2+102)}{4(x^4+3x^2+2)^2}$$

input
```
Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

output
```
(x*(102 + 103*x^2))/(4*(2 + 3*x^2 + x^4)^2) + (-1/2*(x*(244 + 15*x^2))/(2
+ 3*x^2 + x^4) + (40*x + 413*ArcTan[x] - 382*Sqrt[2]*ArcTan[x/Sqrt[2]])/2)
/4
```

3.93.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

3.93. $\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$

3.93.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.71

method	result	size
risch	$5x+\frac{-\frac{15}{8}x^{7}-\frac{289}{8}x^{5}-\frac{139}{2}x^{3}-\frac{71}{2}x}{\left(x^{4}+3x^{2}+2\right)^{2}}+\frac{413\arctan\left(x\right)}{8}-\frac{191\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{4}$	53
default	$5x-\frac{16\left(-\frac{1}{32}x^{3}+\frac{25}{16}x\right)}{\left(x^{2}+2\right)^{2}}-\frac{191\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{4}+\frac{-\frac{19}{8}x^{3}-\frac{21}{8}x}{\left(x^{2}+1\right)^{2}}+\frac{413\arctan\left(x\right)}{8}$	56

input
```
int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
```

output
```
5*x+(-15/8*x^7-289/8*x^5-139/2*x^3-71/2*x)/(x^4+3*x^2+2)^2+413/8*arctan(x)
-191/4*arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.39

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$$
$$=\frac{40\,x^{9}+225\,x^{7}+231\,x^{5}-76\,x^{3}-382\,\sqrt{2}(x^{8}+6\,x^{6}+13\,x^{4}+12\,x^{2}+4)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+413\,(x^{8}+6\,x^{6}}{8\,(x^{8}+6\,x^{6}+13\,x^{4}+12\,x^{2}+4)}$$

input
```
integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

output
```
1/8*(40*x^9 + 225*x^7 + 231*x^5 - 76*x^3 - 382*sqrt(2)*(x^8 + 6*x^6 + 13*x
^4 + 12*x^2 + 4)*arctan(1/2*sqrt(2)*x) + 413*(x^8 + 6*x^6 + 13*x^4 + 12*x^
2 + 4)*arctan(x) - 124*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
```

3.93.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = 5x + \frac{-15x^7-289x^5-556x^3-284x}{8x^8+48x^6+104x^4+96x^2+32} + \frac{413\,\mathrm{atan}\,(x)}{8} - \frac{191\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{4}$$

input
```
integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

output
```
5*x + (-15*x**7 - 289*x**5 - 556*x**3 - 284*x)/(8*x**8 + 48*x**6 + 104*x**
4 + 96*x**2 + 32) + 413*atan(x)/8 - 191*sqrt(2)*atan(sqrt(2)*x/2)/4
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = -\frac{191}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 5\,x - \frac{15\,x^7+289\,x^5+556\,x^3+284\,x}{8\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)} + \frac{413}{8}\arctan(x)$$

input
```
integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

output
```
-191/4*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x - 1/8*(15*x^7 + 289*x^5 + 556*x
^3 + 284*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) + 413/8*arctan(x)
```

3.93.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.71

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = -\frac{191}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 5\,x - \frac{15\,x^7+289\,x^5+556\,x^3+284\,x}{8\,(x^4+3\,x^2+2)^2} + \frac{413}{8}\arctan(x)$$

input
```
integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

output
```
-191/4*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x - 1/8*(15*x^7 + 289*x^5 + 556*x
^3 + 284*x)/(x^4 + 3*x^2 + 2)^2 + 413/8*arctan(x)
```

3.93.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = 5\,x+\frac{413\,\mathrm{atan}(x)}{8}-\frac{191\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{4} - \frac{\frac{15\,x^7}{8}+\frac{289\,x^5}{8}+\frac{139\,x^3}{2}+\frac{71\,x}{2}}{x^8+6\,x^6+13\,x^4+12\,x^2+4}$$

input
```
int((x^6*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^3,x)
```

output
```
5*x + (413*atan(x))/8 - (191*2^(1/2)*atan((2^(1/2)*x)/2))/4 - ((71*x)/2 +
(139*x^3)/2 + (289*x^5)/8 + (15*x^7)/8)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4
)
```

3.94 $\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$

3.94.1 Optimal result

Integrand size = 31, antiderivative size = 72

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = -\frac{x(50+51x^2)}{4(2+3x^2+x^4)^2}+\frac{x(254+125x^2)}{8(2+3x^2+x^4)} - \frac{369\arctan(x)}{8}+\frac{267\arctan\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

output
```
-1/4*x*(51*x^2+50)/(x^4+3*x^2+2)^2+1/8*x*(125*x^2+254)/(x^4+3*x^2+2)-369/8
*arctan(x)+267/8*arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.94.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{1}{8}\left(\frac{x(408+910x^2+629x^4+125x^6)}{(2+3x^2+x^4)^2}-369\arctan(x) + 267\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)$$

input
```
Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

output
```
((x*(408 + 910*x^2 + 629*x^4 + 125*x^6))/(2 + 3*x^2 + x^4)^2 - 369*ArcTan[
x] + 267*Sqrt[2]*ArcTan[x/Sqrt[2]])/8
```

3.94.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2197, 27, 2206, 27, 1480, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^3}dx$$

$$\Big\downarrow \text{2197}$$

$$-\frac{1}{8}\int -\frac{2(20x^6-48x^4-147x^2+50)}{(x^4+3x^2+2)^2}dx-\frac{x(51x^2+50)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{20x^6-48x^4-147x^2+50}{(x^4+3x^2+2)^2}dx-\frac{x(51x^2+50)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{2206}$$

$$\frac{1}{4}\left(\frac{x(125x^2+254)}{2(x^4+3x^2+2)}-\frac{1}{4}\int \frac{6(68-55x^2)}{x^4+3x^2+2}dx\right)-\frac{x(51x^2+50)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{x(125x^2+254)}{2(x^4+3x^2+2)}-\frac{3}{2}\int \frac{68-55x^2}{x^4+3x^2+2}dx\right)-\frac{x(51x^2+50)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{4}\left(\frac{x(125x^2+254)}{2(x^4+3x^2+2)}-\frac{3}{2}\left(123\int \frac{1}{x^2+1}dx-178\int \frac{1}{x^2+2}dx\right)\right)-\frac{x(51x^2+50)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(\frac{x(125x^2+254)}{2(x^4+3x^2+2)}-\frac{3}{2}\left(123\arctan(x)-89\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)\right)-\frac{x(51x^2+50)}{4(x^4+3x^2+2)^2}$$

input
```
Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

output
```
-1/4*(x*(50 + 51*x^2))/(2 + 3*x^2 + x^4)^2 + ((x*(254 + 125*x^2))/(2*(2 +
3*x^2 + x^4)) - (3*(123*ArcTan[x] - 89*Sqrt[2]*ArcTan[x/Sqrt[2]]))/2)/4
```

3.94.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

3.94. $\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$

3.94.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{\frac{125}{8}x^7+\frac{629}{8}x^5+\frac{455}{4}x^3+51x}{\left(x^4+3x^2+2\right)^2}+\frac{267\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{8}-\frac{369\arctan\left(x\right)}{8}$	50
default	$\frac{\frac{51}{4}x^3+\frac{77}{2}x}{\left(x^2+2\right)^2}+\frac{267\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{8}-\frac{-\frac{23}{8}x^3-\frac{25}{8}x}{\left(x^2+1\right)^2}-\frac{369\arctan\left(x\right)}{8}$	54

input
```
int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
```

output
```
(125/8*x^7+629/8*x^5+455/4*x^3+51*x)/(x^4+3*x^2+2)^2+267/8*arctan(1/2*x*2^
(1/2))*2^(1/2)-369/8*arctan(x)
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.38

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$$
$$= \frac{125\,x^7+629\,x^5+910\,x^3+267\,\sqrt{2}(x^8+6\,x^6+13\,x^4+12\,x^2+4)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-369\,(x^8+6\,x^6+13\,x}{8\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}$$

input
```
integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

output
```
1/8*(125*x^7 + 629*x^5 + 910*x^3 + 267*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 + 12*
x^2 + 4)*arctan(1/2*sqrt(2)*x) - 369*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*a
rctan(x) + 408*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
```

3.94.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{125x^7+629x^5+910x^3+408x}{8x^8+48x^6+104x^4+96x^2+32} - \frac{369\,\mathrm{atan}\,(x)}{8} + \frac{267\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{8}$$

input
```
integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

output
```
(125*x**7 + 629*x**5 + 910*x**3 + 408*x)/(8*x**8 + 48*x**6 + 104*x**4 + 96
*x**2 + 32) - 369*atan(x)/8 + 267*sqrt(2)*atan(sqrt(2)*x/2)/8
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{267}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + \frac{125\,x^7+629\,x^5+910\,x^3+408\,x}{8\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)} - \frac{369}{8}\arctan\left(x\right)$$

input
```
integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

output
```
267/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/8*(125*x^7 + 629*x^5 + 910*x^3 + 4
08*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) - 369/8*arctan(x)
```

3.94.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{267}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + \frac{125\,x^7+629\,x^5+910\,x^3+408\,x}{8\,(x^4+3\,x^2+2)^2} - \frac{369}{8}\arctan\left(x\right)$$

input
```
integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

output
```
267/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/8*(125*x^7 + 629*x^5 + 910*x^3 + 4
08*x)/(x^4 + 3*x^2 + 2)^2 - 369/8*arctan(x)
```

3.94.9 Mupad [B] (verification not implemented)

Time = 8.56 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{267\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{8} - \frac{369\,\mathrm{atan}(x)}{8} + \frac{\frac{125\,x^7}{8}+\frac{629\,x^5}{8}+\frac{455\,x^3}{4}+51\,x}{x^8+6\,x^6+13\,x^4+12\,x^2+4}$$

input
```
int((x^4*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^3,x)
```

output
```
(267*2^(1/2)*atan((2^(1/2)*x)/2))/8 - (369*atan(x))/8 + (51*x + (455*x^3)/
4 + (629*x^5)/8 + (125*x^7)/8)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4)
```

3.95 $\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$

3.95.1 Optimal result

Integrand size = 31, antiderivative size = 72

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{x(24+25x^2)}{4(2+3x^2+x^4)^2} - \frac{x(211+130x^2)}{8(2+3x^2+x^4)} + \frac{317\arctan(x)}{8} - \frac{447\arctan\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}$$

output
```
1/4*x*(25*x^2+24)/(x^4+3*x^2+2)^2-1/8*x*(130*x^2+211)/(x^4+3*x^2+2)+317/8*
arctan(x)-447/16*arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.95.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{1}{16}\left(-\frac{2x(374+843x^2+601x^4+130x^6)}{(2+3x^2+x^4)^2} + 634\arctan(x) - 447\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)$$

input
```
Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

output
```
((-2*x*(374 + 843*x^2 + 601*x^4 + 130*x^6))/(2 + 3*x^2 + x^4)^2 + 634*ArcT
an[x] - 447*Sqrt[2]*ArcTan[x/Sqrt[2]])/16
```

3.95.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2197, 27, 2206, 27, 1480, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^3}dx$$

$$\Big\downarrow \text{2197}$$

$$\frac{x(25x^2+24)}{4(x^4+3x^2+2)^2}-\frac{1}{8}\int \frac{2(-20x^4-77x^2+24)}{(x^4+3x^2+2)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x(25x^2+24)}{4(x^4+3x^2+2)^2}-\frac{1}{4}\int \frac{-20x^4-77x^2+24}{(x^4+3x^2+2)^2}dx$$

$$\Big\downarrow \text{2206}$$

$$\frac{1}{4}\left(\frac{1}{4}\int \frac{2(187-130x^2)}{x^4+3x^2+2}dx-\frac{x(130x^2+211)}{2(x^4+3x^2+2)}\right)+\frac{x(25x^2+24)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{187-130x^2}{x^4+3x^2+2}dx-\frac{x(130x^2+211)}{2(x^4+3x^2+2)}\right)+\frac{x(25x^2+24)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(317\int \frac{1}{x^2+1}dx-447\int \frac{1}{x^2+2}dx\right)-\frac{x(130x^2+211)}{2(x^4+3x^2+2)}\right)+\frac{x(25x^2+24)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(317\arctan(x)-\frac{447\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}\right)-\frac{x(130x^2+211)}{2(x^4+3x^2+2)}\right)+\frac{x(25x^2+24)}{4(x^4+3x^2+2)^2}$$

input
```
Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

output
```
(x*(24 + 25*x^2))/(4*(2 + 3*x^2 + x^4)^2) + (-1/2*(x*(211 + 130*x^2))/(2 +
 3*x^2 + x^4) + (317*ArcTan[x] - (447*ArcTan[x/Sqrt[2]])/Sqrt[2])/2)/4
```

3.95.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

3.95. $\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$

3.95.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{-\frac{65}{4}x^7-\frac{601}{8}x^5-\frac{843}{8}x^3-\frac{187}{4}x}{(x^4+3x^2+2)^2}-\frac{447\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{16}+\frac{317\arctan(x)}{8}$	50
default	$-\frac{\frac{103}{8}x^3+\frac{129}{4}x}{(x^2+2)^2}-\frac{447\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{16}+\frac{-\frac{27}{8}x^3-\frac{29}{8}x}{(x^2+1)^2}+\frac{317\arctan(x)}{8}$	53

input
```
int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-65/4*x^7-601/8*x^5-843/8*x^3-187/4*x)/(x^4+3*x^2+2)^2-447/16*arctan(1/2*
x*2^(1/2))*2^(1/2)+317/8*arctan(x)
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.38

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = -\frac{260\,x^7+1202\,x^5+1686\,x^3+447\,\sqrt{2}(x^8+6\,x^6+13\,x^4+12\,x^2+4)\arctan\left(\frac{1}{2}\sqrt{2}x\right)-634\,(x^8+6\,x^6+}{16\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}$$

input
```
integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

output
```
-1/16*(260*x^7 + 1202*x^5 + 1686*x^3 + 447*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 +
 12*x^2 + 4)*arctan(1/2*sqrt(2)*x) - 634*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 +
4)*arctan(x) + 748*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
```

3.95.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{-130x^7-601x^5-843x^3-374x}{8x^8+48x^6+104x^4+96x^2+32} + \frac{317\operatorname{atan}(x)}{8} - \frac{447\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{16}$$

input
```
integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

output
```
(-130*x**7 - 601*x**5 - 843*x**3 - 374*x)/(8*x**8 + 48*x**6 + 104*x**4 + 9
6*x**2 + 32) + 317*atan(x)/8 - 447*sqrt(2)*atan(sqrt(2)*x/2)/16
```

3.95.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = -\frac{447}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{130\,x^7+601\,x^5+843\,x^3+374\,x}{8\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)} + \frac{317}{8}\arctan(x)$$

input
```
integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

output
```
-447/16*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/8*(130*x^7 + 601*x^5 + 843*x^3 +
 374*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) + 317/8*arctan(x)
```

3.95.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = -\frac{447}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{130\,x^7+601\,x^5+843\,x^3+374\,x}{8\,(x^4+3\,x^2+2)^2} + \frac{317}{8}\arctan(x)$$

input
```
integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

output
```
-447/16*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/8*(130*x^7 + 601*x^5 + 843*x^3 +
 374*x)/(x^4 + 3*x^2 + 2)^2 + 317/8*arctan(x)
```

3.95.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{317\,\mathrm{atan}(x)}{8} - \frac{447\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{16} - \frac{\frac{65\,x^7}{4}+\frac{601\,x^5}{8}+\frac{843\,x^3}{8}+\frac{187\,x}{4}}{x^8+6\,x^6+13\,x^4+12\,x^2+4}$$

input
```
int((x^2*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^3,x)
```

output
```
(317*atan(x))/8 - (447*2^(1/2)*atan((2^(1/2)*x)/2))/16 - ((187*x)/4 + (843
*x^3)/8 + (601*x^5)/8 + (65*x^7)/4)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4)
```

3.96 $\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^3} dx$

3.96.1 Optimal result

Integrand size = 28, antiderivative size = 72

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^3} dx = -\frac{x(11+12x^2)}{4(2+3x^2+x^4)^2}+\frac{x(335+217x^2)}{16(2+3x^2+x^4)} - \frac{257\arctan(x)}{8}+\frac{731\arctan\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}$$

output
```
-1/4*x*(12*x^2+11)/(x^4+3*x^2+2)^2+1/16*x*(217*x^2+335)/(x^4+3*x^2+2)-257/
8*arctan(x)+731/32*arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.96.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^3} dx = \frac{1}{32}\left(\frac{2x(626+1391x^2+986x^4+217x^6)}{(2+3x^2+x^4)^2} - 1028\arctan(x) + 731\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^3,x]
```

output
```
((2*x*(626 + 1391*x^2 + 986*x^4 + 217*x^6))/(2 + 3*x^2 + x^4)^2 - 1028*Arc
Tan[x] + 731*Sqrt[2]*ArcTan[x/Sqrt[2]])/32
```

3.96.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2206, 27, 1492, 1480, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{(x^4+3x^2+2)^3}dx$$

$$\Big\downarrow \text{2206}$$

$$-\frac{1}{8}\int -\frac{2(19-40x^2)}{(x^4+3x^2+2)^2}dx-\frac{x(12x^2+11)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{19-40x^2}{(x^4+3x^2+2)^2}dx-\frac{x(12x^2+11)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{1492}$$

$$\frac{1}{4}\left(\frac{x(217x^2+335)}{4(x^4+3x^2+2)}-\frac{1}{4}\int \frac{297-217x^2}{x^4+3x^2+2}dx\right)-\frac{x(12x^2+11)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{4}\left(\frac{1}{4}\left(731\int \frac{1}{x^2+2}dx-514\int \frac{1}{x^2+1}dx\right)+\frac{x(217x^2+335)}{4(x^4+3x^2+2)}\right)-\frac{x(12x^2+11)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(\frac{1}{4}\left(\frac{731\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-514\arctan(x)\right)+\frac{x(217x^2+335)}{4(x^4+3x^2+2)}\right)-\frac{x(12x^2+11)}{4(x^4+3x^2+2)^2}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^3,x]
```

output
```
-1/4*(x*(11 + 12*x^2))/(2 + 3*x^2 + x^4)^2 + ((x*(335 + 217*x^2))/(4*(2 +
3*x^2 + x^4)) + (-514*ArcTan[x] + (731*ArcTan[x/Sqrt[2]])/Sqrt[2])/4)/4
```

3.96.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

3.96. $\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^3} dx$

3.96.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{\frac{217}{16}x^7+\frac{493}{8}x^5+\frac{1391}{16}x^3+\frac{313}{8}x}{\left(x^4+3x^2+2\right)^2}-\frac{257\arctan\left(x\right)}{8}+\frac{731\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{32}$	50
default	$\frac{\frac{155}{16}x^3+\frac{181}{8}x}{\left(x^2+2\right)^2}+\frac{731\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{32}-\frac{-\frac{31}{8}x^3-\frac{33}{8}x}{\left(x^2+1\right)^2}-\frac{257\arctan\left(x\right)}{8}$	53

input
```
int((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
```

output
```
(217/16*x^7+493/8*x^5+1391/16*x^3+313/8*x)/(x^4+3*x^2+2)^2-257/8*arctan(x)
+731/32*arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.38

$$\int \frac{4+x^2+3x^4+5x^6}{\left(2+3x^2+x^4\right)^3}\,dx$$
$$=\frac{434\,x^7+1972\,x^5+2782\,x^3+731\,\sqrt{2}(x^8+6\,x^6+13\,x^4+12\,x^2+4)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-1028\,(x^8+6\,x^6+}{32\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

output
```
1/32*(434*x^7 + 1972*x^5 + 2782*x^3 + 731*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 +
12*x^2 + 4)*arctan(1/2*sqrt(2)*x) - 1028*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 +
4)*arctan(x) + 1252*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
```

3.96.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^3}\,dx = \frac{217x^7+986x^5+1391x^3+626x}{16x^8+96x^6+208x^4+192x^2+64} - \frac{257\,\mathrm{atan}\,(x)}{8} + \frac{731\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{32}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

output
```
(217*x**7 + 986*x**5 + 1391*x**3 + 626*x)/(16*x**8 + 96*x**6 + 208*x**4 +
192*x**2 + 64) - 257*atan(x)/8 + 731*sqrt(2)*atan(sqrt(2)*x/2)/32
```

3.96.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^3}\,dx = \frac{731}{32}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + \frac{217\,x^7+986\,x^5+1391\,x^3+626\,x}{16\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)} - \frac{257}{8}\arctan\left(x\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

output
```
731/32*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/16*(217*x^7 + 986*x^5 + 1391*x^3
+ 626*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) - 257/8*arctan(x)
```

3.96.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^3}\,dx = \frac{731}{32}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + \frac{217\,x^7+986\,x^5+1391\,x^3+626\,x}{16\,(x^4+3\,x^2+2)^2} - \frac{257}{8}\arctan\left(x\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

output
```
731/32*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/16*(217*x^7 + 986*x^5 + 1391*x^3
+ 626*x)/(x^4 + 3*x^2 + 2)^2 - 257/8*arctan(x)
```

3.96.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^3} \, dx = \frac{731\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{32} - \frac{257\,\mathrm{atan}(x)}{8} + \frac{\frac{217\,x^7}{16}+\frac{493\,x^5}{8}+\frac{1391\,x^3}{16}+\frac{313\,x}{8}}{x^8+6\,x^6+13\,x^4+12\,x^2+4}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(3*x^2 + x^4 + 2)^3,x)
```

output
```
(731*2^(1/2)*atan((2^(1/2)*x)/2))/32 - (257*atan(x))/8 + ((313*x)/8 + (139
1*x^3)/16 + (493*x^5)/8 + (217*x^7)/16)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4
)
```

3.97 $\int \frac{4+x^2+3x^4+5x^6}{x^2(2+3x^2+x^4)^3}\,dx$

3.97.1 Optimal result

Integrand size = 31, antiderivative size = 79

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^3}\,dx=-\frac{1}{2x}+\frac{x(9+11x^2)}{8\left(2+3x^2+x^4\right)^2}-\frac{x(547+347x^2)}{32\left(2+3x^2+x^4\right)}+\frac{189\arctan(x)}{8}-\frac{1119\arctan\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

output
```
-1/2/x+1/8*x*(11*x^2+9)/(x^4+3*x^2+2)^2-1/32*x*(347*x^2+547)/(x^4+3*x^2+2)
+189/8*arctan(x)-1119/64*arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.97.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.80

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^3}\,dx=\frac{1}{64}\left(-\frac{2(64+1250x^2+2499x^4+1684x^6+363x^8)}{x\left(2+3x^2+x^4\right)^2}+1512\arctan(x)-1119\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^3),x]
```

output
```
((-2*(64 + 1250*x^2 + 2499*x^4 + 1684*x^6 + 363*x^8))/(x*(2 + 3*x^2 + x^4)
^2) + 1512*ArcTan[x] - 1119*Sqrt[2]*ArcTan[x/Sqrt[2]])/64
```

3.97.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2198, 25, 2198, 25, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^2 \left(x^4+3x^2+2\right)^3} \, dx$$

$$\Big\downarrow \text{2198}$$

$$\frac{x \left(11x^2+9\right)}{8 \left(x^4+3x^2+2\right)^2}-\frac{1}{8} \int -\frac{55x^4-29x^2+16}{x^2 \left(x^4+3x^2+2\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{8} \int \frac{55x^4-29x^2+16}{x^2 \left(x^4+3x^2+2\right)^2}dx+\frac{x \left(11x^2+9\right)}{8 \left(x^4+3x^2+2\right)^2}$$

$$\Big\downarrow \text{2198}$$

$$\frac{1}{8} \left(-\frac{1}{4} \int -\frac{-347x^4+441x^2+32}{x^2 \left(x^4+3x^2+2\right)}dx-\frac{x \left(347x^2+547\right)}{4 \left(x^4+3x^2+2\right)}\right)+\frac{x \left(11x^2+9\right)}{8 \left(x^4+3x^2+2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{8} \left(\frac{1}{4} \int \frac{-347x^4+441x^2+32}{x^2 \left(x^4+3x^2+2\right)}dx-\frac{x \left(347x^2+547\right)}{4 \left(x^4+3x^2+2\right)}\right)+\frac{x \left(11x^2+9\right)}{8 \left(x^4+3x^2+2\right)^2}$$

$$\Big\downarrow \text{2195}$$

$$\frac{1}{8} \left(\frac{1}{4} \int \left(-\frac{1119}{x^2+2}+\frac{16}{x^2}+\frac{756}{x^2+1}\right)dx-\frac{x \left(347x^2+547\right)}{4 \left(x^4+3x^2+2\right)}\right)+\frac{x \left(11x^2+9\right)}{8 \left(x^4+3x^2+2\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8} \left(\frac{1}{4} \left(756 \arctan(x)-\frac{1119 \arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{16}{x}\right)-\frac{x \left(347x^2+547\right)}{4 \left(x^4+3x^2+2\right)}\right)+\frac{x \left(11x^2+9\right)}{8 \left(x^4+3x^2+2\right)^2}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^3),x]
```

output
```
(x*(9 + 11*x^2))/(8*(2 + 3*x^2 + x^4)^2) + (-1/4*(x*(547 + 347*x^2))/(2 +
3*x^2 + x^4) + (-16/x + 756*ArcTan[x] - (1119*ArcTan[x/Sqrt[2]])/Sqrt[2])/
4)/8
```

3.97.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

3.97.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.71

method	result	size
risch	$\frac{-\frac{363}{32}x^8-\frac{421}{8}x^6-\frac{2499}{32}x^4-\frac{625}{16}x^2-2}{x(x^4+3x^2+2)^2}-\frac{1119\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{64}+\frac{189\arctan(x)}{8}$	56
default	$-\frac{\frac{207}{16}x^3+\frac{233}{8}x}{2(x^2+2)^2}-\frac{1119\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{64}-\frac{1}{2x}+\frac{-\frac{35}{8}x^3-\frac{37}{8}x}{(x^2+1)^2}+\frac{189\arctan(x)}{8}$	58

input
```
int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-363/32*x^8-421/8*x^6-2499/32*x^4-625/16*x^2-2)/x/(x^4+3*x^2+2)^2-1119/64
*arctan(1/2*x*2^(1/2))*2^(1/2)+189/8*arctan(x)
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.37

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^3}\,dx = -\frac{726\,x^8+3368\,x^6+4998\,x^4+1119\,\sqrt{2}(x^9+6\,x^7+13\,x^5+12\,x^3+4\,x)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+2500\,x^2-15}{64\left(x^9+6\,x^7+13\,x^5+12\,x^3+4\,x\right)}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

output
```
-1/64*(726*x^8 + 3368*x^6 + 4998*x^4 + 1119*sqrt(2)*(x^9 + 6*x^7 + 13*x^5
+ 12*x^3 + 4*x)*arctan(1/2*sqrt(2)*x) + 2500*x^2 - 1512*(x^9 + 6*x^7 + 13*
x^5 + 12*x^3 + 4*x)*arctan(x) + 128)/(x^9 + 6*x^7 + 13*x^5 + 12*x^3 + 4*x)
```

3.97.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^3}\,dx = \frac{-363x^8-1684x^6-2499x^4-1250x^2-64}{32x^9+192x^7+416x^5+384x^3+128x} + \frac{189\,\mathrm{atan}\,(x)}{8} - \frac{1119\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{64}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+3*x**2+2)**3,x)
```

output
```
(-363*x**8 - 1684*x**6 - 2499*x**4 - 1250*x**2 - 64)/(32*x**9 + 192*x**7 +
 416*x**5 + 384*x**3 + 128*x) + 189*atan(x)/8 - 1119*sqrt(2)*atan(sqrt(2)*
x/2)/64
```

3.97.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.82

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^3}\,dx = -\frac{1119}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{363\,x^8+1684\,x^6+2499\,x^4+1250\,x^2+64}{32\left(x^9+6\,x^7+13\,x^5+12\,x^3+4\,x\right)} + \frac{189}{8}\arctan\left(x\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

output
```
-1119/64*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/32*(363*x^8 + 1684*x^6 + 2499*x
^4 + 1250*x^2 + 64)/(x^9 + 6*x^7 + 13*x^5 + 12*x^3 + 4*x) + 189/8*arctan(x
)
```

3.97.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.70

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^3}\,dx = -\frac{1119}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{347\,x^7+1588\,x^5+2291\,x^3+1058\,x}{32\left(x^4+3\,x^2+2\right)^2} - \frac{1}{2\,x} + \frac{189}{8}\arctan\left(x\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

output
```
-1119/64*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/32*(347*x^7 + 1588*x^5 + 2291*x
^3 + 1058*x)/(x^4 + 3*x^2 + 2)^2 - 1/2/x + 189/8*arctan(x)
```

3.97.9 Mupad [B] (verification not implemented)

Time = 8.84 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.82

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^3}\,dx = \frac{189\,\mathrm{atan}(x)}{8} - \frac{1119\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{64} - \frac{\frac{363\,x^8}{32}+\frac{421\,x^6}{8}+\frac{2499\,x^4}{32}+\frac{625\,x^2}{16}+2}{x^9+6\,x^7+13\,x^5+12\,x^3+4\,x}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^2*(3*x^2 + x^4 + 2)^3),x)
```

output
```
(189*atan(x))/8 - (1119*2^(1/2)*atan((2^(1/2)*x)/2))/64 - ((625*x^2)/16 +
(2499*x^4)/32 + (421*x^6)/8 + (363*x^8)/32 + 2)/(4*x + 12*x^3 + 13*x^5 + 6
*x^7 + x^9)
```

3.97. $\int \frac{4+x^2+3x^4+5x^6}{x^2(2+3x^2+x^4)^3}\,dx$

3.98 $\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^3}\,dx$

3.98.1 Optimal result

Integrand size = 31, antiderivative size = 86

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^3}\,dx=-\frac{1}{6x^3}+\frac{17}{8x}-\frac{x(5+9x^2)}{16\left(2+3x^2+x^4\right)^2}+\frac{x(951+571x^2)}{64\left(2+3x^2+x^4\right)}-\frac{113\arctan(x)}{8}+\frac{1611\arctan\left(\frac{x}{\sqrt{2}}\right)}{64\sqrt{2}}$$

output
```
-1/6/x^3+17/8/x-1/16*x*(9*x^2+5)/(x^4+3*x^2+2)^2+1/64*x*(571*x^2+951)/(x^4
+3*x^2+2)-113/8*arctan(x)+1611/128*arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.98.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^3}\,dx=\frac{1}{384}\left(-\frac{64}{x^3}+\frac{816}{x}-\frac{24x(5+9x^2)}{\left(2+3x^2+x^4\right)^2}+\frac{6x(951+571x^2)}{2+3x^2+x^4}-5424\arctan(x)+4833\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^3),x]
```

output
```
(-64/x^3 + 816/x - (24*x*(5 + 9*x^2))/(2 + 3*x^2 + x^4)^2 + (6*x*(951 + 57
1*x^2))/(2 + 3*x^2 + x^4) - 5424*ArcTan[x] + 4833*Sqrt[2]*ArcTan[x/Sqrt[2]
])/384
```

3.98.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2198, 27, 2198, 25, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^4\left(x^4+3x^2+2\right)^3}dx$$

$$\Big\downarrow \text{ 2198}$$

$$-\frac{1}{8}\int -\frac{-45x^6+73x^4-40x^2+32}{2x^4\left(x^4+3x^2+2\right)^2}dx-\frac{x\left(9x^2+5\right)}{16\left(x^4+3x^2+2\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{16}\int \frac{-45x^6+73x^4-40x^2+32}{x^4\left(x^4+3x^2+2\right)^2}dx-\frac{x\left(9x^2+5\right)}{16\left(x^4+3x^2+2\right)^2}$$

$$\Big\downarrow \text{ 2198}$$

$$\frac{1}{16}\left(\frac{x\left(571x^2+951\right)}{4\left(x^4+3x^2+2\right)}-\frac{1}{4}\int -\frac{571x^6-573x^4-176x^2+64}{x^4\left(x^4+3x^2+2\right)}dx\right)-\frac{x\left(9x^2+5\right)}{16\left(x^4+3x^2+2\right)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{16}\left(\frac{1}{4}\int \frac{571x^6-573x^4-176x^2+64}{x^4\left(x^4+3x^2+2\right)}dx+\frac{x\left(571x^2+951\right)}{4\left(x^4+3x^2+2\right)}\right)-\frac{x\left(9x^2+5\right)}{16\left(x^4+3x^2+2\right)^2}$$

$$\Big\downarrow \text{ 2195}$$

$$\frac{1}{16}\left(\frac{1}{4}\int \left(\frac{1611}{x^2+2}-\frac{136}{x^2}+\frac{32}{x^4}-\frac{904}{x^2+1}\right)dx+\frac{x\left(571x^2+951\right)}{4\left(x^4+3x^2+2\right)}\right)-\frac{x\left(9x^2+5\right)}{16\left(x^4+3x^2+2\right)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{16}\left(\frac{1}{4}\left(-904\arctan(x)+\frac{1611\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{32}{3x^3}+\frac{136}{x}\right)+\frac{x(571x^2+951)}{4(x^4+3x^2+2)}\right)-\frac{x(9x^2+5)}{16(x^4+3x^2+2)^2}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^3),x]
```

output
```
-1/16*(x*(5 + 9*x^2))/(2 + 3*x^2 + x^4)^2 + ((x*(951 + 571*x^2))/(4*(2 + 3
*x^2 + x^4)) + (-32/(3*x^3) + 136/x - 904*ArcTan[x] + (1611*ArcTan[x/Sqrt[
2]])/Sqrt[2])/4)/16
```

3.98.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

3.98.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.71

method	result	size
risch	$\frac{\frac{707}{64}x^{10}+\frac{1301}{24}x^{8}+\frac{5663}{64}x^{6}+\frac{5063}{96}x^{4}+\frac{13}{2}x^{2}-\frac{2}{3}}{x^{3}(x^{4}+3x^{2}+2)^{2}}+\frac{1611\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{128}-\frac{113\arctan(x)}{8}$	61
default	$\frac{\frac{259}{8}x^{3}+\frac{285}{4}x}{8(x^{2}+2)^{2}}+\frac{1611\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{128}-\frac{1}{6x^{3}}+\frac{17}{8x}-\frac{-\frac{39}{8}x^{3}-\frac{41}{8}x}{(x^{2}+1)^{2}}-\frac{113\arctan(x)}{8}$	64

input
```
int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
```

output
```
(707/64*x^10+1301/24*x^8+5663/64*x^6+5063/96*x^4+13/2*x^2-2/3)/x^3/(x^4+3*
x^2+2)^2+1611/128*arctan(1/2*x*2^(1/2))*2^(1/2)-113/8*arctan(x)
```

3.98.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.38

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^3}\,dx = \frac{4242\,x^{10}+20816\,x^{8}+33978\,x^{6}+20252\,x^{4}+4833\,\sqrt{2}(x^{11}+6\,x^{9}+13\,x^{7}+12\,x^{5}+4\,x^{3})\arctan\left(\frac{1}{2}\sqrt{2}x\right)}{384\,(x^{11}+6\,x^{9}+13\,x^{7}+12\,x^{5}+4\,x^{3}}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

output
```
1/384*(4242*x^10 + 20816*x^8 + 33978*x^6 + 20252*x^4 + 4833*sqrt(2)*(x^11
+ 6*x^9 + 13*x^7 + 12*x^5 + 4*x^3)*arctan(1/2*sqrt(2)*x) + 2496*x^2 - 5424
*(x^11 + 6*x^9 + 13*x^7 + 12*x^5 + 4*x^3)*arctan(x) - 256)/(x^11 + 6*x^9 +
 13*x^7 + 12*x^5 + 4*x^3)
```

3.98.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^3}\,dx = -\frac{113\,\mathrm{atan}\,(x)}{8} + \frac{1611\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{128} + \frac{2121x^{10}+10408x^8+16989x^6+10126x^4+1248x^2-128}{192x^{11}+1152x^9+2496x^7+2304x^5+768x^3}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+3*x**2+2)**3,x)
```

output
```
-113*atan(x)/8 + 1611*sqrt(2)*atan(sqrt(2)*x/2)/128 + (2121*x**10 + 10408*
x**8 + 16989*x**6 + 10126*x**4 + 1248*x**2 - 128)/(192*x**11 + 1152*x**9 +
 2496*x**7 + 2304*x**5 + 768*x**3)
```

3.98.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.84

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^3}\,dx = \frac{1611}{128}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + \frac{2121\,x^{10}+10408\,x^8+16989\,x^6+10126\,x^4+1248\,x^2-128}{192\,(x^{11}+6\,x^9+13\,x^7+12\,x^5+4\,x^3)} - \frac{113}{8}\arctan(x)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

output
```
1611/128*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/192*(2121*x^10 + 10408*x^8 + 16
989*x^6 + 10126*x^4 + 1248*x^2 - 128)/(x^11 + 6*x^9 + 13*x^7 + 12*x^5 + 4*
x^3) - 113/8*arctan(x)
```

3.98.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.72

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^3}\,dx = \frac{1611}{128}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + \frac{571\,x^7+2664\,x^5+3959\,x^3+1882\,x}{64\,\left(x^4+3\,x^2+2\right)^2} + \frac{51\,x^2-4}{24\,x^3} - \frac{113}{8}\arctan\left(x\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

output
```
1611/128*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/64*(571*x^7 + 2664*x^5 + 3959*x
^3 + 1882*x)/(x^4 + 3*x^2 + 2)^2 + 1/24*(51*x^2 - 4)/x^3 - 113/8*arctan(x)
```

3.98.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^3}\,dx = \frac{\frac{707\,x^{10}}{64}+\frac{1301\,x^8}{24}+\frac{5663\,x^6}{64}+\frac{5063\,x^4}{96}+\frac{13\,x^2}{2}-\frac{2}{3}}{x^{11}+6\,x^9+13\,x^7+12\,x^5+4\,x^3} - \frac{113\,\mathrm{atan}(x)}{8} + \frac{1611\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{128}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^4*(3*x^2 + x^4 + 2)^3),x)
```

output
```
((13*x^2)/2 + (5063*x^4)/96 + (5663*x^6)/64 + (1301*x^8)/24 + (707*x^10)/6
4 - 2/3)/(4*x^3 + 12*x^5 + 13*x^7 + 6*x^9 + x^11) - (113*atan(x))/8 + (161
1*2^(1/2)*atan((2^(1/2)*x)/2))/128
```

3.99 $\int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^3} \, dx$

3.99.1 Optimal result

Integrand size = 31, antiderivative size = 93

$$\int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^3} \, dx = -\frac{1}{10x^5}+\frac{17}{24x^3}-\frac{93}{16x}-\frac{x(3-5x^2)}{32(2+3x^2+x^4)^2} -\frac{x(1771+999x^2)}{128(2+3x^2+x^4)}+\frac{29\arctan(x)}{8}-\frac{2207\arctan\left(\frac{x}{\sqrt{2}}\right)}{128\sqrt{2}}$$

output
```
-1/10/x^5+17/24/x^3-93/16/x-1/32*x*(-5*x^2+3)/(x^4+3*x^2+2)^2-1/128*x*(999
*x^2+1771)/(x^4+3*x^2+2)+29/8*arctan(x)-2207/256*arctan(1/2*x*2^(1/2))*2^(
1/2)
```

3.99.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.78

$$\int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^3} \, dx = \frac{-\frac{2(768-3136x^2+30816x^4+170702x^6+246477x^8+137120x^{10}+26145x^{12})}{x^5(2+3x^2+x^4)^2}+13920\arctan(x)-33105\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)}{3840}$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^3),x]
```

output
```
((-2*(768 - 3136*x^2 + 30816*x^4 + 170702*x^6 + 246477*x^8 + 137120*x^10 +
 26145*x^12))/(x^5*(2 + 3*x^2 + x^4)^2) + 13920*ArcTan[x] - 33105*Sqrt[2]*
ArcTan[x/Sqrt[2]])/3840
```

3.99.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2198, 27, 2198, 25, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^6\left(x^4+3x^2+2\right)^3}dx$$

$$\Big\downarrow \text{ 2198}$$

$$-\frac{1}{8}\int -\frac{25x^8-81x^6+136x^4-80x^2+64}{4x^6\left(x^4+3x^2+2\right)^2}dx-\frac{x\left(3-5x^2\right)}{32\left(x^4+3x^2+2\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{32}\int \frac{25x^8-81x^6+136x^4-80x^2+64}{x^6\left(x^4+3x^2+2\right)^2}dx-\frac{x\left(3-5x^2\right)}{32\left(x^4+3x^2+2\right)^2}$$

$$\Big\downarrow \text{ 2198}$$

$$\frac{1}{32}\left(-\frac{1}{4}\int -\frac{-999x^8+681x^6+736x^4-352x^2+128}{x^6\left(x^4+3x^2+2\right)}dx-\frac{x\left(999x^2+1771\right)}{4\left(x^4+3x^2+2\right)}\right)-\frac{x\left(3-5x^2\right)}{32\left(x^4+3x^2+2\right)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{32}\left(\frac{1}{4}\int \frac{-999x^8+681x^6+736x^4-352x^2+128}{x^6\left(x^4+3x^2+2\right)}dx-\frac{x\left(999x^2+1771\right)}{4\left(x^4+3x^2+2\right)}\right)-\frac{x\left(3-5x^2\right)}{32\left(x^4+3x^2+2\right)^2}$$

$$\Big\downarrow \text{ 2195}$$

$$\frac{1}{32}\left(\frac{1}{4}\int \left(-\frac{2207}{x^2+2}+\frac{744}{x^2}-\frac{272}{x^4}+\frac{64}{x^6}+\frac{464}{x^2+1}\right)dx-\frac{x\left(999x^2+1771\right)}{4\left(x^4+3x^2+2\right)}\right)-\frac{x\left(3-5x^2\right)}{32\left(x^4+3x^2+2\right)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{32}\left(\frac{1}{4}\left(464\arctan(x)-\frac{2207\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{64}{5x^5}+\frac{272}{3x^3}-\frac{744}{x}\right)-\frac{x(999x^2+1771)}{4(x^4+3x^2+2)}\right)-\frac{x(3-5x^2)}{32(x^4+3x^2+2)^2}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^3),x]
```

output
```
-1/32*(x*(3 - 5*x^2))/(2 + 3*x^2 + x^4)^2 + (-1/4*(x*(1771 + 999*x^2))/(2
+ 3*x^2 + x^4) + (-64/(5*x^5) + 272/(3*x^3) - 744/x + 464*ArcTan[x] - (220
7*ArcTan[x/Sqrt[2]])/Sqrt[2])/4)/32
```

3.99.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

3.99.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.71

method	result	size
risch	$\frac{-\frac{1743}{128}x^{12}-\frac{857}{12}x^{10}-\frac{82159}{640}x^8-\frac{85351}{960}x^6-\frac{321}{20}x^4+\frac{49}{30}x^2-\frac{2}{5}}{x^5(x^4+3x^2+2)^2}+\frac{29\arctan(x)}{8}-\frac{2207\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{256}$	66
default	$-\frac{\frac{311}{8}x^3+\frac{337}{4}x}{16(x^2+2)^2}-\frac{2207\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{256}-\frac{1}{10x^5}+\frac{17}{24x^3}-\frac{93}{16x}+\frac{-\frac{43}{8}x^3-\frac{45}{8}x}{(x^2+1)^2}+\frac{29\arctan(x)}{8}$	68

input
```
int((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1743/128*x^12-857/12*x^10-82159/640*x^8-85351/960*x^6-321/20*x^4+49/30*x
^2-2/5)/x^5/(x^4+3*x^2+2)^2+29/8*arctan(x)-2207/256*arctan(1/2*x*2^(1/2))*
2^(1/2)
```

3.99.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.33

$$\int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^3}\,dx =$$
$$-\frac{52290\,x^{12}+274240\,x^{10}+492954\,x^8+341404\,x^6+61632\,x^4+33105\,\sqrt{2}(x^{13}+6\,x^{11}+13\,x^9+12\,x^7+}{3840\,(x^{13}+6\,x^{11}+13\,x^9}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

output
```
-1/3840*(52290*x^12 + 274240*x^10 + 492954*x^8 + 341404*x^6 + 61632*x^4 +
33105*sqrt(2)*(x^13 + 6*x^11 + 13*x^9 + 12*x^7 + 4*x^5)*arctan(1/2*sqrt(2)
*x) - 6272*x^2 - 13920*(x^13 + 6*x^11 + 13*x^9 + 12*x^7 + 4*x^5)*arctan(x)
 + 1536)/(x^13 + 6*x^11 + 13*x^9 + 12*x^7 + 4*x^5)
```

3.99.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.88

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^3}\,dx$$
$$= \frac{29\,\mathrm{atan}\,(x)}{8} - \frac{2207\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{256} + \frac{-26145x^{12}-137120x^{10}-246477x^8-170702x^6-30816x^4+3136x^2-768}{1920x^{13}+11520x^{11}+24960x^9+23040x^7+7680x^5}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**6/(x**4+3*x**2+2)**3,x)
```

output
```
29*atan(x)/8 - 2207*sqrt(2)*atan(sqrt(2)*x/2)/256 + (-26145*x**12 - 137120
*x**10 - 246477*x**8 - 170702*x**6 - 30816*x**4 + 3136*x**2 - 768)/(1920*x
**13 + 11520*x**11 + 24960*x**9 + 23040*x**7 + 7680*x**5)
```

3.99.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^3}\,dx$$
$$= -\frac{2207}{256}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{26145\,x^{12}+137120\,x^{10}+246477\,x^8+170702\,x^6+30816\,x^4-3136\,x^2+768}{1920\left(x^{13}+6\,x^{11}+13\,x^9+12\,x^7+4\,x^5\right)} + \frac{29}{8}\arctan\left(x\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

output
```
-2207/256*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/1920*(26145*x^12 + 137120*x^10
 + 246477*x^8 + 170702*x^6 + 30816*x^4 - 3136*x^2 + 768)/(x^13 + 6*x^11 +
13*x^9 + 12*x^7 + 4*x^5) + 29/8*arctan(x)
```

3.99.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.72

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^3}\,dx = -\frac{2207}{256}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{999\,x^7+4768\,x^5+7291\,x^3+3554\,x}{128\left(x^4+3\,x^2+2\right)^2} - \frac{1395\,x^4-170\,x^2+24}{240\,x^5} + \frac{29}{8}\arctan\left(x\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

output
```
-2207/256*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/128*(999*x^7 + 4768*x^5 + 7291
*x^3 + 3554*x)/(x^4 + 3*x^2 + 2)^2 - 1/240*(1395*x^4 - 170*x^2 + 24)/x^5 +
 29/8*arctan(x)
```

3.99.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^3}\,dx = \frac{29\,\mathrm{atan}(x)}{8} - \frac{2207\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{256} - \frac{\frac{1743\,x^{12}}{128}+\frac{857\,x^{10}}{12}+\frac{82159\,x^8}{640}+\frac{85351\,x^6}{960}+\frac{321\,x^4}{20}-\frac{49\,x^2}{30}+\frac{2}{5}}{x^{13}+6\,x^{11}+13\,x^9+12\,x^7+4\,x^5}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^6*(3*x^2 + x^4 + 2)^3),x)
```

output
```
(29*atan(x))/8 - (2207*2^(1/2)*atan((2^(1/2)*x)/2))/256 - ((321*x^4)/20 -
(49*x^2)/30 + (85351*x^6)/960 + (82159*x^8)/640 + (857*x^10)/12 + (1743*x^
12)/128 + 2/5)/(4*x^5 + 12*x^7 + 13*x^9 + 6*x^11 + x^13)
```

3.100 $\int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} \, dx$

3.100.1 Optimal result

Integrand size = 31, antiderivative size = 86

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} \, dx = 19x^2+\frac{19x^4}{4}-\frac{17x^6}{6}+\frac{5x^8}{8}-\frac{25(15+7x^2)}{8(3+2x^2+x^4)}+\frac{201\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}-\frac{183}{4}\log(3+2x^2+x^4)$$

output
```
19*x^2+19/4*x^4-17/6*x^6+5/8*x^8-25/8*(7*x^2+15)/(x^4+2*x^2+3)-183/4*ln(x^
4+2*x^2+3)+201/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

3.100.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} \, dx = \frac{1}{48}\left(912x^2+228x^4-136x^6+30x^8-\frac{150(15+7x^2)}{3+2x^2+x^4}+603\sqrt{2}\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)-2196\log(3+2x^2+x^4)\right)$$

input
```
Integrate[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
(912*x^2 + 228*x^4 - 136*x^6 + 30*x^8 - (150*(15 + 7*x^2))/(3 + 2*x^2 + x^
4) + 603*Sqrt[2]*ArcTan[(1 + x^2)/Sqrt[2]] - 2196*Log[3 + 2*x^2 + x^4])/48
```

3.100.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2194, 2191, 27, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9(5x^6+3x^4+x^2+4)}{(x^4+2x^2+3)^2}dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{x^8(5x^6+3x^4+x^2+4)}{(x^4+2x^2+3)^2}dx^2$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{2}\left(\frac{1}{8}\int -\frac{2(-20x^{10}+28x^8-100x^4+200x^2+75)}{x^4+2x^2+3}dx^2-\frac{25(7x^2+15)}{4(x^4+2x^2+3)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{1}{4}\int \frac{-20x^{10}+28x^8-100x^4+200x^2+75}{x^4+2x^2+3}dx^2-\frac{25(7x^2+15)}{4(x^4+2x^2+3)}\right)$$

$$\Big\downarrow \text{2188}$$

$$\frac{1}{2}\left(-\frac{1}{4}\int \left(-20x^6+68x^4-76x^2+\frac{3(244x^2+177)}{x^4+2x^2+3}-152\right)dx^2-\frac{25(7x^2+15)}{4(x^4+2x^2+3)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{201\arctan\left(\frac{x^2+1}{\sqrt{2}}\right)}{\sqrt{2}}+5x^8-\frac{68x^6}{3}+38x^4+152x^2-366\log\left(x^4+2x^2+3\right)\right)-\frac{25(7x^2+15)}{4(x^4+2x^2+3)}\right)$$

input
```
Int[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
((-25*(15 + 7*x^2))/(4*(3 + 2*x^2 + x^4)) + (152*x^2 + 38*x^4 - (68*x^6)/3
 + 5*x^8 + (201*ArcTan[(1 + x^2)/Sqrt[2]])/Sqrt[2] - 366*Log[3 + 2*x^2 + x
^4])/4)/2
```

3.100.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.100.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{5x^8}{8}-\frac{17x^6}{6}+\frac{19x^4}{4}+19x^2+\frac{-\frac{175x^2}{8}-\frac{375}{8}}{x^4+2x^2+3}-\frac{183\ln\left(x^4+2x^2+3\right)}{4}+\frac{201\arctan\left(\frac{\left(x^2+1\right)\sqrt{2}}{2}\right)\sqrt{2}}{16}$	71
default	$\frac{5x^8}{8}-\frac{17x^6}{6}+\frac{19x^4}{4}+19x^2-\frac{\frac{175x^2}{4}+\frac{375}{4}}{2(x^4+2x^2+3)}-\frac{183\ln\left(x^4+2x^2+3\right)}{4}+\frac{201\sqrt{2}\arctan\left(\frac{\left(2x^2+2\right)\sqrt{2}}{4}\right)}{16}$	74

3.100. $\int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

input
```
int(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
5/8*x^8-17/6*x^6+19/4*x^4+19*x^2+(-175/8*x^2-375/8)/(x^4+2*x^2+3)-183/4*ln
(x^4+2*x^2+3)+201/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

3.100.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.10

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$$
$$= \frac{30\,x^{12} - 76\,x^{10} + 46\,x^8 + 960\,x^6 + 2508\,x^4 + 603\,\sqrt{2}(x^4+2\,x^2+3)\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right) + 1686\,x^2 - 21}{48\,(x^4+2\,x^2+3)}$$

input
```
integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
1/48*(30*x^12 - 76*x^10 + 46*x^8 + 960*x^6 + 2508*x^4 + 603*sqrt(2)*(x^4 +
 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1686*x^2 - 2196*(x^4 + 2*x^2 +
 3)*log(x^4 + 2*x^2 + 3) - 2250)/(x^4 + 2*x^2 + 3)
```

3.100.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.01

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5x^8}{8} - \frac{17x^6}{6} + \frac{19x^4}{4} + 19x^2 + \frac{-175x^2-375}{8x^4+16x^2+24} - \frac{183\log(x^4+2x^2+3)}{4} + \frac{201\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

input
```
integrate(x**9*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

output
```
5*x**8/8 - 17*x**6/6 + 19*x**4/4 + 19*x**2 + (-175*x**2 - 375)/(8*x**4 + 1
6*x**2 + 24) - 183*log(x**4 + 2*x**2 + 3)/4 + 201*sqrt(2)*atan(sqrt(2)*x**
2/2 + sqrt(2)/2)/16
```

3.100.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5}{8}\,x^8 - \frac{17}{6}\,x^6 + \frac{19}{4}\,x^4 + 19\,x^2 + \frac{201}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right) - \frac{25\,(7\,x^2+15)}{8\,(x^4+2\,x^2+3)} - \frac{183}{4}\,\log\left(x^4+2\,x^2+3\right)$$

input
```
integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
5/8*x^8 - 17/6*x^6 + 19/4*x^4 + 19*x^2 + 201/16*sqrt(2)*arctan(1/2*sqrt(2)
*(x^2 + 1)) - 25/8*(7*x^2 + 15)/(x^4 + 2*x^2 + 3) - 183/4*log(x^4 + 2*x^2
+ 3)
```

3.100.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5}{8}\,x^8 - \frac{17}{6}\,x^6 + \frac{19}{4}\,x^4 + 19\,x^2 + \frac{201}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right) + \frac{366\,x^4+557\,x^2+723}{8\,(x^4+2\,x^2+3)} - \frac{183}{4}\,\log\left(x^4+2\,x^2+3\right)$$

input
```
integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
5/8*x^8 - 17/6*x^6 + 19/4*x^4 + 19*x^2 + 201/16*sqrt(2)*arctan(1/2*sqrt(2)
*(x^2 + 1)) + 1/8*(366*x^4 + 557*x^2 + 723)/(x^4 + 2*x^2 + 3) - 183/4*log(
x^4 + 2*x^2 + 3)
```

3.100.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.87

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{201\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16} - \frac{\frac{175\,x^2}{8}+\frac{375}{8}}{x^4+2\,x^2+3} - \frac{183\,\ln\left(x^4+2\,x^2+3\right)}{4} + 19\,x^2 + \frac{19\,x^4}{4} - \frac{17\,x^6}{6} + \frac{5\,x^8}{8}$$

input
```
int((x^9*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

output
```
(201*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/16 - ((175*x^2)/8 + 375/8)
/(2*x^2 + x^4 + 3) - (183*log(2*x^2 + x^4 + 3))/4 + 19*x^2 + (19*x^4)/4 -
(17*x^6)/6 + (5*x^8)/8
```

3.101 $\int \frac{x^7\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^2}\,dx$

3.101.1 Optimal result

Integrand size = 31, antiderivative size = 81

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{19x^2}{2}-\frac{17x^4}{4}+\frac{5x^6}{6}+\frac{25(3+5x^2)}{8(3+2x^2+x^4)} - \frac{455\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{19}{2}\log\left(3+2x^2+x^4\right)$$

output
```
19/2*x^2-17/4*x^4+5/6*x^6+25/8*(5*x^2+3)/(x^4+2*x^2+3)+19/2*ln(x^4+2*x^2+3
)-455/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

3.101.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{1}{48}\left(456x^2-204x^4+40x^6+\frac{150(3+5x^2)}{3+2x^2+x^4} - 1365\sqrt{2}\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)+456\log\left(3+2x^2+x^4\right)\right)$$

input
```
Integrate[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
(456*x^2 - 204*x^4 + 40*x^6 + (150*(3 + 5*x^2))/(3 + 2*x^2 + x^4) - 1365*S
qrt[2]*ArcTan[(1 + x^2)/Sqrt[2]] + 456*Log[3 + 2*x^2 + x^4])/48
```

3.101.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2194, 2191, 27, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 (5x^6+3x^4+x^2+4)}{(x^4+2x^2+3)^2} \, dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{x^6 (5x^6+3x^4+x^2+4)}{(x^4+2x^2+3)^2} dx^2$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{2}\left(\frac{1}{8}\int -\frac{2(-20x^8+28x^6-100x^2+75)}{x^4+2x^2+3}dx^2+\frac{25(5x^2+3)}{4(x^4+2x^2+3)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{25(5x^2+3)}{4(x^4+2x^2+3)}-\frac{1}{4}\int \frac{-20x^8+28x^6-100x^2+75}{x^4+2x^2+3}dx^2\right)$$

$$\Big\downarrow \text{2188}$$

$$\frac{1}{2}\left(\frac{25(5x^2+3)}{4(x^4+2x^2+3)}-\frac{1}{4}\int \left(-20x^4+68x^2+\frac{303-152x^2}{x^4+2x^2+3}-76\right)dx^2\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(-\frac{455\arctan\left(\frac{x^2+1}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{20x^6}{3}-34x^4+76x^2+76\log\left(x^4+2x^2+3\right)\right)+\frac{25(5x^2+3)}{4(x^4+2x^2+3)}\right)$$

input
```
Int[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
((25*(3 + 5*x^2))/(4*(3 + 2*x^2 + x^4)) + (76*x^2 - 34*x^4 + (20*x^6)/3 -
(455*ArcTan[(1 + x^2)/Sqrt[2]])/Sqrt[2] + 76*Log[3 + 2*x^2 + x^4])/4)/2
```

3.101.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.101.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{5x^6}{6}-\frac{17x^4}{4}+\frac{19x^2}{2}+\frac{\frac{125x^2}{8}+\frac{75}{8}}{x^4+2x^2+3}+\frac{19\ln\left(x^4+2x^2+3\right)}{2}-\frac{455\arctan\left(\frac{\left(x^2+1\right)\sqrt{2}}{2}\right)\sqrt{2}}{16}$	66
default	$\frac{5x^6}{6}-\frac{17x^4}{4}+\frac{19x^2}{2}+\frac{\frac{125x^2}{4}+\frac{75}{4}}{2x^4+4x^2+6}+\frac{19\ln\left(x^4+2x^2+3\right)}{2}-\frac{455\sqrt{2}\,\arctan\left(\frac{\left(2x^2+2\right)\sqrt{2}}{4}\right)}{16}$	69

input
```
int(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
5/6*x^6-17/4*x^4+19/2*x^2+(125/8*x^2+75/8)/(x^4+2*x^2+3)+19/2*ln(x^4+2*x^2
+3)-455/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

3.101.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$$
$$= \frac{40\,x^{10} - 124\,x^8 + 168\,x^6 + 300\,x^4 - 1365\,\sqrt{2}(x^4 + 2\,x^2 + 3)\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2 + 1)\right) + 2118\,x^2 + 456\,(x^4 +}{48\,(x^4 + 2\,x^2 + 3)}$$

input
```
integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
1/48*(40*x^10 - 124*x^8 + 168*x^6 + 300*x^4 - 1365*sqrt(2)*(x^4 + 2*x^2 +
3)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 2118*x^2 + 456*(x^4 + 2*x^2 + 3)*log(x^
4 + 2*x^2 + 3) + 450)/(x^4 + 2*x^2 + 3)
```

3.101.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5x^6}{6} - \frac{17x^4}{4} + \frac{19x^2}{2} + \frac{125x^2+75}{8x^4+16x^2+24} + \frac{19\log\left(x^4+2x^2+3\right)}{2} - \frac{455\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

input
```
integrate(x**7*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

output
```
5*x**6/6 - 17*x**4/4 + 19*x**2/2 + (125*x**2 + 75)/(8*x**4 + 16*x**2 + 24)
 + 19*log(x**4 + 2*x**2 + 3)/2 - 455*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)
/2)/16
```

3.101.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.81

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5}{6}\,x^6 - \frac{17}{4}\,x^4 + \frac{19}{2}\,x^2 - \frac{455}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right) + \frac{25\,(5\,x^2+3)}{8\,(x^4+2\,x^2+3)} + \frac{19}{2}\,\log\left(x^4+2\,x^2+3\right)$$

input
```
integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
5/6*x^6 - 17/4*x^4 + 19/2*x^2 - 455/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1
)) + 25/8*(5*x^2 + 3)/(x^4 + 2*x^2 + 3) + 19/2*log(x^4 + 2*x^2 + 3)
```

3.101.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5}{6}\,x^6 - \frac{17}{4}\,x^4 + \frac{19}{2}\,x^2 - \frac{455}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right) - \frac{76\,x^4+27\,x^2+153}{8\,(x^4+2\,x^2+3)} + \frac{19}{2}\,\log\left(x^4+2\,x^2+3\right)$$

input
```
integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
5/6*x^6 - 17/4*x^4 + 19/2*x^2 - 455/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1
)) - 1/8*(76*x^4 + 27*x^2 + 153)/(x^4 + 2*x^2 + 3) + 19/2*log(x^4 + 2*x^2
+ 3)
```

3.101.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{19\,\ln\left(x^4+2\,x^2+3\right)}{2} + \frac{\frac{125\,x^2}{8}+\frac{75}{8}}{x^4+2\,x^2+3} - \frac{455\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16} + \frac{19\,x^2}{2} - \frac{17\,x^4}{4} + \frac{5\,x^6}{6}$$

input
```
int((x^7*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

output
```
(19*log(2*x^2 + x^4 + 3))/2 + ((125*x^2)/8 + 75/8)/(2*x^2 + x^4 + 3) - (45
5*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/16 + (19*x^2)/2 - (17*x^4)/4
+ (5*x^6)/6
```

3.101. $\int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

3.102 $\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

3.102.1 Optimal result

Integrand size = 31, antiderivative size = 74

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = -\frac{17x^2}{2}+\frac{5x^4}{4}+\frac{25(3-x^2)}{8(3+2x^2+x^4)} + \frac{203\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{19}{4}\log\left(3+2x^2+x^4\right)$$

output
```
-17/2*x^2+5/4*x^4+25/8*(-x^2+3)/(x^4+2*x^2+3)+19/4*ln(x^4+2*x^2+3)+203/16*
arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

3.102.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{1}{16}\left(-136x^2+20x^4-\frac{50(-3+x^2)}{3+2x^2+x^4} + 203\sqrt{2}\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)+76\log\left(3+2x^2+x^4\right)\right)$$

input
```
Integrate[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
(-136*x^2 + 20*x^4 - (50*(-3 + x^2))/(3 + 2*x^2 + x^4) + 203*Sqrt[2]*ArcTa
n[(1 + x^2)/Sqrt[2]] + 76*Log[3 + 2*x^2 + x^4])/16
```

3.102.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2194, 2191, 27, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(5x^6+3x^4+x^2+4)}{(x^4+2x^2+3)^2}dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{x^4(5x^6+3x^4+x^2+4)}{(x^4+2x^2+3)^2}dx^2$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{2}\left(\frac{1}{8}\int \frac{2(20x^6-28x^4+75)}{x^4+2x^2+3}dx^2+\frac{25(3-x^2)}{4(x^4+2x^2+3)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{4}\int \frac{20x^6-28x^4+75}{x^4+2x^2+3}dx^2+\frac{25(3-x^2)}{4(x^4+2x^2+3)}\right)$$

$$\Big\downarrow \text{2188}$$

$$\frac{1}{2}\left(\frac{1}{4}\int \left(20x^2+\frac{76x^2+279}{x^4+2x^2+3}-68\right)dx^2+\frac{25(3-x^2)}{4(x^4+2x^2+3)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{203\arctan\left(\frac{x^2+1}{\sqrt{2}}\right)}{\sqrt{2}}+10x^4-68x^2+38\log\left(x^4+2x^2+3\right)\right)+\frac{25(3-x^2)}{4(x^4+2x^2+3)}\right)$$

input
```
Int[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
((25*(3 - x^2))/(4*(3 + 2*x^2 + x^4)) + (-68*x^2 + 10*x^4 + (203*ArcTan[(1
 + x^2)/Sqrt[2]])/Sqrt[2] + 38*Log[3 + 2*x^2 + x^4])/4)/2
```

3.102.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.102.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{5x^4}{4}-\frac{17x^2}{2}+\frac{289}{20}+\frac{-\frac{25x^2}{8}+\frac{75}{8}}{x^4+2x^2+3}+\frac{19\ln\left(x^4+2x^2+3\right)}{4}+\frac{203\arctan\left(\frac{\left(x^2+1\right)\sqrt{2}}{2}\right)\sqrt{2}}{16}$	62
default	$\frac{5x^4}{4}-\frac{17x^2}{2}+\frac{-\frac{25x^2}{4}+\frac{75}{4}}{2x^4+4x^2+6}+\frac{19\ln\left(x^4+2x^2+3\right)}{4}+\frac{203\sqrt{2}\arctan\left(\frac{\left(2x^2+2\right)\sqrt{2}}{4}\right)}{16}$	64

input
```
int(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
5/4*x^4-17/2*x^2+289/20+(-25/8*x^2+75/8)/(x^4+2*x^2+3)+19/4*ln(x^4+2*x^2+3
)+203/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

3.102.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{20\,x^8-96\,x^6-212\,x^4+203\,\sqrt{2}(x^4+2\,x^2+3)\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right)-458\,x^2+76\,(x^4+2\,x^2+3)\log(}{16\,(x^4+2\,x^2+3)}$$

input
```
integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
1/16*(20*x^8 - 96*x^6 - 212*x^4 + 203*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2
*sqrt(2)*(x^2 + 1)) - 458*x^2 + 76*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3)
+ 150)/(x^4 + 2*x^2 + 3)
```

3.102.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5x^4}{4}-\frac{17x^2}{2}+\frac{75-25x^2}{8x^4+16x^2+24} + \frac{19\log(x^4+2x^2+3)}{4}+\frac{203\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

input
```
integrate(x**5*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

output
```
5*x**4/4 - 17*x**2/2 + (75 - 25*x**2)/(8*x**4 + 16*x**2 + 24) + 19*log(x**
4 + 2*x**2 + 3)/4 + 203*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2)/16
```

3.102.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.80

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5}{4}\,x^4 - \frac{17}{2}\,x^2 + \frac{203}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right) - \frac{25\,(x^2-3)}{8\,(x^4+2\,x^2+3)} + \frac{19}{4}\,\log\left(x^4+2\,x^2+3\right)$$

input
```
integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
5/4*x^4 - 17/2*x^2 + 203/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/8*(
x^2 - 3)/(x^4 + 2*x^2 + 3) + 19/4*log(x^4 + 2*x^2 + 3)
```

3.102.8 Giac [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5}{4}\,x^4 - \frac{17}{2}\,x^2 + \frac{203}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right) - \frac{38\,x^4+101\,x^2+39}{8\,(x^4+2\,x^2+3)} + \frac{19}{4}\,\log\left(x^4+2\,x^2+3\right)$$

input
```
integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
5/4*x^4 - 17/2*x^2 + 203/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/8*(3
8*x^4 + 101*x^2 + 39)/(x^4 + 2*x^2 + 3) + 19/4*log(x^4 + 2*x^2 + 3)
```

3.102.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{19\,\ln\left(x^4+2\,x^2+3\right)}{4} - \frac{\frac{25\,x^2}{8}-\frac{75}{8}}{x^4+2\,x^2+3} + \frac{203\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16} - \frac{17\,x^2}{2} + \frac{5\,x^4}{4}$$

input
```
int((x^5*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

output
```
(19*log(2*x^2 + x^4 + 3))/4 - ((25*x^2)/8 - 75/8)/(2*x^2 + x^4 + 3) + (203
*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/16 - (17*x^2)/2 + (5*x^4)/4
```

3.102. $\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

3.103 $\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

3.103.1 Optimal result

Integrand size = 31, antiderivative size = 65

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5x^2}{2} - \frac{25(3+x^2)}{8(3+2x^2+x^4)} - \frac{17\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}} - \frac{17}{4}\log\left(3+2x^2+x^4\right)$$

output
```
5/2*x^2-25/8*(x^2+3)/(x^4+2*x^2+3)-17/4*ln(x^4+2*x^2+3)-17/16*arctan(1/2*(
x^2+1)*2^(1/2))*2^(1/2)
```

3.103.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{1}{16}\left(40x^2 - \frac{50(3+x^2)}{3+2x^2+x^4} - 17\sqrt{2}\arctan\left(\frac{1+x^2}{\sqrt{2}}\right) - 68\log\left(3+2x^2+x^4\right)\right)$$

input
```
Integrate[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
(40*x^2 - (50*(3 + x^2))/(3 + 2*x^2 + x^4) - 17*Sqrt[2]*ArcTan[(1 + x^2)/S
qrt[2]] - 68*Log[3 + 2*x^2 + x^4])/16
```

3.103.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2194, 2191, 27, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(5x^6+3x^4+x^2+4\right)}{\left(x^4+2x^2+3\right)^2}dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{x^2\left(5x^6+3x^4+x^2+4\right)}{\left(x^4+2x^2+3\right)^2}dx^2$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{2}\left(\frac{1}{8}\int -\frac{2\left(-20x^4+28x^2+25\right)}{x^4+2x^2+3}dx^2-\frac{25\left(x^2+3\right)}{4\left(x^4+2x^2+3\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{1}{4}\int \frac{-20x^4+28x^2+25}{x^4+2x^2+3}dx^2-\frac{25\left(x^2+3\right)}{4\left(x^4+2x^2+3\right)}\right)$$

$$\Big\downarrow \text{2188}$$

$$\frac{1}{2}\left(-\frac{1}{4}\int \left(\frac{17\left(4x^2+5\right)}{x^4+2x^2+3}-20\right)dx^2-\frac{25\left(x^2+3\right)}{4\left(x^4+2x^2+3\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(-\frac{17\arctan\left(\frac{x^2+1}{\sqrt{2}}\right)}{\sqrt{2}}+20x^2-34\log\left(x^4+2x^2+3\right)\right)-\frac{25\left(x^2+3\right)}{4\left(x^4+2x^2+3\right)}\right)$$

input
```
Int[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
((-25*(3 + x^2))/(4*(3 + 2*x^2 + x^4)) + (20*x^2 - (17*ArcTan[(1 + x^2)/Sq
rt[2]])/Sqrt[2] - 34*Log[3 + 2*x^2 + x^4])/4)/2
```

3.103.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.103.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

method	result	size
risch	$\frac{5x^2}{2}+\frac{-\frac{25x^2}{8}-\frac{75}{8}}{x^4+2x^2+3}-\frac{17\ln\left(x^4+2x^2+3\right)}{4}-\frac{17\arctan\left(\frac{\left(x^2+1\right)\sqrt{2}}{2}\right)\sqrt{2}}{16}$	56
default	$\frac{5x^2}{2}-\frac{\frac{25x^2}{4}+\frac{75}{4}}{2\left(x^4+2x^2+3\right)}-\frac{17\ln\left(x^4+2x^2+3\right)}{4}-\frac{17\sqrt{2}\arctan\left(\frac{\left(2x^2+2\right)\sqrt{2}}{4}\right)}{16}$	59

input
```
int(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
5/2*x^2+(-25/8*x^2-75/8)/(x^4+2*x^2+3)-17/4*ln(x^4+2*x^2+3)-17/16*arctan(1
/2*(x^2+1)*2^(1/2))*2^(1/2)
```

3.103.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.23

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$$
$$= \frac{40\,x^6+80\,x^4-17\,\sqrt{2}(x^4+2\,x^2+3)\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right)+70\,x^2-68\,(x^4+2\,x^2+3)\log\left(x^4+2\,x^2+\right.}{16\,(x^4+2\,x^2+3)}$$

input
```
integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
1/16*(40*x^6 + 80*x^4 - 17*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x
^2 + 1)) + 70*x^2 - 68*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3) - 150)/(x^4
+ 2*x^2 + 3)
```

3.103.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5x^2}{2}+\frac{-25x^2-75}{8x^4+16x^2+24}-\frac{17\log\left(x^4+2x^2+3\right)}{4}-\frac{17\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

input
```
integrate(x**3*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

output
```
5*x**2/2 + (-25*x**2 - 75)/(8*x**4 + 16*x**2 + 24) - 17*log(x**4 + 2*x**2
+ 3)/4 - 17*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2)/16
```

3.103.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5}{2}\,x^2 - \frac{17}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right) - \frac{25\,(x^2+3)}{8\,(x^4+2\,x^2+3)} - \frac{17}{4}\,\log\left(x^4+2\,x^2+3\right)$$

input
```
integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
5/2*x^2 - 17/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/8*(x^2 + 3)/(x^
4 + 2*x^2 + 3) - 17/4*log(x^4 + 2*x^2 + 3)
```

3.103.8 Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5}{2}\,x^2 - \frac{17}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right) - \frac{25\,(x^2+3)}{8\,(x^4+2\,x^2+3)} - \frac{17}{4}\,\log\left(x^4+2\,x^2+3\right)$$

input
```
integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
5/2*x^2 - 17/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/8*(x^2 + 3)/(x^
4 + 2*x^2 + 3) - 17/4*log(x^4 + 2*x^2 + 3)
```

3.103.9 Mupad [B] (verification not implemented)

Time = 8.61 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5\,x^2}{2} - \frac{\frac{25\,x^2}{8}+\frac{75}{8}}{x^4+2\,x^2+3} - \frac{17\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16} - \frac{17\,\ln\left(x^4+2\,x^2+3\right)}{4}$$

input
```
int((x^3*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

output
```
(5*x^2)/2 - ((25*x^2)/8 + 75/8)/(2*x^2 + x^4 + 3) - (17*2^(1/2)*atan(2^(1/
2)/2 + (2^(1/2)*x^2)/2))/16 - (17*log(2*x^2 + x^4 + 3))/4
```

3.103. $\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

3.104 $\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

3.104.1 Optimal result

Integrand size = 29, antiderivative size = 58

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx=\frac{25(1+x^2)}{8(3+2x^2+x^4)}-\frac{23\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{5}{4}\log\left(3+2x^2+x^4\right)$$

output
```
25/8*(x^2+1)/(x^4+2*x^2+3)+5/4*ln(x^4+2*x^2+3)-23/16*arctan(1/2*(x^2+1)*2^
(1/2))*2^(1/2)
```

3.104.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx=\frac{25(1+x^2)}{8(3+2x^2+x^4)}-\frac{23\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{5}{4}\log\left(3+2x^2+x^4\right)$$

input
```
Integrate[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
(25*(1 + x^2))/(8*(3 + 2*x^2 + x^4)) - (23*ArcTan[(1 + x^2)/Sqrt[2]])/(8*S
qrt[2]) + (5*Log[3 + 2*x^2 + x^4])/4
```

3.104.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2194, 2191, 27, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(5x^6+3x^4+x^2+4)}{(x^4+2x^2+3)^2}dx$$

$$\Big\downarrow \text{ 2194}$$

$$\frac{1}{2}\int \frac{5x^6+3x^4+x^2+4}{(x^4+2x^2+3)^2}dx^2$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{1}{2}\left(\frac{1}{8}\int -\frac{2(3-20x^2)}{x^4+2x^2+3}dx^2+\frac{25(x^2+1)}{4(x^4+2x^2+3)}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\frac{25(x^2+1)}{4(x^4+2x^2+3)}-\frac{1}{4}\int \frac{3-20x^2}{x^4+2x^2+3}dx^2\right)$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(10\int \frac{2(x^2+1)}{x^4+2x^2+3}dx^2-23\int \frac{1}{x^4+2x^2+3}dx^2\right)+\frac{25(x^2+1)}{4(x^4+2x^2+3)}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(20\int \frac{x^2+1}{x^4+2x^2+3}dx^2-23\int \frac{1}{x^4+2x^2+3}dx^2\right)+\frac{25(x^2+1)}{4(x^4+2x^2+3)}\right)$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(46\int \frac{1}{-x^4-8}d(2x^2+2)+20\int \frac{x^2+1}{x^4+2x^2+3}dx^2\right)+\frac{25(x^2+1)}{4(x^4+2x^2+3)}\right)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(20\int \frac{x^2+1}{x^4+2x^2+3}dx^2-\frac{23\arctan\left(\frac{2x^2+2}{2\sqrt{2}}\right)}{\sqrt{2}}\right)+\frac{25(x^2+1)}{4(x^4+2x^2+3)}\right)$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(10\log\left(x^4+2x^2+3\right)-\frac{23\arctan\left(\frac{2x^2+2}{2\sqrt{2}}\right)}{\sqrt{2}}\right)+\frac{25\left(x^2+1\right)}{4\left(x^4+2x^2+3\right)}\right)$$

input
```
Int[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
((25*(1 + x^2))/(4*(3 + 2*x^2 + x^4)) + ((-23*ArcTan[(2 + 2*x^2)/(2*Sqrt[2
])])/Sqrt[2] + 10*Log[3 + 2*x^2 + x^4])/4)/2
```

3.104.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.104.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{\frac{25x^2}{8}+\frac{25}{8}}{x^4+2x^2+3}+\frac{5\ln\left(x^4+2x^2+3\right)}{4}-\frac{23\arctan\left(\frac{\left(x^2+1\right)\sqrt{2}}{2}\right)\sqrt{2}}{16}$	51
default	$\frac{\frac{25x^2}{4}+\frac{25}{4}}{2x^4+4x^2+6}+\frac{5\ln\left(x^4+2x^2+3\right)}{4}-\frac{23\sqrt{2}\,\arctan\left(\frac{\left(2x^2+2\right)\sqrt{2}}{4}\right)}{16}$	54

input
```
int(x*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
(25/8*x^2+25/8)/(x^4+2*x^2+3)+5/4*ln(x^4+2*x^2+3)-23/16*arctan(1/2*(x^2+1)
*2^(1/2))*2^(1/2)
```

3.104.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.21

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = -\frac{23\sqrt{2}(x^4+2x^2+3)\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right)-50x^2-20(x^4+2x^2+3)\log(x^4+2x^2+3)-50}{16(x^4+2x^2+3)}$$

input
```
integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
-1/16*(23*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 50*x^2
 - 20*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3) - 50)/(x^4 + 2*x^2 + 3)
```

3.104.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{25x^2+25}{8x^4+16x^2+24}+\frac{5\log(x^4+2x^2+3)}{4} - \frac{23\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

input
```
integrate(x*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

output
```
(25*x**2 + 25)/(8*x**4 + 16*x**2 + 24) + 5*log(x**4 + 2*x**2 + 3)/4 - 23*s
qrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2)/16
```

3.104.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = -\frac{23}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right) + \frac{25(x^2+1)}{8(x^4+2x^2+3)}+\frac{5}{4}\log(x^4+2x^2+3)$$

input
```
integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
-23/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 25/8*(x^2 + 1)/(x^4 + 2*x^2
 + 3) + 5/4*log(x^4 + 2*x^2 + 3)
```

3.104.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = -\frac{23}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right) + \frac{25\,(x^2+1)}{8\,(x^4+2\,x^2+3)} + \frac{5}{4}\log\left(x^4+2\,x^2+3\right)$$

input
```
integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
-23/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 25/8*(x^2 + 1)/(x^4 + 2*x^2
 + 3) + 5/4*log(x^4 + 2*x^2 + 3)
```

3.104.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.19

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5\,\ln\left(x^4+2\,x^2+3\right)}{4} + \frac{25\,x^2}{8\,\left(x^4+2\,x^2+3\right)} + \frac{25}{8\,\left(x^4+2\,x^2+3\right)} - \frac{23\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

input
```
int((x*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

output
```
(5*log(2*x^2 + x^4 + 3))/4 + (25*x^2)/(8*(2*x^2 + x^4 + 3)) + 25/(8*(2*x^2
 + x^4 + 3)) - (23*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/16
```

3.105 $\int \frac{4+x^2+3x^4+5x^6}{x(3+2x^2+x^4)^2} \, dx$

3.105.1 Optimal result

Integrand size = 31, antiderivative size = 66

$$\int \frac{4+x^2+3x^4+5x^6}{x\left(3+2x^2+x^4\right)^2} \, dx = \frac{25(1-x^2)}{24\left(3+2x^2+x^4\right)} + \frac{89 \arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{72\sqrt{2}} + \frac{4\log(x)}{9} - \frac{1}{9}\log\left(3+2x^2+x^4\right)$$

output
```
25/24*(-x^2+1)/(x^4+2*x^2+3)+4/9*ln(x)-1/9*ln(x^4+2*x^2+3)+89/144*arctan(1
/2*(x^2+1)*2^(1/2))*2^(1/2)
```

3.105.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09

$$\int \frac{4+x^2+3x^4+5x^6}{x\left(3+2x^2+x^4\right)^2} \, dx = \frac{1}{288}\left(178\sqrt{2}\arctan\left(\frac{1+x^2}{\sqrt{2}}\right) + 128\log(x) + \frac{4(75-75x^2-8(3+2x^2+x^4)\log\left(3+2x^2+x^4\right))}{3+2x^2+x^4}\right)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(3 + 2*x^2 + x^4)^2),x]
```

output
```
(178*Sqrt[2]*ArcTan[(1 + x^2)/Sqrt[2]] + 128*Log[x] + (4*(75 - 75*x^2 - 8*
(3 + 2*x^2 + x^4)*Log[3 + 2*x^2 + x^4]))/(3 + 2*x^2 + x^4))/288
```

3.105.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2194, 2177, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x\left(x^4+2x^2+3\right)^2}dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{5x^6+3x^4+x^2+4}{x^2\left(x^4+2x^2+3\right)^2}dx^2$$

$$\Big\downarrow \text{2177}$$

$$\frac{1}{2}\left(\frac{1}{8}\int \frac{2\left(35x^2+16\right)}{3x^2\left(x^4+2x^2+3\right)}dx^2+\frac{25\left(1-x^2\right)}{12\left(x^4+2x^2+3\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{12}\int \frac{35x^2+16}{x^2\left(x^4+2x^2+3\right)}dx^2+\frac{25\left(1-x^2\right)}{12\left(x^4+2x^2+3\right)}\right)$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{2}\left(\frac{1}{12}\int \left(\frac{73-16x^2}{3\left(x^4+2x^2+3\right)}+\frac{16}{3x^2}\right)dx^2+\frac{25\left(1-x^2\right)}{12\left(x^4+2x^2+3\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{12}\left(\frac{89\arctan\left(\frac{x^2+1}{\sqrt{2}}\right)}{3\sqrt{2}}+\frac{16\log\left(x^2\right)}{3}-\frac{8}{3}\log\left(x^4+2x^2+3\right)\right)+\frac{25\left(1-x^2\right)}{12\left(x^4+2x^2+3\right)}\right)$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(3 + 2*x^2 + x^4)^2),x]
```

output
```
((25*(1 - x^2))/(12*(3 + 2*x^2 + x^4)) + ((89*ArcTan[(1 + x^2)/Sqrt[2]])/(
3*Sqrt[2]) + (16*Log[x^2])/3 - (8*Log[3 + 2*x^2 + x^4])/3)/12)/2
```

3.105.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.105. $\int \frac{4+x^2+3x^4+5x^6}{x(3+2x^2+x^4)^2} \, dx$

3.105.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{4\ln(x)}{9}-\frac{\frac{75x^2}{4}-\frac{75}{4}}{18(x^4+2x^2+3)}-\frac{\ln(x^4+2x^2+3)}{9}+\frac{89\sqrt{2}\,\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{144}$	58
risch	$\frac{-\frac{25x^2}{24}+\frac{25}{24}}{x^4+2x^2+3}+\frac{4\ln(x)}{9}-\frac{\ln(7921x^4+15842x^2+23763)}{9}+\frac{89\sqrt{2}\,\arctan\left(\frac{(89x^2+89)\sqrt{2}}{178}\right)}{144}$	59

input
```
int((5*x^6+3*x^4+x^2+4)/x/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
4/9*ln(x)-1/18*(75/4*x^2-75/4)/(x^4+2*x^2+3)-1/9*ln(x^4+2*x^2+3)+89/144*2^
(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.27

$$\int \frac{4+x^2+3x^4+5x^6}{x\,(3+2x^2+x^4)^2}\,dx$$
$$=\frac{89\,\sqrt{2}(x^4+2\,x^2+3)\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right)-150\,x^2-16\,(x^4+2\,x^2+3)\log\left(x^4+2\,x^2+3\right)+64\,(x^4+2}{144\,(x^4+2\,x^2+3)}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
1/144*(89*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 150*x^
2 - 16*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3) + 64*(x^4 + 2*x^2 + 3)*log(x
) + 150)/(x^4 + 2*x^2 + 3)
```

3.105.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int \frac{4+x^2+3x^4+5x^6}{x(3+2x^2+x^4)^2}\,dx = \frac{25-25x^2}{24x^4+48x^2+72} + \frac{4\log(x)}{9} - \frac{\log(x^4+2x^2+3)}{9} + \frac{89\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{144}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x/(x**4+2*x**2+3)**2,x)
```

output
```
(25 - 25*x**2)/(24*x**4 + 48*x**2 + 72) + 4*log(x)/9 - log(x**4 + 2*x**2 +
 3)/9 + 89*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2)/144
```

3.105.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{x(3+2x^2+x^4)^2}\,dx = \frac{89}{144}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right) - \frac{25(x^2-1)}{24(x^4+2x^2+3)} - \frac{1}{9}\log(x^4+2x^2+3) + \frac{2}{9}\log(x^2)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
89/144*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/24*(x^2 - 1)/(x^4 + 2*x^
2 + 3) - 1/9*log(x^4 + 2*x^2 + 3) + 2/9*log(x^2)
```

3.105.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.94

$$\int \frac{4+x^2+3x^4+5x^6}{x(3+2x^2+x^4)^2}\,dx = \frac{89}{144}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right) + \frac{8x^4-59x^2+99}{72(x^4+2x^2+3)} - \frac{1}{9}\log(x^4+2x^2+3) + \frac{2}{9}\log(x^2)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
89/144*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/72*(8*x^4 - 59*x^2 + 99)/
(x^4 + 2*x^2 + 3) - 1/9*log(x^4 + 2*x^2 + 3) + 2/9*log(x^2)
```

3.105.9 Mupad [B] (verification not implemented)

Time = 8.66 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

$$\int \frac{4+x^2+3x^4+5x^6}{x\left(3+2x^2+x^4\right)^2}\,dx = \frac{4\,\ln\left(x\right)}{9} - \frac{\ln\left(x^4+2\,x^2+3\right)}{9} - \frac{\frac{25\,x^2}{24}-\frac{25}{24}}{x^4+2\,x^2+3} + \frac{89\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{144}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x*(2*x^2 + x^4 + 3)^2),x)
```

output
```
(4*log(x))/9 - log(2*x^2 + x^4 + 3)/9 - ((25*x^2)/24 - 25/24)/(2*x^2 + x^4
 + 3) + (89*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/144
```

3.106 $\int \frac{4+x^2+3x^4+5x^6}{x^3(3+2x^2+x^4)^2} dx$

3.106.1 Optimal result

Integrand size = 31, antiderivative size = 71

$$\int \frac{4+x^2+3x^4+5x^6}{x^3(3+2x^2+x^4)^2} dx = -\frac{2}{9x^2} - \frac{25(5+x^2)}{72(3+2x^2+x^4)} - \frac{71\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{216\sqrt{2}} - \frac{13\log(x)}{27} + \frac{13}{108}\log\left(3+2x^2+x^4\right)$$

output
```
-2/9/x^2-25/72*(x^2+5)/(x^4+2*x^2+3)-13/27*ln(x)+13/108*ln(x^4+2*x^2+3)-71
/432*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

3.106.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.42

$$\int \frac{4+x^2+3x^4+5x^6}{x^3(3+2x^2+x^4)^2} dx = \frac{1}{864}\left(-\frac{192}{x^2} - \frac{300(5+x^2)}{3+2x^2+x^4} - 416\log(x) + \sqrt{2}\left(-71i+52\sqrt{2}\right)\log\left(-i+\sqrt{2}-ix^2\right) + \sqrt{2}\left(71i+52\sqrt{2}\right)\log\left(i+\sqrt{2}+ix^2\right)\right)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(3 + 2*x^2 + x^4)^2),x]
```

output
```
(-192/x^2 - (300*(5 + x^2))/(3 + 2*x^2 + x^4) - 416*Log[x] + Sqrt[2]*(-71*
I + 52*Sqrt[2])*Log[-I + Sqrt[2] - I*x^2] + Sqrt[2]*(71*I + 52*Sqrt[2])*Lo
g[I + Sqrt[2] + I*x^2])/864
```

3.106.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2194, 2177, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^3\left(x^4+2x^2+3\right)^2}dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{5x^6+3x^4+x^2+4}{x^4\left(x^4+2x^2+3\right)^2}dx^2$$

$$\Big\downarrow \text{2177}$$

$$\frac{1}{2}\left(\frac{1}{8}\int \frac{2\left(-25x^4-20x^2+48\right)}{9x^4\left(x^4+2x^2+3\right)}dx^2-\frac{25\left(x^2+5\right)}{36\left(x^4+2x^2+3\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{36}\int \frac{-25x^4-20x^2+48}{x^4\left(x^4+2x^2+3\right)}dx^2-\frac{25\left(x^2+5\right)}{36\left(x^4+2x^2+3\right)}\right)$$

$$\Big\downarrow \text{2159}$$

$$\frac{1}{2}\left(\frac{1}{36}\int \left(\frac{52x^2-19}{3\left(x^4+2x^2+3\right)}-\frac{52}{3x^2}+\frac{16}{x^4}\right)dx^2-\frac{25\left(x^2+5\right)}{36\left(x^4+2x^2+3\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{36}\left(-\frac{71\arctan\left(\frac{x^2+1}{\sqrt{2}}\right)}{3\sqrt{2}}-\frac{16}{x^2}-\frac{52\log\left(x^2\right)}{3}+\frac{26}{3}\log\left(x^4+2x^2+3\right)\right)-\frac{25\left(x^2+5\right)}{36\left(x^4+2x^2+3\right)}\right)$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(3 + 2*x^2 + x^4)^2),x]
```

output
```
((-25*(5 + x^2))/(36*(3 + 2*x^2 + x^4)) + (-16/x^2 - (71*ArcTan[(1 + x^2)/
Sqrt[2]])/(3*Sqrt[2]) - (52*Log[x^2])/3 + (26*Log[3 + 2*x^2 + x^4])/3)/36)
/2
```

3.106.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.106.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{2}{9x^2}-\frac{13\ln(x)}{27}+\frac{-\frac{75x^2}{4}-\frac{375}{4}}{54x^4+108x^2+162}+\frac{13\ln(x^4+2x^2+3)}{108}-\frac{71\sqrt{2}\,\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{432}$	63
risch	$\frac{-\frac{41}{72}x^4-\frac{157}{72}x^2-\frac{2}{3}}{x^2(x^4+2x^2+3)}-\frac{13\ln(x)}{27}+\frac{13\ln(5041x^4+10082x^2+15123)}{108}-\frac{71\sqrt{2}\,\arctan\left(\frac{(71x^2+71)\sqrt{2}}{142}\right)}{432}$	67

input
```
int((5*x^6+3*x^4+x^2+4)/x^3/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/9/x^2-13/27*ln(x)+1/54*(-75/4*x^2-375/4)/(x^4+2*x^2+3)+13/108*ln(x^4+2*
x^2+3)-71/432*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.48

$$\int \frac{4+x^2+3x^4+5x^6}{x^3(3+2x^2+x^4)^2}\,dx = -\frac{246\,x^4+71\,\sqrt{2}(x^6+2\,x^4+3\,x^2)\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right)+942\,x^2-52\,(x^6+2\,x^4+3\,x^2)\log\left(x^4+2\,x^2\right.}{432\,(x^6+2\,x^4+3\,x^2)}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
-1/432*(246*x^4 + 71*sqrt(2)*(x^6 + 2*x^4 + 3*x^2)*arctan(1/2*sqrt(2)*(x^2
 + 1)) + 942*x^2 - 52*(x^6 + 2*x^4 + 3*x^2)*log(x^4 + 2*x^2 + 3) + 208*(x^
6 + 2*x^4 + 3*x^2)*log(x) + 288)/(x^6 + 2*x^4 + 3*x^2)
```

3.106.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

$$\int \frac{4+x^2+3x^4+5x^6}{x^3(3+2x^2+x^4)^2}\,dx = \frac{-41x^4-157x^2-48}{72x^6+144x^4+216x^2} - \frac{13\log(x)}{27} + \frac{13\log(x^4+2x^2+3)}{108} - \frac{71\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{432}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**3/(x**4+2*x**2+3)**2,x)
```

output
```
(-41*x**4 - 157*x**2 - 48)/(72*x**6 + 144*x**4 + 216*x**2) - 13*log(x)/27
+ 13*log(x**4 + 2*x**2 + 3)/108 - 71*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)
/2)/432
```

3.106.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int \frac{4+x^2+3x^4+5x^6}{x^3(3+2x^2+x^4)^2}\,dx = -\frac{71}{432}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right) - \frac{41\,x^4+157\,x^2+48}{72\,(x^6+2\,x^4+3\,x^2)} + \frac{13}{108}\,\log\left(x^4+2\,x^2+3\right) - \frac{13}{54}\,\log\left(x^2\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
-71/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/72*(41*x^4 + 157*x^2 + 4
8)/(x^6 + 2*x^4 + 3*x^2) + 13/108*log(x^4 + 2*x^2 + 3) - 13/54*log(x^2)
```

3.106.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int \frac{4+x^2+3x^4+5x^6}{x^3(3+2x^2+x^4)^2}\,dx = -\frac{71}{432}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right) - \frac{41\,x^4+157\,x^2+48}{72\,(x^6+2\,x^4+3\,x^2)} + \frac{13}{108}\,\log\left(x^4+2\,x^2+3\right) - \frac{13}{54}\,\log\left(x^2\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
-71/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/72*(41*x^4 + 157*x^2 + 4
8)/(x^6 + 2*x^4 + 3*x^2) + 13/108*log(x^4 + 2*x^2 + 3) - 13/54*log(x^2)
```

3.106.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\int \frac{4+x^2+3x^4+5x^6}{x^3(3+2x^2+x^4)^2} dx = \frac{13 \ln(x^4+2x^2+3)}{108} - \frac{13 \ln(x)}{27} - \frac{\frac{41x^4}{72}+\frac{157x^2}{72}+\frac{2}{3}}{x^6+2x^4+3x^2} - \frac{71\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{432}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^3*(2*x^2 + x^4 + 3)^2),x)
```

output
```
(13*log(2*x^2 + x^4 + 3))/108 - (13*log(x))/27 - ((157*x^2)/72 + (41*x^4)/
72 + 2/3)/(3*x^2 + 2*x^4 + x^6) - (71*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^
2)/2))/432
```

3.107 $\int \frac{4+x^2+3x^4+5x^6}{x^5\left(3+2x^2+x^4\right)^2}\,dx$

3.107.1 Optimal result

Integrand size = 31, antiderivative size = 80

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(3+2x^2+x^4\right)^2}\,dx=-\frac{1}{9x^4}+\frac{13}{54x^2}+\frac{25(7+5x^2)}{216\left(3+2x^2+x^4\right)}+\frac{125\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{216\sqrt{2}}+\frac{13\log(x)}{27}-\frac{13}{108}\log\left(3+2x^2+x^4\right)$$

output
```
-1/9/x^4+13/54/x^2+25/216*(5*x^2+7)/(x^4+2*x^2+3)+13/27*ln(x)-13/108*ln(x^
4+2*x^2+3)+125/432*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

3.107.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(3+2x^2+x^4\right)^2}\,dx=\frac{1}{864}\left(-\frac{96}{x^4}+\frac{208}{x^2}+\frac{700}{3+2x^2+x^4}+\frac{500x^2}{3+2x^2+x^4}+250\sqrt{2}\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)+416\log(x)-104\log\left(3+2x^2+x^4\right)\right)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(3 + 2*x^2 + x^4)^2),x]
```

output
```
(-96/x^4 + 208/x^2 + 700/(3 + 2*x^2 + x^4) + (500*x^2)/(3 + 2*x^2 + x^4) +
 250*Sqrt[2]*ArcTan[(1 + x^2)/Sqrt[2]] + 416*Log[x] - 104*Log[3 + 2*x^2 +
x^4])/864
```

3.107.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2194, 2177, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^5\left(x^4+2x^2+3\right)^2}dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{5x^6+3x^4+x^2+4}{x^6\left(x^4+2x^2+3\right)^2}dx^2$$

$$\Big\downarrow \text{2177}$$

$$\frac{1}{2}\left(\frac{1}{8}\int \frac{2\left(125x^6+100x^4-60x^2+144\right)}{27x^6\left(x^4+2x^2+3\right)}dx^2+\frac{25\left(5x^2+7\right)}{108\left(x^4+2x^2+3\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{108}\int \frac{125x^6+100x^4-60x^2+144}{x^6\left(x^4+2x^2+3\right)}dx^2+\frac{25\left(5x^2+7\right)}{108\left(x^4+2x^2+3\right)}\right)$$

$$\Big\downarrow \text{2159}$$

$$\frac{1}{2}\left(\frac{1}{108}\int \left(\frac{73-52x^2}{x^4+2x^2+3}+\frac{52}{x^2}-\frac{52}{x^4}+\frac{48}{x^6}\right)dx^2+\frac{25\left(5x^2+7\right)}{108\left(x^4+2x^2+3\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{108}\left(\frac{125\arctan\left(\frac{x^2+1}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{24}{x^4}+\frac{52}{x^2}+52\log\left(x^2\right)-26\log\left(x^4+2x^2+3\right)\right)+\frac{25\left(5x^2+7\right)}{108\left(x^4+2x^2+3\right)}\right)$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(3 + 2*x^2 + x^4)^2),x]
```

output
```
((25*(7 + 5*x^2))/(108*(3 + 2*x^2 + x^4)) + (-24/x^4 + 52/x^2 + (125*ArcTa
n[(1 + x^2)/Sqrt[2]])/Sqrt[2] + 52*Log[x^2] - 26*Log[3 + 2*x^2 + x^4])/108
)/2
```

3.107.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.107.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{1}{9x^4}+\frac{13}{54x^2}+\frac{13\ln(x)}{27}-\frac{-\frac{125x^2}{4}-\frac{175}{4}}{54(x^4+2x^2+3)}-\frac{13\ln(x^4+2x^2+3)}{108}+\frac{125\sqrt{2}\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{432}$	68
risch	$\frac{\frac{59}{72}x^6+\frac{85}{72}x^4+\frac{1}{2}x^2-\frac{1}{3}}{x^4(x^4+2x^2+3)}+\frac{13\ln(x)}{27}-\frac{13\ln(15625x^4+31250x^2+46875)}{108}+\frac{125\sqrt{2}\arctan\left(\frac{(125x^2+125)\sqrt{2}}{250}\right)}{432}$	72

input
```
int((5*x^6+3*x^4+x^2+4)/x^5/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/9/x^4+13/54/x^2+13/27*ln(x)-1/54*(-125/4*x^2-175/4)/(x^4+2*x^2+3)-13/10
8*ln(x^4+2*x^2+3)+125/432*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

3.107.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.38

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(3+2x^2+x^4\right)^2}\,dx$$
$$= \frac{354\,x^6+510\,x^4+125\,\sqrt{2}(x^8+2\,x^6+3\,x^4)\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right)+216\,x^2-52\,(x^8+2\,x^6+3\,x^4)\log\left(x^4 \right.}{432\,(x^8+2\,x^6+3\,x^4)}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
1/432*(354*x^6 + 510*x^4 + 125*sqrt(2)*(x^8 + 2*x^6 + 3*x^4)*arctan(1/2*sq
rt(2)*(x^2 + 1)) + 216*x^2 - 52*(x^8 + 2*x^6 + 3*x^4)*log(x^4 + 2*x^2 + 3)
 + 208*(x^8 + 2*x^6 + 3*x^4)*log(x) - 144)/(x^8 + 2*x^6 + 3*x^4)
```

3.107.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(3+2x^2+x^4\right)^2}\,dx = \frac{13\log(x)}{27} - \frac{13\log(x^4+2x^2+3)}{108} + \frac{125\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{432} + \frac{59x^6+85x^4+36x^2-24}{72x^8+144x^6+216x^4}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**5/(x**4+2*x**2+3)**2,x)
```

output
```
13*log(x)/27 - 13*log(x**4 + 2*x**2 + 3)/108 + 125*sqrt(2)*atan(sqrt(2)*x*
*2/2 + sqrt(2)/2)/432 + (59*x**6 + 85*x**4 + 36*x**2 - 24)/(72*x**8 + 144*
x**6 + 216*x**4)
```

3.107.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int \frac{4+x^2+3x^4+5x^6}{x^5(3+2x^2+x^4)^2}\,dx = \frac{125}{432}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right)+\frac{59\,x^6+85\,x^4+36\,x^2-24}{72\,(x^8+2\,x^6+3\,x^4)} - \frac{13}{108}\log\left(x^4+2\,x^2+3\right)+\frac{13}{54}\log\left(x^2\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
125/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/72*(59*x^6 + 85*x^4 + 36
*x^2 - 24)/(x^8 + 2*x^6 + 3*x^4) - 13/108*log(x^4 + 2*x^2 + 3) + 13/54*log
(x^2)
```

3.107.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int \frac{4+x^2+3x^4+5x^6}{x^5(3+2x^2+x^4)^2}\,dx = \frac{125}{432}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right)+\frac{26\,x^4+177\,x^2+253}{216\,(x^4+2\,x^2+3)} - \frac{39\,x^4-26\,x^2+12}{108\,x^4}-\frac{13}{108}\log\left(x^4+2\,x^2+3\right)+\frac{13}{54}\log\left(x^2\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
125/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/216*(26*x^4 + 177*x^2 +
253)/(x^4 + 2*x^2 + 3) - 1/108*(39*x^4 - 26*x^2 + 12)/x^4 - 13/108*log(x^4
 + 2*x^2 + 3) + 13/54*log(x^2)
```

3.107.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

$$\int \frac{4+x^2+3x^4+5x^6}{x^5(3+2x^2+x^4)^2}\,dx = \frac{13\,\ln(x)}{27}-\frac{13\,\ln\left(x^4+2\,x^2+3\right)}{108} + \frac{\frac{59\,x^6}{72}+\frac{85\,x^4}{72}+\frac{x^2}{2}-\frac{1}{3}}{x^8+2\,x^6+3\,x^4}+\frac{125\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{432}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^5*(2*x^2 + x^4 + 3)^2),x)
```

output
```
(13*log(x))/27 - (13*log(2*x^2 + x^4 + 3))/108 + (x^2/2 + (85*x^4)/72 + (5
9*x^6)/72 - 1/3)/(3*x^4 + 2*x^6 + x^8) + (125*2^(1/2)*atan(2^(1/2)/2 + (2^
(1/2)*x^2)/2))/432
```

3.107. $\int \frac{4+x^2+3x^4+5x^6}{x^5(3+2x^2+x^4)^2}\,dx$

3.108 $\int \frac{4+x^2+3x^4+5x^6}{x^7\left(3+2x^2+x^4\right)^2}\,dx$

3.108.1 Optimal result

Integrand size = 31, antiderivative size = 87

$$\int \frac{4+x^2+3x^4+5x^6}{x^7\left(3+2x^2+x^4\right)^2}\,dx=-\frac{2}{27x^6}+\frac{13}{108x^4}-\frac{13}{54x^2}+\frac{25(1-7x^2)}{648\left(3+2x^2+x^4\right)}-\frac{1237\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{1944\sqrt{2}}+\frac{61\log(x)}{243}-\frac{61}{972}\log\left(3+2x^2+x^4\right)$$

output
```
-2/27/x^6+13/108/x^4-13/54/x^2+25/648*(-7*x^2+1)/(x^4+2*x^2+3)+61/243*ln(x
)-61/972*ln(x^4+2*x^2+3)-1237/3888*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

3.108.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.31

$$\int \frac{4+x^2+3x^4+5x^6}{x^7\left(3+2x^2+x^4\right)^2}\,dx=\frac{-\frac{576}{x^6}+\frac{936}{x^4}-\frac{1872}{x^2}-\frac{300(-1+7x^2)}{3+2x^2+x^4}+1952\log(x)-\sqrt{2}\left(1237i+244\sqrt{2}\right)\log\left(-i+\sqrt{2}-ix^2\right)+\sqrt{2}\left(1237i-\right.}{7776}$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^7*(3 + 2*x^2 + x^4)^2),x]
```

output
```
(-576/x^6 + 936/x^4 - 1872/x^2 - (300*(-1 + 7*x^2))/(3 + 2*x^2 + x^4) + 19
52*Log[x] - Sqrt[2]*(1237*I + 244*Sqrt[2])*Log[-I + Sqrt[2] - I*x^2] + Sqr
t[2]*(1237*I - 244*Sqrt[2])*Log[I + Sqrt[2] + I*x^2])/7776
```

3.108.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2194, 2177, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^7\left(x^4+2x^2+3\right)^2}dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{5x^6+3x^4+x^2+4}{x^8\left(x^4+2x^2+3\right)^2}dx^2$$

$$\Big\downarrow \text{2177}$$

$$\frac{1}{2}\left(\frac{1}{8}\int \frac{2\left(-175x^8+400x^6+300x^4-180x^2+432\right)}{81x^8\left(x^4+2x^2+3\right)}dx^2+\frac{25\left(1-7x^2\right)}{324\left(x^4+2x^2+3\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{324}\int \frac{-175x^8+400x^6+300x^4-180x^2+432}{x^8\left(x^4+2x^2+3\right)}dx^2+\frac{25\left(1-7x^2\right)}{324\left(x^4+2x^2+3\right)}\right)$$

$$\Big\downarrow \text{2159}$$

$$\frac{1}{2}\left(\frac{1}{324}\int \left(\frac{-244x^2-1481}{3\left(x^4+2x^2+3\right)}+\frac{244}{3x^2}+\frac{156}{x^4}-\frac{156}{x^6}+\frac{144}{x^8}\right)dx^2+\frac{25\left(1-7x^2\right)}{324\left(x^4+2x^2+3\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{324}\left(-\frac{1237\arctan\left(\frac{x^2+1}{\sqrt{2}}\right)}{3\sqrt{2}}-\frac{48}{x^6}+\frac{78}{x^4}-\frac{156}{x^2}+\frac{244\log\left(x^2\right)}{3}-\frac{122}{3}\log\left(x^4+2x^2+3\right)\right)+\frac{25\left(1-7x^2\right)}{324\left(x^4+2x^2+3\right)}\right)$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^7*(3 + 2*x^2 + x^4)^2),x]
```

output
```
((25*(1 - 7*x^2))/(324*(3 + 2*x^2 + x^4)) + (-48/x^6 + 78/x^4 - 156/x^2 -
(1237*ArcTan[(1 + x^2)/Sqrt[2]])/(3*Sqrt[2]) + (244*Log[x^2])/3 - (122*Log
[3 + 2*x^2 + x^4])/3)/324)/2
```

3.108.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.108.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84

method	result
default	$-\frac{2}{27x^6}+\frac{13}{108x^4}-\frac{13}{54x^2}+\frac{61\ln(x)}{243}-\frac{\frac{525x^2}{4}-\frac{75}{4}}{486(x^4+2x^2+3)}-\frac{61\ln(x^4+2x^2+3)}{972}-\frac{1237\sqrt{2}\,\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{3888}$
risch	$\frac{-\frac{331}{648}x^8-\frac{209}{648}x^6-\frac{5}{9}x^4+\frac{23}{108}x^2-\frac{2}{9}}{x^6(x^4+2x^2+3)}+\frac{61\ln(x)}{243}-\frac{61\ln(1530169x^4+3060338x^2+4590507)}{972}-\frac{1237\sqrt{2}\,\arctan\left(\frac{(1237x^2+1237)\sqrt{2}}{2474}\right)}{3888}$

input
```
int((5*x^6+3*x^4+x^2+4)/x^7/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/27/x^6+13/108/x^4-13/54/x^2+61/243*ln(x)-1/486*(525/4*x^2-75/4)/(x^4+2*
x^2+3)-61/972*ln(x^4+2*x^2+3)-1237/3888*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/
2))
```

3.108.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.32

$$\int \frac{4+x^2+3x^4+5x^6}{x^7\left(3+2x^2+x^4\right)^2}\,dx =$$
$$-\frac{1986\,x^8+1254\,x^6+2160\,x^4+1237\,\sqrt{2}(x^{10}+2\,x^8+3\,x^6)\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right)-828\,x^2+244\,(x^{10}+}{3888\,(x^{10}+2\,x^8+3\,x^6)}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^7/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
-1/3888*(1986*x^8 + 1254*x^6 + 2160*x^4 + 1237*sqrt(2)*(x^10 + 2*x^8 + 3*x
^6)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 828*x^2 + 244*(x^10 + 2*x^8 + 3*x^6)*l
og(x^4 + 2*x^2 + 3) - 976*(x^10 + 2*x^8 + 3*x^6)*log(x) + 864)/(x^10 + 2*x
^8 + 3*x^6)
```

3.108.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int \frac{4+x^2+3x^4+5x^6}{x^7(3+2x^2+x^4)^2}\,dx = \frac{61\log(x)}{243} - \frac{61\log(x^4+2x^2+3)}{972} - \frac{1237\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{3888} + \frac{-331x^8-209x^6-360x^4+138x^2-144}{648x^{10}+1296x^8+1944x^6}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**7/(x**4+2*x**2+3)**2,x)
```

output
```
61*log(x)/243 - 61*log(x**4 + 2*x**2 + 3)/972 - 1237*sqrt(2)*atan(sqrt(2)*
x**2/2 + sqrt(2)/2)/3888 + (-331*x**8 - 209*x**6 - 360*x**4 + 138*x**2 - 1
44)/(648*x**10 + 1296*x**8 + 1944*x**6)
```

3.108.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87

$$\int \frac{4+x^2+3x^4+5x^6}{x^7(3+2x^2+x^4)^2}\,dx = -\frac{1237}{3888}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right) - \frac{331\,x^8+209\,x^6+360\,x^4-138\,x^2+144}{648\,(x^{10}+2\,x^8+3\,x^6)} - \frac{61}{972}\log\left(x^4+2\,x^2+3\right) + \frac{61}{486}\log\left(x^2\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^7/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
-1237/3888*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/648*(331*x^8 + 209*x^
6 + 360*x^4 - 138*x^2 + 144)/(x^10 + 2*x^8 + 3*x^6) - 61/972*log(x^4 + 2*x
^2 + 3) + 61/486*log(x^2)
```

3.108.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97

$$\int \frac{4+x^2+3x^4+5x^6}{x^7\left(3+2x^2+x^4\right)^2}\,dx = -\frac{1237}{3888}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right) + \frac{122\,x^4-281\,x^2+441}{1944\,\left(x^4+2\,x^2+3\right)} - \frac{671\,x^6+702\,x^4-351\,x^2+216}{2916\,x^6} - \frac{61}{972}\log\left(x^4+2\,x^2+3\right) + \frac{61}{486}\log\left(x^2\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^7/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
-1237/3888*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/1944*(122*x^4 - 281*x
^2 + 441)/(x^4 + 2*x^2 + 3) - 1/2916*(671*x^6 + 702*x^4 - 351*x^2 + 216)/x
^6 - 61/972*log(x^4 + 2*x^2 + 3) + 61/486*log(x^2)
```

3.108.9 Mupad [B] (verification not implemented)

Time = 8.64 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.90

$$\int \frac{4+x^2+3x^4+5x^6}{x^7\left(3+2x^2+x^4\right)^2}\,dx = \frac{61\,\ln\left(x\right)}{243} - \frac{61\,\ln\left(x^4+2\,x^2+3\right)}{972} - \frac{\frac{331\,x^8}{648}+\frac{209\,x^6}{648}+\frac{5\,x^4}{9}-\frac{23\,x^2}{108}+\frac{2}{9}}{x^{10}+2\,x^8+3\,x^6} - \frac{1237\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{3888}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^7*(2*x^2 + x^4 + 3)^2),x)
```

output
```
(61*log(x))/243 - (61*log(2*x^2 + x^4 + 3))/972 - ((5*x^4)/9 - (23*x^2)/10
8 + (209*x^6)/648 + (331*x^8)/648 + 2/9)/(3*x^6 + 2*x^8 + x^10) - (1237*2^
(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/3888
```

3.109 $\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

3.109.1 Optimal result

Integrand size = 31, antiderivative size = 248

$$\begin{aligned}\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx &= 38x+\frac{19x^3}{3}-\frac{17x^5}{5}+\frac{5x^7}{7}+\frac{25x(3+5x^2)}{8(3+2x^2+x^4)}\\ &\quad+\frac{1}{16}\sqrt{\frac{1}{2}\left(262771+618291\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)\\ &\quad-\frac{1}{16}\sqrt{\frac{1}{2}\left(262771+618291\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)\\ &\quad-\frac{1}{32}\sqrt{\frac{1}{2}\left(-262771+618291\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)\\ &\quad+\frac{1}{32}\sqrt{\frac{1}{2}\left(-262771+618291\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)\end{aligned}$$

output
```
38*x+19/3*x^3-17/5*x^5+5/7*x^7+25/8*x*(5*x^2+3)/(x^4+2*x^2+3)-1/64*ln(x^2+
3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-525542+1236582*3^(1/2))^(1/2)+1/64*ln(x^
2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-525542+1236582*3^(1/2))^(1/2)+1/32*arc
tan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(525542+1236582*3^(1/
2))^(1/2)-1/32*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(525
542+1236582*3^(1/2))^(1/2)
```

3.109.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.58

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = 38x+\frac{19x^3}{3}-\frac{17x^5}{5}+\frac{5x^7}{7}+\frac{25x(3+5x^2)}{8(3+2x^2+x^4)} - \frac{(352i+1339\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{16\sqrt{2-2i\sqrt{2}}} - \frac{(-352i+1339\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{16\sqrt{2+2i\sqrt{2}}}$$

input
```
Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
38*x + (19*x^3)/3 - (17*x^5)/5 + (5*x^7)/7 + (25*x*(3 + 5*x^2))/(8*(3 + 2*
x^2 + x^4)) - ((352*I + 1339*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(16*S
qrt[2 - (2*I)*Sqrt[2]]) - ((-352*I + 1339*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqr
t[2]]])/(16*Sqrt[2 + (2*I)*Sqrt[2]])
```

3.109.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2197, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8(5x^6+3x^4+x^2+4)}{(x^4+2x^2+3)^2}dx$$

$$\Big\downarrow \text{2197}$$

$$\frac{1}{48}\int -\frac{6(-40x^{10}+56x^8-200x^4+275x^2+75)}{x^4+2x^2+3}dx+\frac{25x(5x^2+3)}{8(x^4+2x^2+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{25x(5x^2+3)}{8(x^4+2x^2+3)}-\frac{1}{8}\int \frac{-40x^{10}+56x^8-200x^4+275x^2+75}{x^4+2x^2+3}dx$$

$$\Big\downarrow \text{2205}$$

$$\frac{25x(5x^2+3)}{8(x^4+2x^2+3)}-\frac{1}{8}\int\left(-40x^6+136x^4-152x^2+\frac{1339x^2+987}{x^4+2x^2+3}-304\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(262771+618291\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{2}\sqrt{\frac{1}{2}\left(262771+618291\sqrt{3}\right)}\arctan\left(\frac{2x+\sqrt{2}}{\sqrt{2(1}}\right.\right.$$

$$\frac{25x(5x^2+3)}{8(x^4+2x^2+3)}$$

input
```
Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
(25*x*(3 + 5*x^2))/(8*(3 + 2*x^2 + x^4)) + (304*x + (152*x^3)/3 - (136*x^5
)/5 + (40*x^7)/7 + (Sqrt[(262771 + 618291*Sqrt[3])/2]*ArcTan[(Sqrt[2*(-1 +
 Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/2 - (Sqrt[(262771 + 618291*Sqrt[
3])/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/2 - (
Sqrt[(-262771 + 618291*Sqrt[3])/2]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x
+ x^2])/4 + (Sqrt[(-262771 + 618291*Sqrt[3])/2]*Log[Sqrt[3] + Sqrt[2*(-1 +
 Sqrt[3])]*x + x^2])/4)/8
```

3.109.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

3.109.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.32

method	result
risch	$\frac{5x^7}{7}-\frac{17x^5}{5}+\frac{19x^3}{3}+38x+\frac{\frac{125}{8}x^3+\frac{75}{8}x}{x^4+2x^2+3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+2\_Z^2+3\right)}\frac{\left(-1339\_R^2-987\right)\ln\left(x-\_R\right)}{\_R^3+\_R}\right)}{32}$
default	$\frac{5x^7}{7}-\frac{17x^5}{5}+\frac{19x^3}{3}+38x-\frac{-\frac{125}{8}x^3-\frac{75}{8}x}{x^4+2x^2+3}+\frac{\left(-505\sqrt{-2+2\sqrt{3}}\,\sqrt{3}-176\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{64}+\frac{\left(-6\right.}{}$

input
```
int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
5/7*x^7-17/5*x^5+19/3*x^3+38*x+(125/8*x^3+75/8*x)/(x^4+2*x^2+3)+1/32*sum((
-1339*_R^2-987)/(_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^2+3))
```

3.109.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.87

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$$
$$= \frac{2400\,x^{11} - 6624\,x^9 + 5632\,x^7 + 135968\,x^5 + 371700\,x^3 - 105\,(x^4+2\,x^2+3)\sqrt{734099i\,\sqrt{2}-262771}\log\left(\right.}{}$$

input
```
integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
1/3360*(2400*x^11 - 6624*x^9 + 5632*x^7 + 135968*x^5 + 371700*x^3 - 105*(x
^4 + 2*x^2 + 3)*sqrt(734099*I*sqrt(2) - 262771)*log(sqrt(734099*I*sqrt(2)
- 262771)*(505*I*sqrt(2) + 329) + 618291*x) + 105*(x^4 + 2*x^2 + 3)*sqrt(7
34099*I*sqrt(2) - 262771)*log(sqrt(734099*I*sqrt(2) - 262771)*(-505*I*sqrt
(2) - 329) + 618291*x) + 105*(x^4 + 2*x^2 + 3)*sqrt(-734099*I*sqrt(2) - 26
2771)*log((505*I*sqrt(2) - 329)*sqrt(-734099*I*sqrt(2) - 262771) + 618291*
x) - 105*(x^4 + 2*x^2 + 3)*sqrt(-734099*I*sqrt(2) - 262771)*log((-505*I*sq
rt(2) + 329)*sqrt(-734099*I*sqrt(2) - 262771) + 618291*x) + 414540*x)/(x^4
 + 2*x^2 + 3)
```

3.109.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.29

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5x^7}{7} - \frac{17x^5}{5} + \frac{19x^3}{3} + 38x + \frac{125x^3+75x}{8x^4+16x^2+24}$$
$$+\mathrm{RootSum}\left(1048576t^4 + 538155008t^2 + 1146851282043, \left(t \mapsto t\log\left(-\frac{16547840t^3}{453886804809} - \frac{11974973632t}{453886804809}\right.\right.\right.$$

input
```
integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

output
```
5*x**7/7 - 17*x**5/5 + 19*x**3/3 + 38*x + (125*x**3 + 75*x)/(8*x**4 + 16*x
**2 + 24) + RootSum(1048576*_t**4 + 538155008*_t**2 + 1146851282043, Lambd
a(_t, _t*log(-16547840*_t**3/453886804809 - 11974973632*_t/453886804809 +
x)))
```

3.109.7 Maxima [F]

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \int \frac{(5\,x^6+3\,x^4+x^2+4)x^8}{(x^4+2\,x^2+3)^2}\,dx$$

input
```
integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
5/7*x^7 - 17/5*x^5 + 19/3*x^3 + 38*x + 25/8*(5*x^3 + 3*x)/(x^4 + 2*x^2 + 3
) - 1/8*integrate((1339*x^2 + 987)/(x^4 + 2*x^2 + 3), x)
```

3.109.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 585 vs. $2(175) = 350$.

Time = 0.59 (sec) , antiderivative size = 585, normalized size of antiderivative = 2.36

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5}{7}\,x^7 - \frac{17}{5}\,x^5 + \frac{19}{3}\,x^3$$
$$+ \frac{1}{20736}\,\sqrt{2}\Big(1339\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\,\sqrt{3}+18\right)^{\frac{3}{2}} + 24102\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\,\sqrt{3}+18}\left(\sqrt{3}-3\right) - 24102\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{\cdots}$$
$$+ \frac{1}{20736}\,\sqrt{2}\Big(1339\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\,\sqrt{3}+18\right)^{\frac{3}{2}} + 24102\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\,\sqrt{3}+18}\left(\sqrt{3}-3\right) - 24102\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{\cdots}$$
$$+ \frac{1}{41472}\,\sqrt{2}\Big(24102\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\,\sqrt{3}+18} - 1339\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\,\sqrt{3}+18\right)^{\frac{3}{2}} + 1339\cdot 3^{\frac{3}{4}}\left(6\,\sqrt{3}+1\cdots\right.$$
$$+ 2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\,\sqrt{3}+\frac{1}{2}} + \sqrt{3}\Big)$$
$$- \frac{1}{41472}\,\sqrt{2}\Big(24102\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\,\sqrt{3}+18} - 1339\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\,\sqrt{3}+18\right)^{\frac{3}{2}} + 1339\cdot 3^{\frac{3}{4}}\left(6\,\sqrt{3}+1\cdots\right.$$
$$- 2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\,\sqrt{3}+\frac{1}{2}} + \sqrt{3}\Big) + 38\,x + \frac{25\,(5\,x^3+3\,x)}{8\,(x^4+2\,x^2+3)}$$

input
```
integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
5/7*x^7 - 17/5*x^5 + 19/3*x^3 + 1/20736*sqrt(2)*(1339*3^(3/4)*sqrt(2)*(6*s
qrt(3) + 18)^(3/2) + 24102*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) -
 3) - 24102*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 1339*3^(3/4)*(-6
*sqrt(3) + 18)^(3/2) - 35532*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 35532*
3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*s
qrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/20736*sqrt(2)*(1339*3^(3/4)*sq
rt(2)*(6*sqrt(3) + 18)^(3/2) + 24102*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*
(sqrt(3) - 3) - 24102*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 1339*3
^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 35532*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18
) + 35532*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*s
qrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/41472*sqrt(2)*(24102
*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 1339*3^(3/4)*sqrt(2
)*(-6*sqrt(3) + 18)^(3/2) + 1339*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 24102*3^
(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 35532*3^(1/4)*sqrt(2)*sqrt(-6*s
qrt(3) + 18) - 35532*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*s
qrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/41472*sqrt(2)*(24102*3^(3/4)*sqrt(2
)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 1339*3^(3/4)*sqrt(2)*(-6*sqrt(3) +
 18)^(3/2) + 1339*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 24102*3^(3/4)*sqrt(6*sq
rt(3) + 18)*(sqrt(3) - 3) - 35532*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) -
35532*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqr...
```

3.109.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.69

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = 38\,x+\frac{\frac{125\,x^3}{8}+\frac{75\,x}{8}}{x^4+2\,x^2+3}+\frac{19\,x^3}{3}-\frac{17\,x^5}{5}+\frac{5\,x^7}{7}$$
$$+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-262771-\sqrt{2}\,734099\mathrm{i}}\,734099\mathrm{i}}{64\,\left(-\frac{1112159985}{64}+\frac{\sqrt{2}\,724555713\mathrm{i}}{128}\right)}+\frac{734099\,\sqrt{2}\,x\,\sqrt{-262771-\sqrt{2}\,734099\mathrm{i}}}{128\,\left(-\frac{1112159985}{64}+\frac{\sqrt{2}\,724555713\mathrm{i}}{128}\right)}\right)\,\sqrt{-262771-\sqrt{2}\,734099\mathrm{i}}\,1\mathrm{i}}{16}$$
$$-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-262771+\sqrt{2}\,734099\mathrm{i}}\,734099\mathrm{i}}{64\,\left(\frac{1112159985}{64}+\frac{\sqrt{2}\,724555713\mathrm{i}}{128}\right)}-\frac{734099\,\sqrt{2}\,x\,\sqrt{-262771+\sqrt{2}\,734099\mathrm{i}}}{128\,\left(\frac{1112159985}{64}+\frac{\sqrt{2}\,724555713\mathrm{i}}{128}\right)}\right)\,\sqrt{-262771+\sqrt{2}\,734099\mathrm{i}}\,1\mathrm{i}}{16}$$

input
```
int((x^8*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

output
```
38*x + (atan((x*(- 2^(1/2)*734099i - 262771)^(1/2)*734099i)/(64*((2^(1/2)*
724555713i)/128 - 1112159985/64)) + (734099*2^(1/2)*x*(- 2^(1/2)*734099i -
 262771)^(1/2))/(128*((2^(1/2)*724555713i)/128 - 1112159985/64)))*(- 2^(1/
2)*734099i - 262771)^(1/2)*1i)/16 - (atan((x*(2^(1/2)*734099i - 262771)^(1
/2)*734099i)/(64*((2^(1/2)*724555713i)/128 + 1112159985/64)) - (734099*2^(
1/2)*x*(2^(1/2)*734099i - 262771)^(1/2))/(128*((2^(1/2)*724555713i)/128 +
1112159985/64)))*(2^(1/2)*734099i - 262771)^(1/2)*1i)/16 + ((75*x)/8 + (12
5*x^3)/8)/(2*x^2 + x^4 + 3) + (19*x^3)/3 - (17*x^5)/5 + (5*x^7)/7
```

3.109. $\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

3.110 $\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

3.110.1 Optimal result

Integrand size = 31, antiderivative size = 237

$$\begin{aligned}\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx &= 19x-\frac{17x^3}{3}+x^5+\frac{25x(3-x^2)}{8(3+2x^2+x^4)}\\ &\quad+\frac{3}{16}\sqrt{\frac{3}{2}\left(-8669+5011\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)\\ &\quad-\frac{3}{16}\sqrt{\frac{3}{2}\left(-8669+5011\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)\\ &\quad+\frac{3}{32}\sqrt{\frac{3}{2}\left(8669+5011\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)\\ &\quad-\frac{3}{32}\sqrt{\frac{3}{2}\left(8669+5011\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)\end{aligned}$$

output
```
19*x-17/3*x^3+x^5+25/8*x*(-x^2+3)/(x^4+2*x^2+3)+3/32*arctan((-2*x+(-2+2*3^
(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-52014+30066*3^(1/2))^(1/2)-3/32*arcta
n((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-52014+30066*3^(1/2))^(
1/2)+3/64*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(52014+30066*3^(1/2))^(1/
2)-3/64*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(52014+30066*3^(1/2))^(1/2)
```

3.110.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.56

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = 19x-\frac{17x^3}{3}+x^5-\frac{25x(-3+x^2)}{8(3+2x^2+x^4)} + \frac{9\left(90i+31\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{16\sqrt{2-2i\sqrt{2}}} + \frac{9\left(-90i+31\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{16\sqrt{2+2i\sqrt{2}}}$$

input
```
Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
19*x - (17*x^3)/3 + x^5 - (25*x*(-3 + x^2))/(8*(3 + 2*x^2 + x^4)) + (9*(90
*I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(16*Sqrt[2 - (2*I)*Sqrt[2]
]) + (9*(-90*I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(16*Sqrt[2 + (
2*I)*Sqrt[2]])
```

3.110.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2197, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(5x^6+3x^4+x^2+4)}{(x^4+2x^2+3)^2}dx$$

$$\Big\downarrow \text{2197}$$

$$\frac{1}{48}\int -\frac{6(-40x^8+56x^6-175x^2+75)}{x^4+2x^2+3}dx+\frac{25x(3-x^2)}{8(x^4+2x^2+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{25x(3-x^2)}{8(x^4+2x^2+3)}-\frac{1}{8}\int \frac{-40x^8+56x^6-175x^2+75}{x^4+2x^2+3}dx$$

$$\Big\downarrow \text{2205}$$

$$\frac{25x(3-x^2)}{8(x^4+2x^2+3)}-\frac{1}{8}\int\left(-40x^4+136x^2+\frac{9(59-31x^2)}{x^4+2x^2+3}-152\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}\left(\frac{3}{2}\sqrt{\frac{3}{2}\left(5011\sqrt{3}-8669\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{3}{2}\sqrt{\frac{3}{2}\left(5011\sqrt{3}-8669\right)}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right.\right.$$

$$\frac{25x(3-x^2)}{8(x^4+2x^2+3)}$$

input
```
Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
(25*x*(3 - x^2))/(8*(3 + 2*x^2 + x^4)) + (152*x - (136*x^3)/3 + 8*x^5 + (3
*Sqrt[(3*(-8669 + 5011*Sqrt[3]))/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/
Sqrt[2*(1 + Sqrt[3])]])/2 - (3*Sqrt[(3*(-8669 + 5011*Sqrt[3]))/2]*ArcTan[(
Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/2 + (3*Sqrt[(3*(8669
 + 5011*Sqrt[3]))/2]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/4 - (3
*Sqrt[(3*(8669 + 5011*Sqrt[3]))/2]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x
+ x^2])/4)/8
```

3.110.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

3.110.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.30

method	result
risch	$x^5-\frac{17x^3}{3}+19x+\frac{-\frac{25}{8}x^3+\frac{75}{8}x}{x^4+2x^2+3}+\frac{9\left(\sum_{\_R=\text{RootOf}\left(\_Z^4+2\_Z^2+3\right)}\frac{\left(31\_R^2-59\right)\ln\left(x-\_R\right)}{\_R^3+\_R}\right)}{32}$
default	$x^5-\frac{17x^3}{3}+19x+\frac{-\frac{25}{8}x^3+\frac{75}{8}x}{x^4+2x^2+3}+\frac{3\left(76\sqrt{-2+2\sqrt{3}}\sqrt{3}+135\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{64}+\frac{3\left(-118\sqrt{3}+\frac{(76\sqrt{}\right.}{}}$

input
```
int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
x^5-17/3*x^3+19*x+(-25/8*x^3+75/8*x)/(x^4+2*x^2+3)+9/32*sum((31*_R^2-59)/(
_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^2+3))
```

3.110.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.89

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$$
$$= \frac{96\,x^9 - 352\,x^7 + 1024\,x^5 + 1716\,x^3 - 3\,(x^4+2\,x^2+3)\sqrt{8073i\,\sqrt{2}+234063}\log\left(\sqrt{8073i\,\sqrt{2}+234063}(7\right.}{}$$

input
```
integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
1/96*(96*x^9 - 352*x^7 + 1024*x^5 + 1716*x^3 - 3*(x^4 + 2*x^2 + 3)*sqrt(80
73*I*sqrt(2) + 234063)*log(sqrt(8073*I*sqrt(2) + 234063)*(76*I*sqrt(2) + 5
9) + 45099*x) + 3*(x^4 + 2*x^2 + 3)*sqrt(8073*I*sqrt(2) + 234063)*log(sqrt
(8073*I*sqrt(2) + 234063)*(-76*I*sqrt(2) - 59) + 45099*x) + 3*(x^4 + 2*x^2
 + 3)*sqrt(-8073*I*sqrt(2) + 234063)*log((76*I*sqrt(2) - 59)*sqrt(-8073*I*
sqrt(2) + 234063) + 45099*x) - 3*(x^4 + 2*x^2 + 3)*sqrt(-8073*I*sqrt(2) +
234063)*log((-76*I*sqrt(2) + 59)*sqrt(-8073*I*sqrt(2) + 234063) + 45099*x)
 + 6372*x)/(x^4 + 2*x^2 + 3)
```

3.110.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1205 vs. 2(199) = 398.

Time = 0.74 (sec) , antiderivative size = 1205, normalized size of antiderivative = 5.08

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

output
```
x**5 - 17*x**3/3 + 19*x + (-25*x**3 + 75*x)/(8*x**4 + 16*x**2 + 24) - 3*sq
rt(26007/2048 + 15033*sqrt(3)/2048)*log(x**2 + x*(-304*sqrt(2)*sqrt(8669 +
 5011*sqrt(3))/299 - 433349*sqrt(6)*sqrt(8669 + 5011*sqrt(3))/1498289 + 15
2*sqrt(3)*sqrt(8669 + 5011*sqrt(3))*sqrt(43440359*sqrt(3) + 75240962)/1498
289) - 2882918249387*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962)/22448699275
21 - 993398584*sqrt(6)*sqrt(43440359*sqrt(3) + 75240962)/1343965233 + 4993
6376949404567/2244869927521 + 17261871038090*sqrt(3)/1343965233) + 3*sqrt(
26007/2048 + 15033*sqrt(3)/2048)*log(x**2 + x*(-152*sqrt(3)*sqrt(8669 + 50
11*sqrt(3))*sqrt(43440359*sqrt(3) + 75240962)/1498289 + 433349*sqrt(6)*sqr
t(8669 + 5011*sqrt(3))/1498289 + 304*sqrt(2)*sqrt(8669 + 5011*sqrt(3))/299
) - 2882918249387*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962)/2244869927521
- 993398584*sqrt(6)*sqrt(43440359*sqrt(3) + 75240962)/1343965233 + 4993637
6949404567/2244869927521 + 17261871038090*sqrt(3)/1343965233) - 2*sqrt(-27
*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962)/1024 + 234063/2048 + 405891*sqr
t(3)/2048)*atan(2996578*sqrt(3)*x/(17641*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(4344
0359*sqrt(3) + 75240962) + 8669 + 15033*sqrt(3)) + 152*sqrt(43440359*sqrt(
3) + 75240962)*sqrt(-2*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962) + 8669 +
15033*sqrt(3))) - 1523344*sqrt(6)*sqrt(8669 + 5011*sqrt(3))/(17641*sqrt(2)
*sqrt(-2*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962) + 8669 + 15033*sqrt(3))
 + 152*sqrt(43440359*sqrt(3) + 75240962)*sqrt(-2*sqrt(2)*sqrt(43440359*...
```

3.110.7 Maxima [F]

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \int \frac{(5\,x^6+3\,x^4+x^2+4)x^6}{(x^4+2\,x^2+3)^2}\,dx$$

input
```
integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
x^5 - 17/3*x^3 + 19*x - 25/8*(x^3 - 3*x)/(x^4 + 2*x^2 + 3) + 9/8*integrate
((31*x^2 - 59)/(x^4 + 2*x^2 + 3), x)
```

3.110.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 576 vs. $2(166) = 332$.

Time = 0.63 (sec) , antiderivative size = 576, normalized size of antiderivative = 2.43

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = x^5 - \frac{17}{3}x^3$$

$$-\frac{1}{2304}\sqrt{2}\Big(31\cdot 3^{\frac{3}{4}}\sqrt{2}\big(6\sqrt{3}+18\big)^{\frac{3}{2}} + 558\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\big(\sqrt{3}-3\big) - 558\cdot 3^{\frac{3}{4}}\big(\sqrt{3}+3\big)\sqrt{-6\sqrt{3}+$$

$$-\frac{1}{2304}\sqrt{2}\Big(31\cdot 3^{\frac{3}{4}}\sqrt{2}\big(6\sqrt{3}+18\big)^{\frac{3}{2}} + 558\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\big(\sqrt{3}-3\big) - 558\cdot 3^{\frac{3}{4}}\big(\sqrt{3}+3\big)\sqrt{-6\sqrt{3}+$$

$$-\frac{1}{4608}\sqrt{2}\Big(558\cdot 3^{\frac{3}{4}}\sqrt{2}\big(\sqrt{3}+3\big)\sqrt{-6\sqrt{3}+18} - 31\cdot 3^{\frac{3}{4}}\sqrt{2}\big(-6\sqrt{3}+18\big)^{\frac{3}{2}} + 31\cdot 3^{\frac{3}{4}}\big(6\sqrt{3}+18\big)^{\frac{3}{2}} + 5$$

$$+ 2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}} + \sqrt{3}\Big)$$

$$+\frac{1}{4608}\sqrt{2}\Big(558\cdot 3^{\frac{3}{4}}\sqrt{2}\big(\sqrt{3}+3\big)\sqrt{-6\sqrt{3}+18} - 31\cdot 3^{\frac{3}{4}}\sqrt{2}\big(-6\sqrt{3}+18\big)^{\frac{3}{2}} + 31\cdot 3^{\frac{3}{4}}\big(6\sqrt{3}+18\big)^{\frac{3}{2}} + 55$$

$$- 2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}} + \sqrt{3}\Big) + 19\,x - \frac{25\,(x^3-3\,x)}{8\,(x^4+2\,x^2+3)}$$

input
```
integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
x^5 - 17/3*x^3 - 1/2304*sqrt(2)*(31*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2)
 + 558*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 558*3^(3/4)*(s
qrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 31*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 2
124*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 2124*3^(1/4)*sqrt(-6*sqrt(3) +
18))*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sq
rt(3) + 1/2)) - 1/2304*sqrt(2)*(31*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2)
+ 558*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 558*3^(3/4)*(sq
rt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 31*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 21
24*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 2124*3^(1/4)*sqrt(-6*sqrt(3) + 1
8))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqr
t(3) + 1/2)) - 1/4608*sqrt(2)*(558*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*s
qrt(3) + 18) - 31*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 31*3^(3/4)*(6*
sqrt(3) + 18)^(3/2) + 558*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 212
4*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 2124*3^(1/4)*sqrt(6*sqrt(3) + 18
))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/4608*sqrt
(2)*(558*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 31*3^(3/4)*
sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 31*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 558*
3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 2124*3^(1/4)*sqrt(2)*sqrt(-6*
sqrt(3) + 18) + 2124*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*s
qrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 19*x - 25/8*(x^3 - 3*x)/(x^4 + 2*x...
```

3.110.9 Mupad [B] (verification not implemented)

Time = 8.62 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.69

$$
\begin{aligned}
&\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx \\
&= 19\,x + \frac{\frac{75\,x}{8}-\frac{25\,x^3}{8}}{x^4+2\,x^2+3} - \frac{17\,x^3}{3} + x^5 \\
&\quad - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{26007-\sqrt{2}\,897\mathrm{i}}\,24219\mathrm{i}}{64\,\left(-\frac{1380483}{16}+\frac{\sqrt{2}\,4286763\mathrm{i}}{128}\right)} - \frac{24219\,\sqrt{2}\,x\,\sqrt{26007-\sqrt{2}\,897\mathrm{i}}}{128\,\left(-\frac{1380483}{16}+\frac{\sqrt{2}\,4286763\mathrm{i}}{128}\right)}\right)\,\sqrt{26007-\sqrt{2}\,897\mathrm{i}}\,3\mathrm{i}}{16} \\
&\quad + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{26007+\sqrt{2}\,897\mathrm{i}}\,24219\mathrm{i}}{64\,\left(\frac{1380483}{16}+\frac{\sqrt{2}\,4286763\mathrm{i}}{128}\right)} + \frac{24219\,\sqrt{2}\,x\,\sqrt{26007+\sqrt{2}\,897\mathrm{i}}}{128\,\left(\frac{1380483}{16}+\frac{\sqrt{2}\,4286763\mathrm{i}}{128}\right)}\right)\,\sqrt{26007+\sqrt{2}\,897\mathrm{i}}\,3\mathrm{i}}{16}
\end{aligned}
$$

input
```
int((x^6*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

output

```
19*x + ((75*x)/8 - (25*x^3)/8)/(2*x^2 + x^4 + 3) - (atan((x*(26007 - 2^(1/
2)*897i)^(1/2)*24219i)/(64*((2^(1/2)*4286763i)/128 - 1380483/16)) - (24219
*2^(1/2)*x*(26007 - 2^(1/2)*897i)^(1/2))/(128*((2^(1/2)*4286763i)/128 - 13
80483/16)))*(26007 - 2^(1/2)*897i)^(1/2)*3i)/16 + (atan((x*(2^(1/2)*897i +
 26007)^(1/2)*24219i)/(64*((2^(1/2)*4286763i)/128 + 1380483/16)) + (24219*
2^(1/2)*x*(2^(1/2)*897i + 26007)^(1/2))/(128*((2^(1/2)*4286763i)/128 + 138
0483/16)))*(2^(1/2)*897i + 26007)^(1/2)*3i)/16 - (17*x^3)/3 + x^5
```

3.110. $\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

3.111 $\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

3.111.1 Optimal result

Integrand size = 31, antiderivative size = 232

$$\begin{aligned}\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx &= -17x+\frac{5x^3}{3}-\frac{25x(3+x^2)}{8(3+2x^2+x^4)}\\ &\quad-\frac{1}{16}\sqrt{\frac{1}{2}\left(14395+26499\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)\\ &\quad+\frac{1}{16}\sqrt{\frac{1}{2}\left(14395+26499\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)\\ &\quad-\frac{1}{32}\sqrt{\frac{1}{2}\left(-14395+26499\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)\\ &\quad+\frac{1}{32}\sqrt{\frac{1}{2}\left(-14395+26499\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)\end{aligned}$$

output
```
-17*x+5/3*x^3-25/8*x*(x^2+3)/(x^4+2*x^2+3)-1/64*ln(x^2+3^(1/2)-x*(-2+2*3^(
1/2))^(1/2))*(-28790+52998*3^(1/2))^(1/2)+1/64*ln(x^2+3^(1/2)+x*(-2+2*3^(1
/2))^(1/2))*(-28790+52998*3^(1/2))^(1/2)-1/32*arctan((-2*x+(-2+2*3^(1/2))^
(1/2))/(2+2*3^(1/2))^(1/2))*(28790+52998*3^(1/2))^(1/2)+1/32*arctan((2*x+(
-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(28790+52998*3^(1/2))^(1/2)
```

3.111.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.56

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = -17x+\frac{5x^3}{3}-\frac{25x(3+x^2)}{8(3+2x^2+x^4)} + \frac{(-356i+127\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{16\sqrt{2-2i\sqrt{2}}} + \frac{(356i+127\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{16\sqrt{2+2i\sqrt{2}}}$$

input
```
Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
-17*x + (5*x^3)/3 - (25*x*(3 + x^2))/(8*(3 + 2*x^2 + x^4)) + ((-356*I + 12
7*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(16*Sqrt[2 - (2*I)*Sqrt[2]]) + (
(356*I + 127*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(16*Sqrt[2 + (2*I)*Sq
rt[2]])
```

3.111.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2197, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(5x^6+3x^4+x^2+4)}{(x^4+2x^2+3)^2}dx$$

$$\Big\downarrow \text{2197}$$

$$\frac{1}{48}\int \frac{6(40x^6-56x^4-25x^2+75)}{x^4+2x^2+3}dx-\frac{25x(x^2+3)}{8(x^4+2x^2+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\int \frac{40x^6-56x^4-25x^2+75}{x^4+2x^2+3}dx-\frac{25x(x^2+3)}{8(x^4+2x^2+3)}$$

$$\Big\downarrow \text{2205}$$

$$\frac{1}{8}\int \left(40x^2+\frac{127x^2+483}{x^4+2x^2+3}-136\right)dx-\frac{25x(x^2+3)}{8(x^4+2x^2+3)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}\left(-\frac{1}{2}\sqrt{\frac{1}{2}\left(14395+26499\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(14395+26499\sqrt{3}\right)}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{}\right.}}{\sqrt{2\left(1+\right.}}\right.\right.$$
$$\frac{25x(x^2+3)}{8(x^4+2x^2+3)}$$

input
```
Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
(-25*x*(3 + x^2))/(8*(3 + 2*x^2 + x^4)) + (-136*x + (40*x^3)/3 - (Sqrt[(14
395 + 26499*Sqrt[3])/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 +
Sqrt[3])]])/2 + (Sqrt[(14395 + 26499*Sqrt[3])/2]*ArcTan[(Sqrt[2*(-1 + Sqrt
[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/2 - (Sqrt[(-14395 + 26499*Sqrt[3])/2]
*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/4 + (Sqrt[(-14395 + 26499*
Sqrt[3])/2]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/4)/8
```

3.111.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

3.111.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.30

method	result
risch	$\frac{5x^3}{3}-17x+\frac{-\frac{25}{8}x^3-\frac{75}{8}x}{x^4+2x^2+3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+2\_Z^2+3\right)}\frac{\left(127\_R^2+483\right)\ln\left(x-\_R\right)}{\_R^3+\_R}\right)}{32}$
default	$\frac{5x^3}{3}-17x+\frac{-\frac{25}{8}x^3-\frac{75}{8}x}{x^4+2x^2+3}+\frac{\left(-17\sqrt{-2+2\sqrt{3}}\,\sqrt{3}-178\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{64}+\frac{\left(322\sqrt{3}+\frac{\left(-17\sqrt{-2+2\sqrt{3}}\right.}{}\right.}{}$

input
```
int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
5/3*x^3-17*x+(-25/8*x^3-75/8*x)/(x^4+2*x^2+3)+1/32*sum((127*_R^2+483)/(_R^
3+_R)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^2+3))
```

3.111.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.88

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$$
$$= \frac{160\,x^7 - 1312\,x^5 - 3084\,x^3 + 3\,(x^4+2\,x^2+3)\sqrt{30817i\,\sqrt{2}-14395}\log\left(\sqrt{30817i\,\sqrt{2}-14395}(17i\,\sqrt{2}+\right.}{}$$

input
```
integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
1/96*(160*x^7 - 1312*x^5 - 3084*x^3 + 3*(x^4 + 2*x^2 + 3)*sqrt(30817*I*sqr
t(2) - 14395)*log(sqrt(30817*I*sqrt(2) - 14395)*(17*I*sqrt(2) + 161) + 264
99*x) - 3*(x^4 + 2*x^2 + 3)*sqrt(30817*I*sqrt(2) - 14395)*log(sqrt(30817*I
*sqrt(2) - 14395)*(-17*I*sqrt(2) - 161) + 26499*x) - 3*(x^4 + 2*x^2 + 3)*s
qrt(-30817*I*sqrt(2) - 14395)*log((17*I*sqrt(2) - 161)*sqrt(-30817*I*sqrt(
2) - 14395) + 26499*x) + 3*(x^4 + 2*x^2 + 3)*sqrt(-30817*I*sqrt(2) - 14395
)*log((-17*I*sqrt(2) + 161)*sqrt(-30817*I*sqrt(2) - 14395) + 26499*x) - 57
96*x)/(x^4 + 2*x^2 + 3)
```

3.111.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.26

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5x^3}{3} - 17x + \frac{-25x^3-75x}{8x^4+16x^2+24}$$
$$+\operatorname{RootSum}\left(1048576t^4+29480960t^2+2106591003, \left(t\mapsto t\log\left(\frac{557056t^3}{816619683}+\frac{166600064t}{816619683}+x\right)\right)\right)$$

input
```
integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

output
```
5*x**3/3 - 17*x + (-25*x**3 - 75*x)/(8*x**4 + 16*x**2 + 24) + RootSum(1048
576*_t**4 + 29480960*_t**2 + 2106591003, Lambda(_t, _t*log(557056*_t**3/81
6619683 + 166600064*_t/816619683 + x)))
```

3.111.7 Maxima [F]

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \int \frac{(5\,x^6+3\,x^4+x^2+4)x^4}{(x^4+2\,x^2+3)^2}\,dx$$

input
```
integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
5/3*x^3 - 17*x - 25/8*(x^3 + 3*x)/(x^4 + 2*x^2 + 3) + 1/8*integrate((127*x
^2 + 483)/(x^4 + 2*x^2 + 3), x)
```

3.111.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(163) = 326$.

Time = 0.59 (sec) , antiderivative size = 573, normalized size of antiderivative = 2.47

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5}{3}x^3$$

$$-\frac{1}{20736}\sqrt{2}\Big(127\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+2286\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-2286\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6}$$

$$-\frac{1}{20736}\sqrt{2}\Big(127\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+2286\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-2286\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6}$$

$$-\frac{1}{41472}\sqrt{2}\Big(2286\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-127\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+127\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}$$

$$+2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\Big)$$

$$+\frac{1}{41472}\sqrt{2}\Big(2286\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-127\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+127\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}$$

$$-2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\Big)-17\,x-\frac{25\,(x^3+3\,x)}{8\,(x^4+2\,x^2+3)}$$

input
```
integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
5/3*x^3 - 1/20736*sqrt(2)*(127*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 22
86*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 2286*3^(3/4)*(sqrt
(3) + 3)*sqrt(-6*sqrt(3) + 18) + 127*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 173
88*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 17388*3^(1/4)*sqrt(-6*sqrt(3) +
18))*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sq
rt(3) + 1/2)) - 1/20736*sqrt(2)*(127*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2
) + 2286*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 2286*3^(3/4)
*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 127*3^(3/4)*(-6*sqrt(3) + 18)^(3/2)
 - 17388*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 17388*3^(1/4)*sqrt(-6*sqrt
(3) + 18))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(
1/6*sqrt(3) + 1/2)) - 1/41472*sqrt(2)*(2286*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*
sqrt(-6*sqrt(3) + 18) - 127*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 127*
3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 2286*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3
) - 3) - 17388*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 17388*3^(1/4)*sqrt(
6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3))
 + 1/41472*sqrt(2)*(2286*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 1
8) - 127*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 127*3^(3/4)*(6*sqrt(3)
+ 18)^(3/2) + 2286*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 17388*3^(1
/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 17388*3^(1/4)*sqrt(6*sqrt(3) + 18))*lo
g(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 17*x - 25/8*(...
```

3.111.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.70

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$$
$$= \frac{5\,x^3}{3} - \frac{\frac{25\,x^3}{8}+\frac{75\,x}{8}}{x^4+2\,x^2+3} - 17\,x + \frac{\operatorname{atan}\left(\frac{x\,\sqrt{-14395-\sqrt{2}\,30817\mathrm{i}}\,30817\mathrm{i}}{64\left(-\frac{1571667}{64}+\frac{\sqrt{2}\,148846111\mathrm{i}}{128}\right)} - \frac{30817\,\sqrt{2}\,x\,\sqrt{-14395-\sqrt{2}\,30817\mathrm{i}}}{128\left(-\frac{1571667}{64}+\frac{\sqrt{2}\,148846111\mathrm{i}}{128}\right)}\right)\sqrt{-14395-\sqrt{2}\,30817\mathrm{i}}\,1\mathrm{i}}{16} - \frac{\operatorname{atan}\left(\frac{x\,\sqrt{-14395+\sqrt{2}\,30817\mathrm{i}}\,30817\mathrm{i}}{64\left(\frac{1571667}{64}+\frac{\sqrt{2}\,148846111\mathrm{i}}{128}\right)} + \frac{30817\,\sqrt{2}\,x\,\sqrt{-14395+\sqrt{2}\,30817\mathrm{i}}}{128\left(\frac{1571667}{64}+\frac{\sqrt{2}\,148846111\mathrm{i}}{128}\right)}\right)\sqrt{-14395+\sqrt{2}\,30817\mathrm{i}}\,1\mathrm{i}}{16}$$

input
```
int((x^4*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

output
```
(atan((x*(- 2^(1/2)*30817i - 14395)^(1/2)*30817i)/(64*((2^(1/2)*14884611i)
/128 - 1571667/64)) - (30817*2^(1/2)*x*(- 2^(1/2)*30817i - 14395)^(1/2))/(
128*((2^(1/2)*14884611i)/128 - 1571667/64)))*(- 2^(1/2)*30817i - 14395)^(1
/2)*1i)/16 - ((75*x)/8 + (25*x^3)/8)/(2*x^2 + x^4 + 3) - 17*x - (atan((x*(
2^(1/2)*30817i - 14395)^(1/2)*30817i)/(64*((2^(1/2)*14884611i)/128 + 15716
67/64)) + (30817*2^(1/2)*x*(2^(1/2)*30817i - 14395)^(1/2))/(128*((2^(1/2)*
14884611i)/128 + 1571667/64)))*(2^(1/2)*30817i - 14395)^(1/2)*1i)/16 + (5*
x^3)/3
```

3.111. $\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

3.112 $\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

3.112.1 Optimal result

Integrand size = 31, antiderivative size = 225

$$\begin{aligned}
&\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx \\
&= 5x+\frac{25x(1+x^2)}{8(3+2x^2+x^4)}+\frac{1}{16}\sqrt{\frac{1}{6}\left(19291+12899\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad-\frac{1}{16}\sqrt{\frac{1}{6}\left(19291+12899\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad-\frac{1}{32}\sqrt{\frac{1}{6}\left(-19291+12899\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) \\
&\quad+\frac{1}{32}\sqrt{\frac{1}{6}\left(-19291+12899\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)
\end{aligned}$$

output
```
5*x+25/8*x*(x^2+1)/(x^4+2*x^2+3)-1/192*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/
2))*(-115746+77394*3^(1/2))^(1/2)+1/192*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1
/2))*(-115746+77394*3^(1/2))^(1/2)+1/96*arctan((-2*x+(-2+2*3^(1/2))^(1/2))
/(2+2*3^(1/2))^(1/2))*(115746+77394*3^(1/2))^(1/2)-1/96*arctan((2*x+(-2+2*
3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(115746+77394*3^(1/2))^(1/2)
```

3.112.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.54

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = 5x+\frac{25(x+x^3)}{8(3+2x^2+x^4)}-\frac{\left(-34i+111\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{16\sqrt{2-2i\sqrt{2}}}-\frac{\left(34i+111\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{16\sqrt{2+2i\sqrt{2}}}$$

input
```
Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
5*x + (25*(x + x^3))/(8*(3 + 2*x^2 + x^4)) - ((-34*I + 111*Sqrt[2])*ArcTan
[x/Sqrt[1 - I*Sqrt[2]]])/(16*Sqrt[2 - (2*I)*Sqrt[2]]) - ((34*I + 111*Sqrt[
2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(16*Sqrt[2 + (2*I)*Sqrt[2]])
```

3.112.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2197, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(5x^6+3x^4+x^2+4\right)}{\left(x^4+2x^2+3\right)^2}dx$$

$$\Big\downarrow \text{2197}$$

$$\frac{1}{48}\int -\frac{6\left(-40x^4+31x^2+25\right)}{x^4+2x^2+3}dx+\frac{25x\left(x^2+1\right)}{8\left(x^4+2x^2+3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{25x\left(x^2+1\right)}{8\left(x^4+2x^2+3\right)}-\frac{1}{8}\int \frac{-40x^4+31x^2+25}{x^4+2x^2+3}dx$$

$$\Big\downarrow \text{2205}$$

$$\frac{25x\left(x^2+1\right)}{8\left(x^4+2x^2+3\right)}-\frac{1}{8}\int \left(\frac{111x^2+145}{x^4+2x^2+3}-40\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}\left(\frac{1}{2}\sqrt{\frac{1}{6}\left(19291+12899\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{2}\sqrt{\frac{1}{6}\left(19291+12899\sqrt{3}\right)}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{3}\right.}}{\sqrt{2\left(1+\sqrt{\right.}}}\right.\right.$$
$$\frac{25x\left(x^2+1\right)}{8\left(x^4+2x^2+3\right)}$$

input
```
Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
(25*x*(1 + x^2))/(8*(3 + 2*x^2 + x^4)) + (40*x + (Sqrt[(19291 + 12899*Sqrt
[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/2 -
(Sqrt[(19291 + 12899*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqr
t[2*(1 + Sqrt[3])]])/2 - (Sqrt[(-19291 + 12899*Sqrt[3])/6]*Log[Sqrt[3] - S
qrt[2*(-1 + Sqrt[3])]*x + x^2])/4 + (Sqrt[(-19291 + 12899*Sqrt[3])/6]*Log[
Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/4)/8
```

3.112.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

3.112.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.28

method	result
risch	$5x+\frac{\frac{25}{8}x^3+\frac{25}{8}x}{x^4+2x^2+3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+2\_Z^2+3\right)}\frac{\left(-111\_R^2-145\right)\ln\left(x-\_R\right)}{\_R^3+\_R}\right)}{32}$
default	$5x-\frac{-\frac{25}{8}x^3-\frac{25}{8}x}{x^4+2x^2+3}-\frac{\left(94\sqrt{-2+2\sqrt{3}}\sqrt{3}-51\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{192}-\frac{\left(290\sqrt{3}+\frac{\left(94\sqrt{-2+2\sqrt{3}}\sqrt{3}-51\sqrt{-2+2\sqrt{}\right.}{2}\right.}{48\sqrt{2}}$

input
```
int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
5*x+(25/8*x^3+25/8*x)/(x^4+2*x^2+3)+1/32*sum((-111*_R^2-145)/(_R^3+_R)*ln(
x-_R),_R=RootOf(_Z^4+2*_Z^2+3))
```

3.112.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.99

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$$
$$=\frac{480\,x^5+1260\,x^3-\sqrt{3}(x^4+2\,x^2+3)\sqrt{7969i\,\sqrt{2}-19291}\log\left(\sqrt{3}\sqrt{7969i\,\sqrt{2}-19291}(94i\,\sqrt{2}+145)+\right.}{}$$

input
```
integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
1/96*(480*x^5 + 1260*x^3 - sqrt(3)*(x^4 + 2*x^2 + 3)*sqrt(7969*I*sqrt(2) -
 19291)*log(sqrt(3)*sqrt(7969*I*sqrt(2) - 19291)*(94*I*sqrt(2) + 145) + 38
697*x) + sqrt(3)*(x^4 + 2*x^2 + 3)*sqrt(7969*I*sqrt(2) - 19291)*log(sqrt(3
)*sqrt(7969*I*sqrt(2) - 19291)*(-94*I*sqrt(2) - 145) + 38697*x) + sqrt(3)*
(x^4 + 2*x^2 + 3)*sqrt(-7969*I*sqrt(2) - 19291)*log(sqrt(3)*(94*I*sqrt(2)
- 145)*sqrt(-7969*I*sqrt(2) - 19291) + 38697*x) - sqrt(3)*(x^4 + 2*x^2 + 3
)*sqrt(-7969*I*sqrt(2) - 19291)*log(sqrt(3)*(-94*I*sqrt(2) + 145)*sqrt(-79
69*I*sqrt(2) - 19291) + 38697*x) + 1740*x)/(x^4 + 2*x^2 + 3)
```

3.112.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.23

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = 5x + \frac{25x^3+25x}{8x^4+16x^2+24} + \operatorname{RootSum}\left(3145728t^4+39507968t^2+166384201, \left(t\mapsto t\log\left(-\frac{9240576t^3}{102792131}-\frac{95003488t}{102792131}+x\right)\right)\right)$$

input
```
integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

output
```
5*x + (25*x**3 + 25*x)/(8*x**4 + 16*x**2 + 24) + RootSum(3145728*_t**4 + 3
9507968*_t**2 + 166384201, Lambda(_t, _t*log(-9240576*_t**3/102792131 - 95
003488*_t/102792131 + x)))
```

3.112.7 Maxima [F]

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \int \frac{(5\,x^6+3\,x^4+x^2+4)x^2}{(x^4+2\,x^2+3)^2}\,dx$$

input
```
integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
5*x + 25/8*(x^3 + x)/(x^4 + 2*x^2 + 3) - 1/8*integrate((111*x^2 + 145)/(x^
4 + 2*x^2 + 3), x)
```

3.112.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(158) = 316$.

Time = 0.61 (sec) , antiderivative size = 566, normalized size of antiderivative = 2.52

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$$

$$= \frac{1}{6912}\sqrt{2}\left(37\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}} + 666\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right) - 666\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+1\right.$$

$$+ \frac{1}{6912}\sqrt{2}\left(37\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}} + 666\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right) - 666\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+\right.$$

$$+ \frac{1}{13824}\sqrt{2}\left(666\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18} - 37\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}} + 37\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}} + \right.$$

$$\left. + 2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}} + \sqrt{3}\right)$$

$$- \frac{1}{13824}\sqrt{2}\left(666\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18} - 37\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}} + 37\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}} + \right.$$

$$\left. - 2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}} + \sqrt{3}\right) + 5\,x + \frac{25\,(x^3+x)}{8\,(x^4+2\,x^2+3)}$$

input
```
integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
1/6912*sqrt(2)*(37*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 666*3^(3/4)*sq
rt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 666*3^(3/4)*(sqrt(3) + 3)*sqrt(
-6*sqrt(3) + 18) + 37*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 1740*3^(1/4)*sqrt(
2)*sqrt(6*sqrt(3) + 18) + 1740*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3
^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1
/6912*sqrt(2)*(37*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 666*3^(3/4)*sqr
t(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 666*3^(3/4)*(sqrt(3) + 3)*sqrt(-
6*sqrt(3) + 18) + 37*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 1740*3^(1/4)*sqrt(2
)*sqrt(6*sqrt(3) + 18) + 1740*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^
(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/
13824*sqrt(2)*(666*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 3
7*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 37*3^(3/4)*(6*sqrt(3) + 18)^(3
/2) + 666*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1740*3^(1/4)*sqrt(2
)*sqrt(-6*sqrt(3) + 18) - 1740*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3
^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/13824*sqrt(2)*(666*3^(3/4
)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 37*3^(3/4)*sqrt(2)*(-6*sqr
t(3) + 18)^(3/2) + 37*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 666*3^(3/4)*sqrt(6*
sqrt(3) + 18)*(sqrt(3) - 3) - 1740*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) -
 1740*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3
) + 1/2) + sqrt(3)) + 5*x + 25/8*(x^3 + x)/(x^4 + 2*x^2 + 3)
```

3.112.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.69

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$$
$$= 5\,x + \frac{\frac{25\,x^3}{8}+\frac{25\,x}{8}}{x^4+2\,x^2+3} + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-57873-\sqrt{2}\,23907\mathrm{i}}\,7969\mathrm{i}}{576\,\left(-\frac{374543}{96}+\frac{\sqrt{2}\,1155505\mathrm{i}}{384}\right)}+\frac{7969\,\sqrt{2}\,x\,\sqrt{-57873-\sqrt{2}\,23907\mathrm{i}}}{1152\,\left(-\frac{374543}{96}+\frac{\sqrt{2}\,1155505\mathrm{i}}{384}\right)}\right)\,\sqrt{-57873-\sqrt{2}\,23907\mathrm{i}}\,1\mathrm{i}}{48} - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-57873+\sqrt{2}\,23907\mathrm{i}}\,7969\mathrm{i}}{576\,\left(\frac{374543}{96}+\frac{\sqrt{2}\,1155505\mathrm{i}}{384}\right)}-\frac{7969\,\sqrt{2}\,x\,\sqrt{-57873+\sqrt{2}\,23907\mathrm{i}}}{1152\,\left(\frac{374543}{96}+\frac{\sqrt{2}\,1155505\mathrm{i}}{384}\right)}\right)\,\sqrt{-57873+\sqrt{2}\,23907\mathrm{i}}\,1\mathrm{i}}{48}$$

input
```
int((x^2*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

output
```
5*x + ((25*x)/8 + (25*x^3)/8)/(2*x^2 + x^4 + 3) + (atan((x*(- 2^(1/2)*2390
7i - 57873)^(1/2)*7969i)/(576*((2^(1/2)*1155505i)/384 - 374543/96)) + (796
9*2^(1/2)*x*(- 2^(1/2)*23907i - 57873)^(1/2))/(1152*((2^(1/2)*1155505i)/38
4 - 374543/96)))*(- 2^(1/2)*23907i - 57873)^(1/2)*1i)/48 - (atan((x*(2^(1/
2)*23907i - 57873)^(1/2)*7969i)/(576*((2^(1/2)*1155505i)/384 + 374543/96))
 - (7969*2^(1/2)*x*(2^(1/2)*23907i - 57873)^(1/2))/(1152*((2^(1/2)*1155505
i)/384 + 374543/96)))*(2^(1/2)*23907i - 57873)^(1/2)*1i)/48
```

3.112. $\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

3.113 $\int \frac{4+x^2+3x^4+5x^6}{\left(3+2x^2+x^4\right)^2}\,dx$

3.113.1 Optimal result

Integrand size = 28, antiderivative size = 224

$$\int \frac{4+x^2+3x^4+5x^6}{\left(3+2x^2+x^4\right)^2}\,dx = \frac{25x(1-x^2)}{24\left(3+2x^2+x^4\right)} - \frac{1}{48}\sqrt{\frac{1}{6}\left(-11567+12897\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) + \frac{1}{48}\sqrt{\frac{1}{6}\left(-11567+12897\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) + \frac{1}{96}\sqrt{\frac{1}{6}\left(11567+12897\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x + x^2\right) - \frac{1}{96}\sqrt{\frac{1}{6}\left(11567+12897\sqrt{3}\right)}\log\left(\sqrt{3} + \sqrt{2\left(-1+\sqrt{3}\right)}x + x^2\right)$$

output
```
25/24*x*(-x^2+1)/(x^4+2*x^2+3)-1/288*arctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2
+2*3^(1/2))^(1/2))*(-69402+77382*3^(1/2))^(1/2)+1/288*arctan((2*x+(-2+2*3^
(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-69402+77382*3^(1/2))^(1/2)+1/576*ln(x
^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(69402+77382*3^(1/2))^(1/2)-1/576*ln(x^
2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(69402+77382*3^(1/2))^(1/2)
```

3.113.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.51

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^2}\,dx = \frac{1}{48}\left(-\frac{50x(-1+x^2)}{3+2x^2+x^4} + \frac{(95+44i\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}} + \frac{(95-44i\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^2,x]
```

output
```
((-50*x*(-1 + x^2))/(3 + 2*x^2 + x^4) + ((95 + (44*I)*Sqrt[2])*ArcTan[x/Sq
rt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + ((95 - (44*I)*Sqrt[2])*ArcTan[x/
Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/48
```

3.113.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2206, 27, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{(x^4+2x^2+3)^2}\,dx$$

$$\Big\downarrow \text{ 2206}$$

$$\frac{1}{48}\int \frac{2(95x^2+7)}{x^4+2x^2+3}dx + \frac{25x(1-x^2)}{24(x^4+2x^2+3)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{24}\int \frac{95x^2+7}{x^4+2x^2+3}dx + \frac{25x(1-x^2)}{24(x^4+2x^2+3)}$$

$$\Big\downarrow \text{ 1483}$$

$$\frac{1}{24}\left(\frac{\int \frac{7\sqrt{2\left(-1+\sqrt{3}\right)}-\left(7-95\sqrt{3}\right)x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx}{2\sqrt{6\left(\sqrt{3}-1\right)}}+\frac{\int \frac{\left(7-95\sqrt{3}\right)x+7\sqrt{2\left(-1+\sqrt{3}\right)}}{x^2+\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx}{2\sqrt{6\left(\sqrt{3}-1\right)}}\right)+\frac{25x\left(1-x^2\right)}{24\left(x^4+2x^2+3\right)}$$

$$\Big\downarrow \; 1142$$

$$\frac{1}{24}\left(\frac{\frac{1}{2}\sqrt{51588\sqrt{3}-46268}\int \frac{1}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx-\frac{1}{2}\left(7-95\sqrt{3}\right)\int -\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx}{2\sqrt{6\left(\sqrt{3}-1\right)}}+\frac{\frac{1}{2}\sqrt{51588\sqrt{3}-4\ldots}}{\ldots}\right.$$

$$\frac{25x\left(1-x^2\right)}{24\left(x^4+2x^2+3\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{24}\left(\frac{\frac{1}{2}\sqrt{51588\sqrt{3}-46268}\int \frac{1}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx+\frac{1}{2}\left(7-95\sqrt{3}\right)\int \frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx}{2\sqrt{6\left(\sqrt{3}-1\right)}}+\frac{\frac{1}{2}\sqrt{51588\sqrt{3}-46\ldots}}{\ldots}\right.$$

$$\frac{25x\left(1-x^2\right)}{24\left(x^4+2x^2+3\right)}$$

$$\Big\downarrow \; 1083$$

$$\frac{1}{24}\left(\frac{\frac{1}{2}\left(7-95\sqrt{3}\right)\int \frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx-\sqrt{51588\sqrt{3}-46268}\int \frac{1}{-\left(2x-\sqrt{2\left(-1+\sqrt{3}\right)}\right)^2-2\left(1+\sqrt{3}\right)}d\left(2x-\sqrt{2\left(-1\ldots\right.}\right.}{2\sqrt{6\left(\sqrt{3}-1\right)}}\right.$$

$$\frac{25x\left(1-x^2\right)}{24\left(x^4+2x^2+3\right)}$$

$$\Big\downarrow \; 217$$

3.113. $\int \frac{4+x^2+3x^4+5x^6}{\left(3+2x^2+x^4\right)^2}\,dx$

$$\frac{1}{24}\left(\frac{\frac{1}{2}(7-95\sqrt{3})\int\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx+\sqrt{\frac{51588\sqrt{3}-46268}{2\left(1+\sqrt{3}\right)}}\arctan\left(\frac{2x-\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{2\sqrt{6\left(\sqrt{3}-1\right)}}+\frac{\frac{1}{2}(7-95\sqrt{3})\int\frac{2x+\sqrt{\cdots}}{x^2+\sqrt{2(\cdots}}}{\cdots}\right.$$

$$\frac{25x(1-x^2)}{24\left(x^4+2x^2+3\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{1}{24}\left(\frac{\sqrt{\frac{51588\sqrt{3}-46268}{2\left(1+\sqrt{3}\right)}}\arctan\left(\frac{2x-\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{2}(7-95\sqrt{3})\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{2\sqrt{6\left(\sqrt{3}-1\right)}}+\frac{\sqrt{\frac{51588\sqrt{3}-46268}{2\left(1+\sqrt{3}\right)}}\cdots}{\cdots}\right.$$

$$\frac{25x(1-x^2)}{24\left(x^4+2x^2+3\right)}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^2,x]
```

output
```
(25*x*(1 - x^2))/(24*(3 + 2*x^2 + x^4)) + ((Sqrt[(-46268 + 51588*Sqrt[3])/
(2*(1 + Sqrt[3]))]*ArcTan[(-Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt
[3])]] - ((7 - 95*Sqrt[3])*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/
2)/(2*Sqrt[6*(-1 + Sqrt[3])]) + (Sqrt[(-46268 + 51588*Sqrt[3])/(2*(1 + Sqr
t[3]))]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]] + ((7
 - 95*Sqrt[3])*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/2)/(2*Sqrt[6
*(-1 + Sqrt[3])]))/24
```

3.113.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

3.113. $\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^2}\,dx$

3.113.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.27

method	result
risch	$\frac{-\frac{25}{24}x^3+\frac{25}{24}x}{x^4+2x^2+3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+2\_Z^2+3\right)}\frac{\left(95\_R^2+7\right)\ln\left(x-\_R\right)}{\_R^3+\_R}\right)}{96}$
default	$\frac{-\frac{25}{24}x^3+\frac{25}{24}x}{x^4+2x^2+3}+\frac{\left(139\sqrt{-2+2\sqrt{3}}\,\sqrt{3}+132\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{576}+\frac{\left(14\sqrt{3}+\frac{\left(139\sqrt{-2+2\sqrt{3}}\,\sqrt{3}+132\sqrt{-2+2\sqrt{3}}\right)\sqrt{\cdots}}{2}\cdots\right.}{144\sqrt{2+2\sqrt{\cdots}}}$

input
```
int((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
(-25/24*x^3+25/24*x)/(x^4+2*x^2+3)+1/96*sum((95*_R^2+7)/(_R^3+_R)*ln(x-_R)
,_R=RootOf(_Z^4+2*_Z^2+3))
```

3.113.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.97

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^2}\,dx =$$

$$-\frac{300\,x^3-\sqrt{3}(x^4+2\,x^2+3)\sqrt{13513i\,\sqrt{2}+11567}\log\left(\sqrt{3}\sqrt{13513i\,\sqrt{2}+11567}(139i\,\sqrt{2}+7)+38691\cdots\right.}{\cdots}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
-1/288*(300*x^3 - sqrt(3)*(x^4 + 2*x^2 + 3)*sqrt(13513*I*sqrt(2) + 11567)*
log(sqrt(3)*sqrt(13513*I*sqrt(2) + 11567)*(139*I*sqrt(2) + 7) + 38691*x) +
 sqrt(3)*(x^4 + 2*x^2 + 3)*sqrt(13513*I*sqrt(2) + 11567)*log(sqrt(3)*sqrt(
13513*I*sqrt(2) + 11567)*(-139*I*sqrt(2) - 7) + 38691*x) + sqrt(3)*(x^4 +
2*x^2 + 3)*sqrt(-13513*I*sqrt(2) + 11567)*log(sqrt(3)*(139*I*sqrt(2) - 7)*
sqrt(-13513*I*sqrt(2) + 11567) + 38691*x) - sqrt(3)*(x^4 + 2*x^2 + 3)*sqrt
(-13513*I*sqrt(2) + 11567)*log(sqrt(3)*(-139*I*sqrt(2) + 7)*sqrt(-13513*I*
sqrt(2) + 11567) + 38691*x) - 300*x)/(x^4 + 2*x^2 + 3)
```

3.113.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1185 vs. $2(178) = 356$.

Time = 0.69 (sec) , antiderivative size = 1185, normalized size of antiderivative = 5.29

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

output
```
(-25*x**3 + 25*x)/(24*x**4 + 48*x**2 + 72) + sqrt(11567/55296 + 1433*sqrt(
3)/6144)*log(x**2 + x*(-556*sqrt(2)*sqrt(11567 + 12897*sqrt(3))/13513 - 10
40345*sqrt(6)*sqrt(11567 + 12897*sqrt(3))/174277161 + 278*sqrt(3)*sqrt(115
67 + 12897*sqrt(3))*sqrt(149179599*sqrt(3) + 316396658)/174277161) - 47610
276200401*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658)/30372528846219921 -
4390831246*sqrt(6)*sqrt(149179599*sqrt(3) + 316396658)/7065021829779 + 128
1046481635939181/30372528846219921 + 200684595453464*sqrt(3)/7065021829779
) - sqrt(11567/55296 + 1433*sqrt(3)/6144)*log(x**2 + x*(-278*sqrt(3)*sqrt(
11567 + 12897*sqrt(3))*sqrt(149179599*sqrt(3) + 316396658)/174277161 + 104
0345*sqrt(6)*sqrt(11567 + 12897*sqrt(3))/174277161 + 556*sqrt(2)*sqrt(1156
7 + 12897*sqrt(3))/13513) - 47610276200401*sqrt(2)*sqrt(149179599*sqrt(3)
+ 316396658)/30372528846219921 - 4390831246*sqrt(6)*sqrt(149179599*sqrt(3)
 + 316396658)/7065021829779 + 1281046481635939181/30372528846219921 + 2006
84595453464*sqrt(3)/7065021829779) + 2*sqrt(-sqrt(2)*sqrt(149179599*sqrt(3
) + 316396658)/27648 + 11567/55296 + 1433*sqrt(3)/2048)*atan(348554322*sqr
t(3)*x/(94591*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658)
+ 11567 + 38691*sqrt(3)) + 278*sqrt(149179599*sqrt(3) + 316396658)*sqrt(-2
*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658) + 11567 + 38691*sqrt(3))) - 7
170732*sqrt(6)*sqrt(11567 + 12897*sqrt(3))/(94591*sqrt(2)*sqrt(-2*sqrt(2)*
sqrt(149179599*sqrt(3) + 316396658) + 11567 + 38691*sqrt(3)) + 278*sqrt...
```

3.113.7 Maxima [F]

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^2}\,dx = \int \frac{5\,x^6+3\,x^4+x^2+4}{(x^4+2\,x^2+3)^2}\,dx$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
-25/24*(x^3 - x)/(x^4 + 2*x^2 + 3) + 1/24*integrate((95*x^2 + 7)/(x^4 + 2*
x^2 + 3), x)
```

3.113.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(155) = 310$.

Time = 0.58 (sec) , antiderivative size = 565, normalized size of antiderivative = 2.52

$$\int \frac{4+x^2+3x^4+5x^6}{\left(3+2x^2+x^4\right)^2}\,dx =$$

$$-\frac{1}{62208}\sqrt{2}\left(95\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+1710\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-1710\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{\vphantom{3}}}\right.$$

$$-\frac{1}{62208}\sqrt{2}\left(95\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+1710\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-1710\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{\vphantom{3}}}\right.$$

$$-\frac{1}{124416}\sqrt{2}\left(1710\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-95\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+95\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}\right.$$

$$\left.+2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)$$

$$+\frac{1}{124416}\sqrt{2}\left(1710\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-95\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+95\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}\right.$$

$$\left.-2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)-\frac{25\left(x^3-x\right)}{24\left(x^4+2x^2+3\right)}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
-1/62208*sqrt(2)*(95*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 1710*3^(3/4)
*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1710*3^(3/4)*(sqrt(3) + 3)*s
qrt(-6*sqrt(3) + 18) + 95*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 252*3^(1/4)*sq
rt(2)*sqrt(6*sqrt(3) + 18) + 252*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3
*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) -
 1/62208*sqrt(2)*(95*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 1710*3^(3/4)
*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1710*3^(3/4)*(sqrt(3) + 3)*s
qrt(-6*sqrt(3) + 18) + 95*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 252*3^(1/4)*sq
rt(2)*sqrt(6*sqrt(3) + 18) + 252*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3
*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) -
 1/124416*sqrt(2)*(1710*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18
) - 95*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 95*3^(3/4)*(6*sqrt(3) + 1
8)^(3/2) + 1710*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 252*3^(1/4)*s
qrt(2)*sqrt(-6*sqrt(3) + 18) - 252*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 +
 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/124416*sqrt(2)*(1710*
3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 95*3^(3/4)*sqrt(2)*(
-6*sqrt(3) + 18)^(3/2) + 95*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 1710*3^(3/4)*
sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 252*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) +
 18) - 252*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*s
qrt(3) + 1/2) + sqrt(3)) - 25/24*(x^3 - x)/(x^4 + 2*x^2 + 3)
```

3.113.9 Mupad [B] (verification not implemented)

Time = 8.65 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.68

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^2}\,dx$$
$$= \frac{\frac{25\,x}{24}-\frac{25\,x^3}{24}}{x^4+2\,x^2+3} - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{34701-\sqrt{2}\,40539\mathrm{i}}\,13513\mathrm{i}}{15552\,\left(-\frac{1878307}{5184}+\frac{\sqrt{2}\,94591\mathrm{i}}{10368}\right)}+\frac{13513\,\sqrt{2}\,x\,\sqrt{34701-\sqrt{2}\,40539\mathrm{i}}}{31104\,\left(-\frac{1878307}{5184}+\frac{\sqrt{2}\,94591\mathrm{i}}{10368}\right)}\right)\,\sqrt{34701-\sqrt{2}\,40539\mathrm{i}}\,1\mathrm{i}}{144} + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{34701+\sqrt{2}\,40539\mathrm{i}}\,13513\mathrm{i}}{15552\,\left(\frac{1878307}{5184}+\frac{\sqrt{2}\,94591\mathrm{i}}{10368}\right)}-\frac{13513\,\sqrt{2}\,x\,\sqrt{34701+\sqrt{2}\,40539\mathrm{i}}}{31104\,\left(\frac{1878307}{5184}+\frac{\sqrt{2}\,94591\mathrm{i}}{10368}\right)}\right)\,\sqrt{34701+\sqrt{2}\,40539\mathrm{i}}\,1\mathrm{i}}{144}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(2*x^2 + x^4 + 3)^2,x)
```

output
```
((25*x)/24 - (25*x^3)/24)/(2*x^2 + x^4 + 3) - (atan((x*(34701 - 2^(1/2)*40
539i)^(1/2)*13513i)/(15552*((2^(1/2)*94591i)/10368 - 1878307/5184)) + (135
13*2^(1/2)*x*(34701 - 2^(1/2)*40539i)^(1/2))/(31104*((2^(1/2)*94591i)/1036
8 - 1878307/5184)))*(34701 - 2^(1/2)*40539i)^(1/2)*1i)/144 + (atan((x*(2^(
1/2)*40539i + 34701)^(1/2)*13513i)/(15552*((2^(1/2)*94591i)/10368 + 187830
7/5184)) - (13513*2^(1/2)*x*(2^(1/2)*40539i + 34701)^(1/2))/(31104*((2^(1/
2)*94591i)/10368 + 1878307/5184)))*(2^(1/2)*40539i + 34701)^(1/2)*1i)/144
```

3.113. $\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^2}\,dx$

3.114 $\int \frac{4+x^2+3x^4+5x^6}{x^2(3+2x^2+x^4)^2}\,dx$

3.114.1 Optimal result

Integrand size = 31, antiderivative size = 229

$$\begin{aligned}\int \frac{4+x^2+3x^4+5x^6}{x^2(3+2x^2+x^4)^2}\,dx = & -\frac{4}{9x}-\frac{25x(5+x^2)}{72(3+2x^2+x^4)}\\ & +\frac{1}{48}\sqrt{\frac{1}{6}\left(-965+699\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)\\ & -\frac{1}{48}\sqrt{\frac{1}{6}\left(-965+699\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)\\ & -\frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)\\ & +\frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)\end{aligned}$$

output
```
-4/9/x-25/72*x*(x^2+5)/(x^4+2*x^2+3)+1/288*arctan((-2*x+(-2+2*3^(1/2))^(1/
2))/(2+2*3^(1/2))^(1/2))*(-5790+4194*3^(1/2))^(1/2)-1/288*arctan((2*x+(-2+
2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-5790+4194*3^(1/2))^(1/2)-1/576*ln
(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(5790+4194*3^(1/2))^(1/2)+1/576*ln(x^
2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(5790+4194*3^(1/2))^(1/2)
```

3.114.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.55

$$\int \frac{4+x^2+3x^4+5x^6}{x^2(3+2x^2+x^4)^2}\,dx = -\frac{4}{9x} - \frac{25x(5+x^2)}{72(3+2x^2+x^4)} - \frac{(26i+19\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{48\sqrt{2-2i\sqrt{2}}} - \frac{(-26i+19\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{48\sqrt{2+2i\sqrt{2}}}$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^2),x]
```

output
```
-4/(9*x) - (25*x*(5 + x^2))/(72*(3 + 2*x^2 + x^4)) - ((26*I + 19*Sqrt[2])*
ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(48*Sqrt[2 - (2*I)*Sqrt[2]]) - ((-26*I + 19
*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(48*Sqrt[2 + (2*I)*Sqrt[2]])
```

3.114.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2198, 27, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^2(x^4+2x^2+3)^2}dx$$

$$\Big\downarrow \text{2198}$$

$$\frac{1}{48}\int \frac{2(-25x^4+85x^2+96)}{3x^2(x^4+2x^2+3)}dx - \frac{25x(x^2+5)}{72(x^4+2x^2+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{72}\int \frac{-25x^4+85x^2+96}{x^2(x^4+2x^2+3)}dx - \frac{25x(x^2+5)}{72(x^4+2x^2+3)}$$

$$\Big\downarrow \text{2195}$$

$$\frac{1}{72}\int \left(\frac{32}{x^2} - \frac{3(19x^2-7)}{x^4+2x^2+3}\right)dx - \frac{25x(x^2+5)}{72(x^4+2x^2+3)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{72}\left(\frac{1}{2}\sqrt{\frac{3}{2}\left(699\sqrt{3}-965\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{2}\sqrt{\frac{3}{2}\left(699\sqrt{3}-965\right)}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)\right.$$
$$\frac{25x\left(x^2+5\right)}{72\left(x^4+2x^2+3\right)}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^2),x]
```

output
```
(-25*x*(5 + x^2))/(72*(3 + 2*x^2 + x^4)) + (-32/x + (Sqrt[(3*(-965 + 699*S
qrt[3]))/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/
2 - (Sqrt[(3*(-965 + 699*Sqrt[3]))/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x
)/Sqrt[2*(1 + Sqrt[3])]])/2 - (Sqrt[(3*(965 + 699*Sqrt[3]))/2]*Log[Sqrt[3]
 - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/4 + (Sqrt[(3*(965 + 699*Sqrt[3]))/2]*L
og[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/4)/72
```

3.114.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

3.114.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.28

method	result
risch	$\frac{-\frac{19}{24}x^4-\frac{21}{8}x^2-\frac{4}{3}}{x(x^4+2x^2+3)}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(3\_Z^4-1930\_Z^2+488601\right)}\_R\ln\left(-96\_R^3+34499\_R+361383x\right)\right)}{96}$
default	$-\frac{4}{9x}-\frac{\frac{25}{8}x^3+\frac{125}{8}x}{9(x^4+2x^2+3)}-\frac{\left(32\sqrt{-2+2\sqrt{3}}\sqrt{3}+39\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{576}-\frac{\left(-14\sqrt{3}+\frac{\left(32\sqrt{-2+2\sqrt{3}}\sqrt{3}+39\sqrt{-2\right.}}{2}\right.}{144}$

input
```
int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
(-19/24*x^4-21/8*x^2-4/3)/x/(x^4+2*x^2+3)+1/96*sum(_R*ln(-96*_R^3+34499*_R
+361383*x),_R=RootOf(3*_Z^4-1930*_Z^2+488601))
```

3.114.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00

$$\int \frac{4+x^2+3x^4+5x^6}{x^2(3+2x^2+x^4)^2}\,dx = -\frac{228\,x^4+\sqrt{3}(x^5+2\,x^3+3\,x)\sqrt{517i\,\sqrt{2}+965}\log\left(\sqrt{3}\sqrt{517i\,\sqrt{2}+965}(32i\,\sqrt{2}-7)+2097\,x\right)-\sqrt{3}(x}{}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
-1/288*(228*x^4 + sqrt(3)*(x^5 + 2*x^3 + 3*x)*sqrt(517*I*sqrt(2) + 965)*lo
g(sqrt(3)*sqrt(517*I*sqrt(2) + 965)*(32*I*sqrt(2) - 7) + 2097*x) - sqrt(3)
*(x^5 + 2*x^3 + 3*x)*sqrt(517*I*sqrt(2) + 965)*log(sqrt(3)*sqrt(517*I*sqrt
(2) + 965)*(-32*I*sqrt(2) + 7) + 2097*x) - sqrt(3)*(x^5 + 2*x^3 + 3*x)*sqr
t(-517*I*sqrt(2) + 965)*log(sqrt(3)*(32*I*sqrt(2) + 7)*sqrt(-517*I*sqrt(2)
 + 965) + 2097*x) + sqrt(3)*(x^5 + 2*x^3 + 3*x)*sqrt(-517*I*sqrt(2) + 965)
*log(sqrt(3)*(-32*I*sqrt(2) - 7)*sqrt(-517*I*sqrt(2) + 965) + 2097*x) + 75
6*x^2 + 384)/(x^5 + 2*x^3 + 3*x)
```

3.114.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1192 vs. 2(184) = 368.

Time = 0.76 (sec) , antiderivative size = 1192, normalized size of antiderivative = 5.21

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+2*x**2+3)**2,x)
```

output
```
(-19*x**4 - 63*x**2 - 32)/(24*x**5 + 48*x**3 + 72*x) - sqrt(965/55296 + 23
3*sqrt(3)/18432)*log(x**2 + x*(-128*sqrt(2)*sqrt(965 + 699*sqrt(3))/517 -
21793*sqrt(6)*sqrt(965 + 699*sqrt(3))/361383 + 64*sqrt(3)*sqrt(965 + 699*s
qrt(3))*sqrt(674535*sqrt(3) + 1198514)/361383) - 8882635459*sqrt(2)*sqrt(6
74535*sqrt(3) + 1198514)/130597672689 - 20458048*sqrt(6)*sqrt(674535*sqrt(
3) + 1198514)/560505033 + 18567565928783/130597672689 + 46950427730*sqrt(3
)/560505033) + sqrt(965/55296 + 233*sqrt(3)/18432)*log(x**2 + x*(-64*sqrt(
3)*sqrt(965 + 699*sqrt(3))*sqrt(674535*sqrt(3) + 1198514)/361383 + 21793*s
qrt(6)*sqrt(965 + 699*sqrt(3))/361383 + 128*sqrt(2)*sqrt(965 + 699*sqrt(3)
)/517) - 8882635459*sqrt(2)*sqrt(674535*sqrt(3) + 1198514)/130597672689 -
20458048*sqrt(6)*sqrt(674535*sqrt(3) + 1198514)/560505033 + 18567565928783
/130597672689 + 46950427730*sqrt(3)/560505033) + 2*sqrt(-sqrt(2)*sqrt(6745
35*sqrt(3) + 1198514)/27648 + 965/55296 + 233*sqrt(3)/6144)*atan(722766*sq
rt(3)*x/(-64*sqrt(674535*sqrt(3) + 1198514)*sqrt(-2*sqrt(2)*sqrt(674535*sq
rt(3) + 1198514) + 965 + 2097*sqrt(3)) + 3619*sqrt(2)*sqrt(-2*sqrt(2)*sqrt
(674535*sqrt(3) + 1198514) + 965 + 2097*sqrt(3))) + 89472*sqrt(6)*sqrt(965
 + 699*sqrt(3))/(-64*sqrt(674535*sqrt(3) + 1198514)*sqrt(-2*sqrt(2)*sqrt(6
74535*sqrt(3) + 1198514) + 965 + 2097*sqrt(3)) + 3619*sqrt(2)*sqrt(-2*sqrt
(2)*sqrt(674535*sqrt(3) + 1198514) + 965 + 2097*sqrt(3))) + 65379*sqrt(2)*
sqrt(965 + 699*sqrt(3))/(-64*sqrt(674535*sqrt(3) + 1198514)*sqrt(-2*sqr...
```

3.114.7 Maxima [F]

$$\int \frac{4 + x^2 + 3x^4 + 5x^6}{x^2 (3 + 2x^2 + x^4)^2} \, dx = \int \frac{5\, x^6 + 3\, x^4 + x^2 + 4}{(x^4 + 2\, x^2 + 3)^2 x^2} \, dx$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
-1/24*(19*x^4 + 63*x^2 + 32)/(x^5 + 2*x^3 + 3*x) - 1/24*integrate((19*x^2
- 7)/(x^4 + 2*x^2 + 3), x)
```

3.114.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 572 vs. 2(160) = 320.

Time = 0.58 (sec) , antiderivative size = 572, normalized size of antiderivative = 2.50

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^2}\,dx$$

$$= \frac{1}{62208}\sqrt{2}\left(19\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}} + 342\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right) - 342\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+\right.$$

$$+\frac{1}{62208}\sqrt{2}\left(19\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}} + 342\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right) - 342\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}\right.$$

$$+\frac{1}{124416}\sqrt{2}\left(342\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18} - 19\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}} + 19\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}} +\right.$$

$$\left. + 2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}} + \sqrt{3}\right)$$

$$-\frac{1}{124416}\sqrt{2}\left(342\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18} - 19\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}} + 19\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}} +\right.$$

$$\left. - 2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}} + \sqrt{3}\right) - \frac{19\,x^4+63\,x^2+32}{24\left(x^5+2\,x^3+3\,x\right)}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
1/62208*sqrt(2)*(19*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 342*3^(3/4)*s
qrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 342*3^(3/4)*(sqrt(3) + 3)*sqrt
(-6*sqrt(3) + 18) + 19*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 252*3^(1/4)*sqrt(
2)*sqrt(6*sqrt(3) + 18) - 252*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^
(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/
62208*sqrt(2)*(19*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 342*3^(3/4)*sqr
t(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 342*3^(3/4)*(sqrt(3) + 3)*sqrt(-
6*sqrt(3) + 18) + 19*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 252*3^(1/4)*sqrt(2)
*sqrt(6*sqrt(3) + 18) - 252*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3
/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/12
4416*sqrt(2)*(342*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 19
*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 19*3^(3/4)*(6*sqrt(3) + 18)^(3/
2) + 342*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 252*3^(1/4)*sqrt(2)*
sqrt(-6*sqrt(3) + 18) + 252*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1
/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/124416*sqrt(2)*(342*3^(3/4)*
sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 19*3^(3/4)*sqrt(2)*(-6*sqrt(
3) + 18)^(3/2) + 19*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 342*3^(3/4)*sqrt(6*sq
rt(3) + 18)*(sqrt(3) - 3) + 252*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 25
2*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) +
1/2) + sqrt(3)) - 1/24*(19*x^4 + 63*x^2 + 32)/(x^5 + 2*x^3 + 3*x)
```

3.114.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.69

$$\int \frac{4+x^2+3x^4+5x^6}{x^2(3+2x^2+x^4)^2}\,dx$$
$$= -\frac{\frac{19\,x^4}{24}+\frac{21\,x^2}{8}+\frac{4}{3}}{x^5+2\,x^3+3\,x} - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{2895-\sqrt{2}\,1551\mathrm{i}}\,517\mathrm{i}}{15552\,\left(\frac{517}{162}+\frac{\sqrt{2}\,3619\mathrm{i}}{10368}\right)}+\frac{517\,\sqrt{2}\,x\,\sqrt{2895-\sqrt{2}\,1551\mathrm{i}}}{31104\,\left(\frac{517}{162}+\frac{\sqrt{2}\,3619\mathrm{i}}{10368}\right)}\right)\,\sqrt{2895-\sqrt{2}\,1551\mathrm{i}}\,1\mathrm{i}}{144} + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{2895+\sqrt{2}\,1551\mathrm{i}}\,517\mathrm{i}}{15552\,\left(-\frac{517}{162}+\frac{\sqrt{2}\,3619\mathrm{i}}{10368}\right)}-\frac{517\,\sqrt{2}\,x\,\sqrt{2895+\sqrt{2}\,1551\mathrm{i}}}{31104\,\left(-\frac{517}{162}+\frac{\sqrt{2}\,3619\mathrm{i}}{10368}\right)}\right)\,\sqrt{2895+\sqrt{2}\,1551\mathrm{i}}\,1\mathrm{i}}{144}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^2*(2*x^2 + x^4 + 3)^2),x)
```

output
```
(atan((x*(2^(1/2)*1551i + 2895)^(1/2)*517i)/(15552*((2^(1/2)*3619i)/10368
- 517/162)) - (517*2^(1/2)*x*(2^(1/2)*1551i + 2895)^(1/2))/(31104*((2^(1/2
)*3619i)/10368 - 517/162)))*(2^(1/2)*1551i + 2895)^(1/2)*1i)/144 - (atan((
x*(2895 - 2^(1/2)*1551i)^(1/2)*517i)/(15552*((2^(1/2)*3619i)/10368 + 517/1
62)) + (517*2^(1/2)*x*(2895 - 2^(1/2)*1551i)^(1/2))/(31104*((2^(1/2)*3619i
)/10368 + 517/162)))*(2895 - 2^(1/2)*1551i)^(1/2)*1i)/144 - ((21*x^2)/8 +
(19*x^4)/24 + 4/3)/(3*x + 2*x^3 + x^5)
```

3.114. $\int \frac{4+x^2+3x^4+5x^6}{x^2(3+2x^2+x^4)^2}\,dx$

3.115 $\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^2}\,dx$

3.115.1 Optimal result

Integrand size = 31, antiderivative size = 238

$$\begin{aligned}\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^2}\,dx &= -\frac{4}{27x^3}+\frac{13}{27x}+\frac{25x(7+5x^2)}{216(3+2x^2+x^4)} \\ &\quad -\frac{1}{432}\sqrt{\frac{1}{6}\left(6073+56673\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\ &\quad +\frac{1}{432}\sqrt{\frac{1}{6}\left(6073+56673\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\ &\quad +\frac{1}{864}\sqrt{\frac{1}{6}\left(-6073+56673\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x \right. \\ &\quad \left. +x^2\right)-\frac{1}{864}\sqrt{\frac{1}{6}\left(-6073+56673\sqrt{3}\right)}\log\left(\sqrt{3} \right. \\ &\quad \left. +\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)\end{aligned}$$

output
```
-4/27/x^3+13/27/x+25/216*x*(5*x^2+7)/(x^4+2*x^2+3)+1/5184*ln(x^2+3^(1/2)-x
*(-2+2*3^(1/2))^(1/2))*(-36438+340038*3^(1/2))^(1/2)-1/5184*ln(x^2+3^(1/2)
+x*(-2+2*3^(1/2))^(1/2))*(-36438+340038*3^(1/2))^(1/2)-1/2592*arctan((-2*x
+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(36438+340038*3^(1/2))^(1/2)+1
/2592*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(36438+340038
*3^(1/2))^(1/2)
```

3.115.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.55

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^2}\,dx = \frac{1}{864}\left(\frac{4(-96+248x^2+351x^4+229x^6)}{x^3(3+2x^2+x^4)} + \frac{2(229+46i\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}} + \frac{2(229-46i\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^2),x]
```

output
```
((4*(-96 + 248*x^2 + 351*x^4 + 229*x^6))/(x^3*(3 + 2*x^2 + x^4)) + (2*(229
 + (46*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + (2
*(229 - (46*I)*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]]
)/864
```

3.115.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2198, 27, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^4(x^4+2x^2+3)^2}\,dx$$

$$\Big\downarrow \text{2198}$$

$$\frac{1}{48}\int \frac{2(125x^6+25x^4-120x^2+288)}{9x^4(x^4+2x^2+3)}\,dx + \frac{25x(5x^2+7)}{216(x^4+2x^2+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{216}\int \frac{125x^6+25x^4-120x^2+288}{x^4\left(x^4+2x^2+3\right)}dx+\frac{25x\left(5x^2+7\right)}{216\left(x^4+2x^2+3\right)}$$

$$\Big\downarrow \text{ 2195}$$

$$\frac{1}{216}\int \left(\frac{229x^2+137}{x^4+2x^2+3}-\frac{104}{x^2}+\frac{96}{x^4}\right)dx+\frac{25x\left(5x^2+7\right)}{216\left(x^4+2x^2+3\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{216}\left(-\frac{1}{2}\sqrt{\frac{1}{6}\left(6073+56673\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{2}\sqrt{\frac{1}{6}\left(6073+56673\sqrt{3}\right)}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{\right.}\right.}}{\sqrt{2\left(1+\right.}}\right.\right.$$
$$\frac{25x\left(5x^2+7\right)}{216\left(x^4+2x^2+3\right)}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^2),x]
```

output
```
(25*x*(7 + 5*x^2))/(216*(3 + 2*x^2 + x^4)) + (-32/x^3 + 104/x - (Sqrt[(607
3 + 56673*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sq
rt[3])]])/2 + (Sqrt[(6073 + 56673*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3]
)] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/2 + (Sqrt[(-6073 + 56673*Sqrt[3])/6]*Log
[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/4 - (Sqrt[(-6073 + 56673*Sqrt[
3])/6]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/4)/216
```

3.115.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

3.115.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.29

method	result
risch	$\frac{\frac{229}{216}x^6+\frac{13}{8}x^4+\frac{31}{27}x^2-\frac{4}{9}}{x^3(x^4+2x^2+3)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(3\_Z^4+12146\_Z^2+3211828929)}\_R\ln\left(825\_R^3+11161024\_R+3926135421x\right)\right)}{864}$
default	$-\frac{4}{27x^3}+\frac{13}{27x}+\frac{\frac{125}{8}x^3+\frac{175}{8}x}{27x^4+54x^2+81}+\frac{\left(275\sqrt{-2+2\sqrt{3}}\sqrt{3}+138\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{5184}+\frac{\left(274\sqrt{3}+\frac{(275\sqrt{-2+}}{}\right.}{}$

input
```
int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
(229/216*x^6+13/8*x^4+31/27*x^2-4/9)/x^3/(x^4+2*x^2+3)+1/864*sum(_R*ln(825
*_R^3+11161024*_R+3926135421*x),_R=RootOf(3*_Z^4+12146*_Z^2+3211828929))
```

3.115.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.03

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^2}\,dx$$
$$=\frac{2748x^6+4212x^4+\sqrt{3}(x^7+2x^5+3x^3)\sqrt{69277i\sqrt{2}-6073}\log\left(\sqrt{3}\sqrt{69277i\sqrt{2}-6073}(275i\sqrt{2}+13\right.}{}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
1/2592*(2748*x^6 + 4212*x^4 + sqrt(3)*(x^7 + 2*x^5 + 3*x^3)*sqrt(69277*I*s
qrt(2) - 6073)*log(sqrt(3)*sqrt(69277*I*sqrt(2) - 6073)*(275*I*sqrt(2) + 1
37) + 170019*x) - sqrt(3)*(x^7 + 2*x^5 + 3*x^3)*sqrt(69277*I*sqrt(2) - 607
3)*log(sqrt(3)*sqrt(69277*I*sqrt(2) - 6073)*(-275*I*sqrt(2) - 137) + 17001
9*x) - sqrt(3)*(x^7 + 2*x^5 + 3*x^3)*sqrt(-69277*I*sqrt(2) - 6073)*log(sqr
t(3)*(275*I*sqrt(2) - 137)*sqrt(-69277*I*sqrt(2) - 6073) + 170019*x) + sqr
t(3)*(x^7 + 2*x^5 + 3*x^3)*sqrt(-69277*I*sqrt(2) - 6073)*log(sqrt(3)*(-275
*I*sqrt(2) + 137)*sqrt(-69277*I*sqrt(2) - 6073) + 170019*x) + 2976*x^2 - 1
152)/(x^7 + 2*x^5 + 3*x^3)
```

3.115.6 Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.25

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^2}\,dx$$
$$= \operatorname{RootSum}\left(2293235712t^4+12437504t^2+4405801, \left(t \mapsto t\log\left(\frac{19707494400t^3}{145412423}+\frac{357152768t}{145412423}+x\right)\right)\right) + \frac{229x^6+351x^4+248x^2-96}{216x^7+432x^5+648x^3}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+2*x**2+3)**2,x)
```

output
```
RootSum(2293235712*_t**4 + 12437504*_t**2 + 4405801, Lambda(_t, _t*log(197
07494400*_t**3/145412423 + 357152768*_t/145412423 + x))) + (229*x**6 + 351
*x**4 + 248*x**2 - 96)/(216*x**7 + 432*x**5 + 648*x**3)
```

3.115.7 Maxima [F]

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^2}\,dx = \int \frac{5\,x^6+3\,x^4+x^2+4}{(x^4+2\,x^2+3)^2x^4}\,dx$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
1/216*(229*x^6 + 351*x^4 + 248*x^2 - 96)/(x^7 + 2*x^5 + 3*x^3) + 1/216*int
egrate((229*x^2 + 137)/(x^4 + 2*x^2 + 3), x)
```

3.115.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 579 vs. $2(167) = 334$.

Time = 0.60 (sec) , antiderivative size = 579, normalized size of antiderivative = 2.43

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^2}\,dx =$$

$$-\frac{1}{559872}\sqrt{2}\left(229\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+4122\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-4122\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\ldots}\right.$$

$$-\frac{1}{559872}\sqrt{2}\left(229\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+4122\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-4122\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\ldots}\right.$$

$$-\frac{1}{1119744}\sqrt{2}\left(4122\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-229\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+229\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\ldots\right.\right.$$

$$\left.+2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)$$

$$+\frac{1}{1119744}\sqrt{2}\left(4122\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-229\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+229\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\ldots\right.\right.$$

$$\left.-2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)+\frac{25\left(5x^3+7x\right)}{216\left(x^4+2x^2+3\right)}+\frac{13x^2-4}{27x^3}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
-1/559872*sqrt(2)*(229*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 4122*3^(3/
4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 4122*3^(3/4)*(sqrt(3) + 3)
*sqrt(-6*sqrt(3) + 18) + 229*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 4932*3^(1/4
)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 4932*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arcta
n(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/
2)) - 1/559872*sqrt(2)*(229*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 4122*
3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 4122*3^(3/4)*(sqrt(3)
 + 3)*sqrt(-6*sqrt(3) + 18) + 229*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 4932*3
^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 4932*3^(1/4)*sqrt(-6*sqrt(3) + 18))*
arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3)
 + 1/2)) - 1/1119744*sqrt(2)*(4122*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*s
qrt(3) + 18) - 229*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 229*3^(3/4)*(
6*sqrt(3) + 18)^(3/2) + 4122*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) -
4932*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 4932*3^(1/4)*sqrt(6*sqrt(3) +
 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/111974
4*sqrt(2)*(4122*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 229*
3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 229*3^(3/4)*(6*sqrt(3) + 18)^(3/
2) + 4122*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 4932*3^(1/4)*sqrt(2
)*sqrt(-6*sqrt(3) + 18) - 4932*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3
^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 25/216*(5*x^3 + 7*x)/(x^...
```

3.115.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.69

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^2}\,dx = \frac{\frac{229\,x^6}{216}+\frac{13\,x^4}{8}+\frac{31\,x^2}{27}-\frac{4}{9}}{x^7+2\,x^5+3\,x^3} - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-18219-\sqrt{2}\,207831\mathrm{i}}\,69277\mathrm{i}}{11337408\,\left(-\frac{19051175}{3779136}+\frac{\sqrt{2}\,9490949\mathrm{i}}{7558272}\right)}+\frac{69277\,\sqrt{2}\,x\,\sqrt{-18219-\sqrt{2}\,207831\mathrm{i}}}{22674816\,\left(-\frac{19051175}{3779136}+\frac{\sqrt{2}\,9490949\mathrm{i}}{7558272}\right)}\right)\,\sqrt{-18219-\sqrt{2}\,207831\mathrm{i}}\,1\mathrm{i}}{1296} + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-18219+\sqrt{2}\,207831\mathrm{i}}\,69277\mathrm{i}}{11337408\,\left(\frac{19051175}{3779136}+\frac{\sqrt{2}\,9490949\mathrm{i}}{7558272}\right)}-\frac{69277\,\sqrt{2}\,x\,\sqrt{-18219+\sqrt{2}\,207831\mathrm{i}}}{22674816\,\left(\frac{19051175}{3779136}+\frac{\sqrt{2}\,9490949\mathrm{i}}{7558272}\right)}\right)\,\sqrt{-18219+\sqrt{2}\,207831\mathrm{i}}\,1\mathrm{i}}{1296}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^4*(2*x^2 + x^4 + 3)^2),x)
```

output

```
((31*x^2)/27 + (13*x^4)/8 + (229*x^6)/216 - 4/9)/(3*x^3 + 2*x^5 + x^7) - (
atan((x*(- 2^(1/2)*207831i - 18219)^(1/2)*69277i)/(11337408*((2^(1/2)*9490
949i)/7558272 - 19051175/3779136)) + (69277*2^(1/2)*x*(- 2^(1/2)*207831i -
 18219)^(1/2))/(22674816*((2^(1/2)*9490949i)/7558272 - 19051175/3779136)))
*(- 2^(1/2)*207831i - 18219)^(1/2)*1i)/1296 + (atan((x*(2^(1/2)*207831i -
18219)^(1/2)*69277i)/(11337408*((2^(1/2)*9490949i)/7558272 + 19051175/3779
136)) - (69277*2^(1/2)*x*(2^(1/2)*207831i - 18219)^(1/2))/(22674816*((2^(1
/2)*9490949i)/7558272 + 19051175/3779136)))*(2^(1/2)*207831i - 18219)^(1/2
)*1i)/1296
```

3.115. $\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^2}\,dx$

3.116 $\int \frac{4+x^2+3x^4+5x^6}{x^6\left(3+2x^2+x^4\right)^2} dx$

3.116.1 Optimal result

Integrand size = 31, antiderivative size = 245

$$\begin{aligned}
&\int \frac{4+x^2+3x^4+5x^6}{x^6\left(3+2x^2+x^4\right)^2} dx \\
&= -\frac{4}{45x^5}+\frac{13}{81x^3}-\frac{13}{27x}+\frac{25x(1-7x^2)}{648\left(3+2x^2+x^4\right)} \\
&+ \frac{\sqrt{\frac{1}{6}\left(-1139381+688419\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{1296} \\
&- \frac{\sqrt{\frac{1}{6}\left(-1139381+688419\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{1296} \\
&- \frac{\sqrt{\frac{1}{6}\left(1139381+688419\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)}{2592} \\
&+ \frac{\sqrt{\frac{1}{6}\left(1139381+688419\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)}{2592}
\end{aligned}$$

output
```
-4/45/x^5+13/81/x^3-13/27/x+25/648*x*(-7*x^2+1)/(x^4+2*x^2+3)+1/7776*arcta
n((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-6836286+4130514*3^(1/
2))^(1/2)-1/7776*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-
6836286+4130514*3^(1/2))^(1/2)-1/15552*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/
2))*(6836286+4130514*3^(1/2))^(1/2)+1/15552*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2)
)^(1/2))*(6836286+4130514*3^(1/2))^(1/2)
```

3.116.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.57

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(3+2x^2+x^4\right)^2}\,dx = \frac{-\frac{4(864-984x^2+3928x^4+2475x^6+2435x^8)}{x^5(3+2x^2+x^4)}-\frac{10i\left(-487i+475\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{10i\left(487i+475\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}}{12960}$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(3 + 2*x^2 + x^4)^2),x]
```

output
```
((-4*(864 - 984*x^2 + 3928*x^4 + 2475*x^6 + 2435*x^8))/(x^5*(3 + 2*x^2 + x
^4)) - ((10*I)*(-487*I + 475*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[
1 - I*Sqrt[2]] + ((10*I)*(487*I + 475*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]
]])/Sqrt[1 + I*Sqrt[2]])/12960
```

3.116.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2198, 27, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^6\left(x^4+2x^2+3\right)^2}dx$$

$$\Big\downarrow \; 2198$$

$$\frac{1}{48}\int \frac{2(-175x^8+775x^6+600x^4-360x^2+864)}{27x^6(x^4+2x^2+3)}dx+\frac{25x(1-7x^2)}{648(x^4+2x^2+3)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{648}\int \frac{-175x^8+775x^6+600x^4-360x^2+864}{x^6(x^4+2x^2+3)}dx+\frac{25x(1-7x^2)}{648(x^4+2x^2+3)}$$

$$\Big\downarrow \; 2195$$

$$\frac{1}{648}\int \left(\frac{463-487x^2}{x^4+2x^2+3}+\frac{312}{x^2}-\frac{312}{x^4}+\frac{288}{x^6}\right)dx+\frac{25x(1-7x^2)}{648(x^4+2x^2+3)}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{648}\left(\frac{1}{2}\sqrt{\frac{1}{6}\left(688419\sqrt{3}-1139381\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{2}\sqrt{\frac{1}{6}\left(688419\sqrt{3}-1139381\right)}\arctan\left(\frac{2x -}{\sqrt{}}\right.\right.$$
$$\frac{25x(1-7x^2)}{648(x^4+2x^2+3)}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(3 + 2*x^2 + x^4)^2),x]
```

output
```
(25*x*(1 - 7*x^2))/(648*(3 + 2*x^2 + x^4)) + (-288/(5*x^5) + 104/x^3 - 312
/x + (Sqrt[(-1139381 + 688419*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] -
 2*x)/Sqrt[2*(1 + Sqrt[3])]])/2 - (Sqrt[(-1139381 + 688419*Sqrt[3])/6]*Arc
Tan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/2 - (Sqrt[(1139
381 + 688419*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/4
+ (Sqrt[(1139381 + 688419*Sqrt[3])/6]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]
*x + x^2])/4)/648
```

3.116.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

3.116.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.30

method	result
risch	$\frac{-\frac{487}{648}x^8-\frac{55}{72}x^6-\frac{491}{405}x^4+\frac{41}{135}x^2-\frac{4}{15}}{x^5(x^4+2x^2+3)}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(3\_Z^4-2278762\_Z^2+473920719561\right)}\_R\ln\left(-2886\_R^3+1211171969\_R+17\right.\right.}{2592}$
default	$-\frac{4}{45x^5}+\frac{13}{81x^3}-\frac{13}{27x}-\frac{\frac{175}{24}x^3-\frac{25}{24}x}{27(x^4+2x^2+3)}-\frac{\left(962\sqrt{-2+2\sqrt{3}}\sqrt{3}+1425\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{15552}-\frac{\left(-926\sqrt{3}\right.}{}$

input
```
int((5*x^6+3*x^4+x^2+4)/x^6/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
(-487/648*x^8-55/72*x^6-491/405*x^4+41/135*x^2-4/15)/x^5/(x^4+2*x^2+3)+1/2
592*sum(_R*ln(-2886*_R^3+1211171969*_R+171119622411*x),_R=RootOf(3*_Z^4-22
78762*_Z^2+473920719561))
```

3.116.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.03

$$\int \frac{4+x^2+3x^4+5x^6}{x^6(3+2x^2+x^4)^2}\,dx =$$
$$-\frac{29220\,x^8+29700\,x^6+47136\,x^4+5\,\sqrt{3}(x^9+2\,x^7+3\,x^5)\sqrt{248569i\,\sqrt{2}+1139381}\log\left(\sqrt{3}\sqrt{248569i\,\sqrt{}\right.}{}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
-1/38880*(29220*x^8 + 29700*x^6 + 47136*x^4 + 5*sqrt(3)*(x^9 + 2*x^7 + 3*x
^5)*sqrt(248569*I*sqrt(2) + 1139381)*log(sqrt(3)*sqrt(248569*I*sqrt(2) + 1
139381)*(962*I*sqrt(2) - 463) + 2065257*x) - 5*sqrt(3)*(x^9 + 2*x^7 + 3*x^
5)*sqrt(248569*I*sqrt(2) + 1139381)*log(sqrt(3)*sqrt(248569*I*sqrt(2) + 11
39381)*(-962*I*sqrt(2) + 463) + 2065257*x) - 5*sqrt(3)*(x^9 + 2*x^7 + 3*x^
5)*sqrt(-248569*I*sqrt(2) + 1139381)*log(sqrt(3)*(962*I*sqrt(2) + 463)*sqr
t(-248569*I*sqrt(2) + 1139381) + 2065257*x) + 5*sqrt(3)*(x^9 + 2*x^7 + 3*x
^5)*sqrt(-248569*I*sqrt(2) + 1139381)*log(sqrt(3)*(-962*I*sqrt(2) - 463)*s
qrt(-248569*I*sqrt(2) + 1139381) + 2065257*x) - 11808*x^2 + 10368)/(x^9 +
2*x^7 + 3*x^5)
```

3.116.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1202 vs. 2(199) = 398.

Time = 0.77 (sec) , antiderivative size = 1202, normalized size of antiderivative = 4.91

$$\int \frac{4+x^2+3x^4+5x^6}{x^6(3+2x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**6/(x**4+2*x**2+3)**2,x)
```

output
```
-sqrt(1139381/40310784 + 2833*sqrt(3)/165888)*log(x**2 + x*(-3848*sqrt(2)*
sqrt(1139381 + 688419*sqrt(3))/248569 - 769085497*sqrt(6)*sqrt(1139381 + 6
88419*sqrt(3))/171119622411 + 1924*sqrt(3)*sqrt(1139381 + 688419*sqrt(3))*
sqrt(784371528639*sqrt(3) + 1359975610922)/171119622411) - 867751090756951
0603*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922)/292819251740832134
52921 - 21752950947364*sqrt(6)*sqrt(784371528639*sqrt(3) + 1359975610922)/
127605100269239577 + 20196165220927340076543947/29281925174083213452921 +
50945036826336313070*sqrt(3)/127605100269239577) + sqrt(1139381/40310784 +
 2833*sqrt(3)/165888)*log(x**2 + x*(-1924*sqrt(3)*sqrt(1139381 + 688419*sq
rt(3))*sqrt(784371528639*sqrt(3) + 1359975610922)/171119622411 + 769085497
*sqrt(6)*sqrt(1139381 + 688419*sqrt(3))/171119622411 + 3848*sqrt(2)*sqrt(1
139381 + 688419*sqrt(3))/248569) - 8677510907569510603*sqrt(2)*sqrt(784371
528639*sqrt(3) + 1359975610922)/29281925174083213452921 - 21752950947364*s
qrt(6)*sqrt(784371528639*sqrt(3) + 1359975610922)/127605100269239577 + 201
96165220927340076543947/29281925174083213452921 + 50945036826336313070*sqr
t(3)/127605100269239577) + 2*sqrt(-sqrt(2)*sqrt(784371528639*sqrt(3) + 135
9975610922)/20155392 + 1139381/40310784 + 2833*sqrt(3)/55296)*atan(3422392
44822*sqrt(3)*x/(-1924*sqrt(784371528639*sqrt(3) + 1359975610922)*sqrt(-2*
sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381 + 2065257*sqr
t(3)) + 115087447*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1...
```

3.116.7 Maxima [F]

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(3+2x^2+x^4\right)^2}\,dx=\int \frac{5\,x^6+3\,x^4+x^2+4}{\left(x^4+2\,x^2+3\right)^2x^6}\,dx$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
-1/3240*(2435*x^8 + 2475*x^6 + 3928*x^4 - 984*x^2 + 864)/(x^9 + 2*x^7 + 3*
x^5) - 1/648*integrate((487*x^2 - 463)/(x^4 + 2*x^2 + 3), x)
```

3.116.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. $2(172) = 344$.

Time = 0.63 (sec) , antiderivative size = 584, normalized size of antiderivative = 2.38

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(3+2x^2+x^4\right)^2}\,dx$$

$$= \frac{1}{1679616}\sqrt{2}\left(487\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}} + 8766\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right) - 8766\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\right.$$

$$+\frac{1}{1679616}\sqrt{2}\left(487\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}} + 8766\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right) - 8766\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-\right.$$

$$+\frac{1}{3359232}\sqrt{2}\left(8766\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18} - 487\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}} + 487\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right.\right.$$

$$\left. + 2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}} + \sqrt{3}\right)$$

$$-\frac{1}{3359232}\sqrt{2}\left(8766\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18} - 487\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}} + 487\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right.\right.$$

$$\left. - 2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}} + \sqrt{3}\right) - \frac{25\left(7\,x^3-x\right)}{648\left(x^4+2\,x^2+3\right)} - \frac{195\,x^4-65\,x^2+36}{405\,x^5}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
1/1679616*sqrt(2)*(487*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 8766*3^(3/
4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 8766*3^(3/4)*(sqrt(3) + 3)
*sqrt(-6*sqrt(3) + 18) + 487*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 16668*3^(1/
4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 16668*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arc
tan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) +
1/2)) + 1/1679616*sqrt(2)*(487*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 87
66*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 8766*3^(3/4)*(sqrt
(3) + 3)*sqrt(-6*sqrt(3) + 18) + 487*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 166
68*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 16668*3^(1/4)*sqrt(-6*sqrt(3) +
18))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sq
rt(3) + 1/2)) + 1/3359232*sqrt(2)*(8766*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt
(-6*sqrt(3) + 18) - 487*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 487*3^(3
/4)*(6*sqrt(3) + 18)^(3/2) + 8766*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) -
3) + 16668*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 16668*3^(1/4)*sqrt(6*sq
rt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1
/3359232*sqrt(2)*(8766*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18)
 - 487*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 487*3^(3/4)*(6*sqrt(3) +
18)^(3/2) + 8766*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 16668*3^(1/4
)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 16668*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(
x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 25/648*(7*x^3 -...
```

3.116.9 Mupad [B] (verification not implemented)

Time = 8.77 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.70

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(3+2x^2+x^4\right)^2}\,dx = -\frac{\frac{487\,x^8}{648}+\frac{55\,x^6}{72}+\frac{491\,x^4}{405}-\frac{41\,x^2}{135}+\frac{4}{15}}{x^9+2\,x^7+3\,x^5} - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{3418143-\sqrt{2}\,745707\mathrm{i}}\,248569\mathrm{i}}{306110016\,\left(\frac{119561689}{51018336}+\frac{\sqrt{2}\,1150874447\mathrm{i}}{204073344}\right)}+\frac{248569\,\sqrt{2}\,x\,\sqrt{3418143-\sqrt{2}\,745707\mathrm{i}}}{612220032\,\left(\frac{119561689}{51018336}+\frac{\sqrt{2}\,1150874447\mathrm{i}}{204073344}\right)}\right)\,\sqrt{3418143-\sqrt{2}\,745707\mathrm{i}}\,1\mathrm{i}}{3888} + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{3418143+\sqrt{2}\,745707\mathrm{i}}\,248569\mathrm{i}}{306110016\,\left(-\frac{119561689}{51018336}+\frac{\sqrt{2}\,1150874447\mathrm{i}}{204073344}\right)}-\frac{248569\,\sqrt{2}\,x\,\sqrt{3418143+\sqrt{2}\,745707\mathrm{i}}}{612220032\,\left(-\frac{119561689}{51018336}+\frac{\sqrt{2}\,1150874447\mathrm{i}}{204073344}\right)}\right)\,\sqrt{3418143+\sqrt{2}\,745707\mathrm{i}}\,1\mathrm{i}}{3888}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^6*(2*x^2 + x^4 + 3)^2),x)
```

output
```
(atan((x*(2^(1/2)*745707i + 3418143)^(1/2)*248569i)/(306110016*((2^(1/2)*1
15087447i)/204073344 - 119561689/51018336)) - (248569*2^(1/2)*x*(2^(1/2)*7
45707i + 3418143)^(1/2))/(612220032*((2^(1/2)*115087447i)/204073344 - 1195
61689/51018336)))*(2^(1/2)*745707i + 3418143)^(1/2)*1i)/3888 - (atan((x*(3
418143 - 2^(1/2)*745707i)^(1/2)*248569i)/(306110016*((2^(1/2)*115087447i)/
204073344 + 119561689/51018336)) + (248569*2^(1/2)*x*(3418143 - 2^(1/2)*74
5707i)^(1/2))/(612220032*((2^(1/2)*115087447i)/204073344 + 119561689/51018
336)))*(3418143 - 2^(1/2)*745707i)^(1/2)*1i)/3888 - ((491*x^4)/405 - (41*x
^2)/135 + (55*x^6)/72 + (487*x^8)/648 + 4/15)/(3*x^5 + 2*x^7 + x^9)
```

3.116. $\int \frac{4+x^2+3x^4+5x^6}{x^6(3+2x^2+x^4)^2}\,dx$

3.117 $\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

3.117.1 Optimal result

Integrand size = 31, antiderivative size = 243

$$\begin{aligned}\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx &= 58x-9x^3+x^5-\frac{25x(15+7x^2)}{16\,(3+2x^2+x^4)^2}+\frac{x(3305+252x^2)}{64\,(3+2x^2+x^4)} \\ &\quad+\frac{3}{256}\sqrt{-8595619+7678611\sqrt{3}}\arctan\left(\frac{\sqrt{2\,(-1+\sqrt{3})}-2x}{\sqrt{2\,(1+\sqrt{3})}}\right) \\ &\quad-\frac{3}{256}\sqrt{-8595619+7678611\sqrt{3}}\arctan\left(\frac{\sqrt{2\,(-1+\sqrt{3})}+2x}{\sqrt{2\,(1+\sqrt{3})}}\right) \\ &\quad+\frac{3}{512}\sqrt{8595619+7678611\sqrt{3}}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) \\ &\quad-\frac{3}{512}\sqrt{8595619+7678611\sqrt{3}}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)\end{aligned}$$

output
```
58*x-9*x^3+x^5-25/16*x*(7*x^2+15)/(x^4+2*x^2+3)^2+1/64*x*(252*x^2+3305)/(x
^4+2*x^2+3)+3/256*arctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*
(-8595619+7678611*3^(1/2))^(1/2)-3/256*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(
2+2*3^(1/2))^(1/2))*(-8595619+7678611*3^(1/2))^(1/2)+3/512*ln(x^2+3^(1/2)-
x*(-2+2*3^(1/2))^(1/2))*(8595619+7678611*3^(1/2))^(1/2)-3/512*ln(x^2+3^(1/
2)+x*(-2+2*3^(1/2))^(1/2))*(8595619+7678611*3^(1/2))^(1/2)
```

3.117.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.64

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = 58x-9x^3+x^5-\frac{25x(15+7x^2)}{16(3+2x^2+x^4)^2}+\frac{x(3305+252x^2)}{64(3+2x^2+x^4)} + \frac{3(4795i+148\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{128\sqrt{2-2i\sqrt{2}}} + \frac{3(-4795i+148\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{128\sqrt{2+2i\sqrt{2}}}$$

input
```
Integrate[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

output
```
58*x - 9*x^3 + x^5 - (25*x*(15 + 7*x^2))/(16*(3 + 2*x^2 + x^4)^2) + (x*(33
05 + 252*x^2))/(64*(3 + 2*x^2 + x^4)) + (3*(4795*I + 148*Sqrt[2])*ArcTan[x
/Sqrt[1 - I*Sqrt[2]]])/(128*Sqrt[2 - (2*I)*Sqrt[2]]) + (3*(-4795*I + 148*S
qrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(128*Sqrt[2 + (2*I)*Sqrt[2]])
```

3.117.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2197, 27, 2206, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}(5x^6+3x^4+x^2+4)}{(x^4+2x^2+3)^3}\,dx$$

$$\Big\downarrow \text{2197}$$

$$\frac{1}{96}\int \frac{6(80x^{12}-112x^{10}+400x^6-800x^4-475x^2+375)}{(x^4+2x^2+3)^2}dx-\frac{25x(7x^2+15)}{16(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\int \frac{80x^{12}-112x^{10}+400x^6-800x^4-475x^2+375}{(x^4+2x^2+3)^2}dx-\frac{25x(7x^2+15)}{16(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{2206}$$

$$\frac{1}{16}\left(\frac{1}{48}\int -\frac{12(-320x^8+1088x^6-1216x^4-2684x^2+2805)}{x^4+2x^2+3}dx+\frac{x(252x^2+3305)}{4(x^4+2x^2+3)}\right)-\frac{25x(7x^2+15)}{16(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\left(\frac{x(252x^2+3305)}{4(x^4+2x^2+3)}-\frac{1}{4}\int\frac{-320x^8+1088x^6-1216x^4-2684x^2+2805}{x^4+2x^2+3}dx\right)-\frac{25x(7x^2+15)}{16(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{2205}$$

$$\frac{1}{16}\left(\frac{x(252x^2+3305)}{4(x^4+2x^2+3)}-\frac{1}{4}\int\left(-320x^4+1728x^2+\frac{3(4647-148x^2)}{x^4+2x^2+3}-3712\right)dx\right)-\frac{25x(7x^2+15)}{16(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{16}\left(\frac{1}{4}\left(\frac{3}{4}\sqrt{7678611\sqrt{3}-8595619}\arctan\left(\frac{\sqrt{2(\sqrt{3}-1)}-2x}{\sqrt{2(1+\sqrt{3})}}\right)-\frac{3}{4}\sqrt{7678611\sqrt{3}-8595619}\arctan\left(\frac{2x+\sqrt{2}}{\sqrt{2(1}}\right.\right.\right.$$

$$\frac{25x(7x^2+15)}{16(x^4+2x^2+3)^2}$$

input
```
Int[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

output
```
(-25*x*(15 + 7*x^2))/(16*(3 + 2*x^2 + x^4)^2) + ((x*(3305 + 252*x^2))/(4*(
3 + 2*x^2 + x^4)) + (3712*x - 576*x^3 + 64*x^5 + (3*Sqrt[-8595619 + 767861
1*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/4
 - (3*Sqrt[-8595619 + 7678611*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*
x)/Sqrt[2*(1 + Sqrt[3])]])/4 + (3*Sqrt[8595619 + 7678611*Sqrt[3]]*Log[Sqrt
[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/8 - (3*Sqrt[8595619 + 7678611*Sqrt[
3]]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/8)/4)/16
```

3.117. $\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

3.117.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

3.117. $\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

3.117.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.34

method	result
risch	$x^{5}-9x^{3}+58x+\frac{\frac{63}{16}x^{7}+\frac{3809}{64}x^{5}+\frac{3333}{32}x^{3}+\frac{8415}{64}x}{(x^{4}+2x^{2}+3)^{2}}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}(\_Z^{4}+2\_Z^{2}+3)}\frac{(148\_R^{2}-4647)\ln(x-\_R)}{\_R^{3}+\_R}\right)}{256}$
default	$x^{5}-9x^{3}+58x+\frac{\frac{63}{16}x^{7}+\frac{3809}{64}x^{5}+\frac{3333}{32}x^{3}+\frac{8415}{64}x}{(x^{4}+2x^{2}+3)^{2}}+\frac{3\left(1697\sqrt{-2+2\sqrt{3}}\sqrt{3}+4795\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^{2}+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{1024}$

input
```
int(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)
```

output
```
x^5-9*x^3+58*x+(63/16*x^7+3809/64*x^5+3333/32*x^3+8415/64*x)/(x^4+2*x^2+3)
^2+3/256*sum((148*_R^2-4647)/(_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^2+3))
```

3.117.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.16

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$$
$$= \frac{512\,x^{13}-2560\,x^{11}+16384\,x^{9}+80864\,x^{7}+276744\,x^{5}+368208\,x^{3}-\sqrt{2}(x^{8}+4\,x^{6}+10\,x^{4}+12\,x^{2}+9)\sqrt{}}{}$$

input
```
integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

output
```
1/512*(512*x^13 - 2560*x^11 + 16384*x^9 + 80864*x^7 + 276744*x^5 + 368208*
x^3 - sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(64586691*I*sqrt(2)
+ 77360571)*log((1549*sqrt(2) + 1697*I)*sqrt(64586691*I*sqrt(2) + 77360571
) + 23035833*x) + sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(6458669
1*I*sqrt(2) + 77360571)*log(-(1549*sqrt(2) + 1697*I)*sqrt(64586691*I*sqrt(
2) + 77360571) + 23035833*x) - sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
*sqrt(-64586691*I*sqrt(2) + 77360571)*log((1549*sqrt(2) - 1697*I)*sqrt(-64
586691*I*sqrt(2) + 77360571) + 23035833*x) + sqrt(2)*(x^8 + 4*x^6 + 10*x^4
 + 12*x^2 + 9)*sqrt(-64586691*I*sqrt(2) + 77360571)*log(-(1549*sqrt(2) - 1
697*I)*sqrt(-64586691*I*sqrt(2) + 77360571) + 23035833*x) + 334584*x)/(x^8
 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

3.117.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1204 vs. $2(221) = 442$.

Time = 0.75 (sec) , antiderivative size = 1204, normalized size of antiderivative = 4.95

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x**10*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

output
```
x**5 - 9*x**3 + 58*x + (252*x**7 + 3809*x**5 + 6666*x**3 + 8415*x)/(64*x**
8 + 256*x**6 + 640*x**4 + 768*x**2 + 576) - 3*sqrt(8595619/262144 + 767861
1*sqrt(3)/262144)*log(x**2 + x*(-6788*sqrt(3)*sqrt(8595619 + 7678611*sqrt(
3))/7176299 - 2313785528*sqrt(8595619 + 7678611*sqrt(3))/18368002813563 +
1697*sqrt(2)*sqrt(8595619 + 7678611*sqrt(3))*sqrt(66002414605209*sqrt(3) +
 125383933330562)/18368002813563) - 121809524025246887927 9*sqrt(2)*sqrt(66
002414605209*sqrt(3) + 125383933330562)/101215058207717485241026490 7 - 134
353410196228*sqrt(6)*sqrt(66002414605209*sqrt(3) + 125383933330562)/395442
840668908030011 + 18391902996311867463806959889/10121505820771748524102649
07 + 520457928682380579298 0*sqrt(3)/395442840668908030011) + 3*sqrt(859561
9/262144 + 7678611*sqrt(3)/262144)*log(x**2 + x*(-1697*sqrt(2)*sqrt(859561
9 + 7678611*sqrt(3))*sqrt(66002414605209*sqrt(3) + 125383933330562)/183680
02813563 + 2313785528*sqrt(8595619 + 7678611*sqrt(3))/18368002813563 + 678
8*sqrt(3)*sqrt(8595619 + 7678611*sqrt(3))/7176299) - 121809524025246887927
9*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562)/10121505820771748
52410264907 - 134353410196228*sqrt(6)*sqrt(66002414605209*sqrt(3) + 125383
933330562)/395442840668908030011 + 18391902996311867463806959889/101215058
2077174852410264907 + 520457928682380579298 0*sqrt(3)/395442840668908030011
) - 2*sqrt(-9*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562)/13107
2 + 77360571/262144 + 207322497*sqrt(3)/262144)*atan(110208016881378*x/...
```

3.117.7 Maxima [F]

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \int \frac{(5\,x^6+3\,x^4+x^2+4)x^{10}}{(x^4+2\,x^2+3)^3}\,dx$$

input
```
integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

output
```
x^5 - 9*x^3 + 58*x + 1/64*(252*x^7 + 3809*x^5 + 6666*x^3 + 8415*x)/(x^8 +
4*x^6 + 10*x^4 + 12*x^2 + 9) + 3/64*integrate((148*x^2 - 4647)/(x^4 + 2*x^
2 + 3), x)
```

3.117.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 588 vs. $2(190) = 380$.

Time = 0.73 (sec) , antiderivative size = 588, normalized size of antiderivative = 2.42

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = x^5 - 9\,x^3$$

$$-\frac{1}{13824}\sqrt{2}\left(37\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}} + 666\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right) - 666\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}\right.$$

$$-\frac{1}{13824}\sqrt{2}\left(37\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}} + 666\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right) - 666\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}\right.$$

$$-\frac{1}{27648}\sqrt{2}\left(666\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18} - 37\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}} + 37\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}} + \right.$$

$$\left. + 2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}} + \sqrt{3}\right)$$

$$+\frac{1}{27648}\sqrt{2}\left(666\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18} - 37\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}} + 37\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}} + \right.$$

$$\left. - 2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}} + \sqrt{3}\right) + 58\,x + \frac{252\,x^7 + 3809\,x^5 + 6666\,x^3 + 8415\,x}{64\,(x^4+2\,x^2+3)^2}$$

input
```
integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

output
```
x^5 - 9*x^3 - 1/13824*sqrt(2)*(37*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) +
 666*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 666*3^(3/4)*(sqr
t(3) + 3)*sqrt(-6*sqrt(3) + 18) + 37*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 418
23*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 41823*3^(1/4)*sqrt(-6*sqrt(3) +
18))*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sq
rt(3) + 1/2)) - 1/13824*sqrt(2)*(37*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2)
 + 666*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 666*3^(3/4)*(s
qrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 37*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 4
1823*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 41823*3^(1/4)*sqrt(-6*sqrt(3)
+ 18))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*
sqrt(3) + 1/2)) - 1/27648*sqrt(2)*(666*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(
-6*sqrt(3) + 18) - 37*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 37*3^(3/4)
*(6*sqrt(3) + 18)^(3/2) + 666*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) +
 41823*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 41823*3^(1/4)*sqrt(6*sqrt(3
) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/276
48*sqrt(2)*(666*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 37*3
^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 37*3^(3/4)*(6*sqrt(3) + 18)^(3/2)
 + 666*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 41823*3^(1/4)*sqrt(2)*
sqrt(-6*sqrt(3) + 18) + 41823*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^
(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 58*x + 1/64*(252*x^7 + 38...
```

3.117.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.76

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = 58\,x+\frac{\frac{63\,x^7}{16}+\frac{3809\,x^5}{64}+\frac{3333\,x^3}{32}+\frac{8415\,x}{64}}{x^8+4\,x^6+10\,x^4+12\,x^2+9}-9\,x^3+x^5$$
$$-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{17191238-\sqrt{2}\,14352598\mathrm{i}}\,193760073\mathrm{i}}{131072\,\left(-\frac{986432531643}{131072}+\frac{\sqrt{2}\,9004030592231\mathrm{i}}{131072}\right)}-\frac{193760073\,\sqrt{2}\,x\,\sqrt{17191238-\sqrt{2}\,14352598\mathrm{i}}}{262144\,\left(-\frac{986432531643}{131072}+\frac{\sqrt{2}\,9004030592231\mathrm{i}}{131072}\right)}\right)\,\sqrt{17191238-\sqrt{2}\,1435259}}{256}$$
$$+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{17191238+\sqrt{2}\,14352598\mathrm{i}}\,193760073\mathrm{i}}{131072\,\left(\frac{986432531643}{131072}+\frac{\sqrt{2}\,9004030592231\mathrm{i}}{131072}\right)}+\frac{193760073\,\sqrt{2}\,x\,\sqrt{17191238+\sqrt{2}\,14352598\mathrm{i}}}{262144\,\left(\frac{986432531643}{131072}+\frac{\sqrt{2}\,9004030592231\mathrm{i}}{131072}\right)}\right)\,\sqrt{17191238+\sqrt{2}\,14352598\mathrm{i}}}{256}$$

input
```
int((x^10*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^3,x)
```

output
```
58*x - (atan((x*(17191238 - 2^(1/2)*14352598i)^(1/2)*193760073i)/(131072*(
(2^(1/2)*900403059231i)/131072 - 986432531643/131072)) - (193760073*2^(1/2
)*x*(17191238 - 2^(1/2)*14352598i)^(1/2))/(262144*((2^(1/2)*900403059231i)
/131072 - 986432531643/131072)))*(17191238 - 2^(1/2)*14352598i)^(1/2)*3i)/
256 + (atan((x*(2^(1/2)*14352598i + 17191238)^(1/2)*193760073i)/(131072*((
2^(1/2)*900403059231i)/131072 + 986432531643/131072)) + (193760073*2^(1/2)
*x*(2^(1/2)*14352598i + 17191238)^(1/2))/(262144*((2^(1/2)*900403059231i)/
131072 + 986432531643/131072)))*(2^(1/2)*14352598i + 17191238)^(1/2)*3i)/2
56 + ((8415*x)/64 + (3333*x^3)/32 + (3809*x^5)/64 + (63*x^7)/16)/(12*x^2 +
 10*x^4 + 4*x^6 + x^8 + 9) - 9*x^3 + x^5
```

3.117. $\int \frac{x^{10}\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^3}\,dx$

3.118 $\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3} dx$

3.118.1 Optimal result

Integrand size = 31, antiderivative size = 242

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3} dx = -27x+\frac{5x^3}{3}+\frac{25x(3+5x^2)}{16(3+2x^2+x^4)^2}-\frac{x(1468+835x^2)}{64(3+2x^2+x^4)} - \frac{21}{256}\sqrt{34271+22721\sqrt{3}}\arctan\left(\frac{\sqrt{2(-1+\sqrt{3})}-2x}{\sqrt{2(1+\sqrt{3})}}\right) + \frac{21}{256}\sqrt{34271+22721\sqrt{3}}\arctan\left(\frac{\sqrt{2(-1+\sqrt{3})}+2x}{\sqrt{2(1+\sqrt{3})}}\right) - \frac{21}{512}\sqrt{-34271+22721\sqrt{3}}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x + x^2\right) + \frac{21}{512}\sqrt{-34271+22721\sqrt{3}}\log\left(\sqrt{3} + \sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)$$

output
```
-27*x+5/3*x^3+25/16*x*(5*x^2+3)/(x^4+2*x^2+3)^2-1/64*x*(835*x^2+1468)/(x^4
+2*x^2+3)-21/512*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-34271+22721*3^(1
/2))^(1/2)+21/512*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-34271+22721*3^(
1/2))^(1/2)-21/256*arctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))
*(34271+22721*3^(1/2))^(1/2)+21/256*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2
*3^(1/2))^(1/2))*(34271+22721*3^(1/2))^(1/2)
```

3.118.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.64

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = -27x+\frac{5x^3}{3}+\frac{25x(3+5x^2)}{16\,(3+2x^2+x^4)^2}-\frac{x(1468+835x^2)}{64\,(3+2x^2+x^4)} + \frac{21\left(-175i+137\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{128\sqrt{2-2i\sqrt{2}}} + \frac{21\left(175i+137\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{128\sqrt{2+2i\sqrt{2}}}$$

input
```
Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

output
```
-27*x + (5*x^3)/3 + (25*x*(3 + 5*x^2))/(16*(3 + 2*x^2 + x^4)^2) - (x*(1468
 + 835*x^2))/(64*(3 + 2*x^2 + x^4)) + (21*(-175*I + 137*Sqrt[2])*ArcTan[x/
Sqrt[1 - I*Sqrt[2]]])/(128*Sqrt[2 - (2*I)*Sqrt[2]]) + (21*(175*I + 137*Sqr
t[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(128*Sqrt[2 + (2*I)*Sqrt[2]])
```

3.118.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2197, 27, 2206, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8\left(5x^6+3x^4+x^2+4\right)}{\left(x^4+2x^2+3\right)^3}dx$$

$$\Big\downarrow \text{2197}$$

$$\frac{1}{96}\int -\frac{6\left(-80x^{10}+112x^8-400x^4+175x^2+75\right)}{\left(x^4+2x^2+3\right)^2}dx+\frac{25x\left(5x^2+3\right)}{16\left(x^4+2x^2+3\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{25x\left(5x^2+3\right)}{16\left(x^4+2x^2+3\right)^2}-\frac{1}{16}\int \frac{-80x^{10}+112x^8-400x^4+175x^2+75}{\left(x^4+2x^2+3\right)^2}dx$$

$$\Big\downarrow \; 2206$$

$$\frac{1}{16}\left(-\frac{1}{48}\int -\frac{12\left(320x^6-1088x^4+381x^2+1368\right)}{x^4+2x^2+3}dx-\frac{x\left(835x^2+1468\right)}{4\left(x^4+2x^2+3\right)}\right)+\frac{25x\left(5x^2+3\right)}{16\left(x^4+2x^2+3\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{16}\left(\frac{1}{4}\int \frac{320x^6-1088x^4+381x^2+1368}{x^4+2x^2+3}dx-\frac{x\left(835x^2+1468\right)}{4\left(x^4+2x^2+3\right)}\right)+\frac{25x\left(5x^2+3\right)}{16\left(x^4+2x^2+3\right)^2}$$

$$\Big\downarrow \; 2205$$

$$\frac{1}{16}\left(\frac{1}{4}\int \left(320x^2+\frac{21\left(137x^2+312\right)}{x^4+2x^2+3}-1728\right)dx-\frac{x\left(835x^2+1468\right)}{4\left(x^4+2x^2+3\right)}\right)+\frac{25x\left(5x^2+3\right)}{16\left(x^4+2x^2+3\right)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{16}\left(\frac{1}{4}\left(-\frac{21}{4}\sqrt{34271+22721\sqrt{3}}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{21}{4}\sqrt{34271+22721\sqrt{3}}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{3}\right.}}{\sqrt{2\left(1+\sqrt{3}\right.}}\right.\right.\right.$$
$$\frac{25x\left(5x^2+3\right)}{16\left(x^4+2x^2+3\right)^2}$$

input
```
Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

output
```
(25*x*(3 + 5*x^2))/(16*(3 + 2*x^2 + x^4)^2) + (-1/4*(x*(1468 + 835*x^2))/(
3 + 2*x^2 + x^4) + (-1728*x + (320*x^3)/3 - (21*Sqrt[34271 + 22721*Sqrt[3]
]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/4 + (21*Sq
rt[34271 + 22721*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1
+ Sqrt[3])]])/4 - (21*Sqrt[-34271 + 22721*Sqrt[3]]*Log[Sqrt[3] - Sqrt[2*(-
1 + Sqrt[3])]*x + x^2])/8 + (21*Sqrt[-34271 + 22721*Sqrt[3]]*Log[Sqrt[3] +
 Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/8)/4)/16
```

3.118.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.118. $\int \frac{x^8\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^3}\,dx$

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

3.118.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.33

method	result
risch	$\frac{5x^3}{3} - 27x + \frac{-\frac{835}{64}x^7 - \frac{1569}{32}x^5 - \frac{4941}{64}x^3 - \frac{513}{8}x}{(x^4+2x^2+3)^2} + \frac{21\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+2\_Z^2+3)} \frac{(137\_R^2+312)\ln(x-\_R)}{\_R^3+\_R}\right)}{256}$
default	$\frac{5x^3}{3} - 27x + \frac{-\frac{835}{64}x^7 - \frac{1569}{32}x^5 - \frac{4941}{64}x^3 - \frac{513}{8}x}{(x^4+2x^2+3)^2} + \frac{21\left(33\sqrt{-2+2\sqrt{3}}\,\sqrt{3}-175\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{1024} + \frac{21(4}{}$

3.118. $\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

input
```
int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)
```

output
```
5/3*x^3-27*x+(-835/64*x^7-1569/32*x^5-4941/64*x^3-513/8*x)/(x^4+2*x^2+3)^2
+21/256*sum((137*_R^2+312)/(_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^2+3))
```

3.118.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.15

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$$
$$= \frac{2560\,x^{11} - 31232\,x^9 - 160328\,x^7 - 459312\,x^5 - 593208\,x^3 + 3\,\sqrt{2}(x^8 + 4\,x^6 + 10\,x^4 + 12\,x^2 + 9)\sqrt{60324}}{}$$

input
```
integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

output
```
1/1536*(2560*x^11 - 31232*x^9 - 160328*x^7 - 459312*x^5 - 593208*x^3 + 3*s
qrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(6032439*I*sqrt(2) - 151135
11)*log((104*sqrt(2) - 33*I)*sqrt(6032439*I*sqrt(2) - 15113511) + 477141*x
) - 3*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(6032439*I*sqrt(2) -
 15113511)*log(-(104*sqrt(2) - 33*I)*sqrt(6032439*I*sqrt(2) - 15113511) +
477141*x) + 3*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-6032439*I*
sqrt(2) - 15113511)*log((104*sqrt(2) + 33*I)*sqrt(-6032439*I*sqrt(2) - 151
13511) + 477141*x) - 3*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-6
032439*I*sqrt(2) - 15113511)*log(-(104*sqrt(2) + 33*I)*sqrt(-6032439*I*sqr
t(2) - 15113511) + 477141*x) - 471744*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 +
9)
```

3.118.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.34

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \frac{5x^3}{3} - 27x + \frac{-835x^7-3138x^5-4941x^3-4104x}{64x^8+256x^6+640x^4+768x^2+576} + 21\,\mathrm{RootSum}\left(17179869184t^4+8983937024t^2+1548731523, \left(t\mapsto t\log\left(-\frac{1107296256t^3}{310800559}+\frac{43885798}{3108005}\right.\right.\right.$$

input
```
integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

output
```
5*x**3/3 - 27*x + (-835*x**7 - 3138*x**5 - 4941*x**3 - 4104*x)/(64*x**8 +
256*x**6 + 640*x**4 + 768*x**2 + 576) + 21*RootSum(17179869184*_t**4 + 898
3937024*_t**2 + 1548731523, Lambda(_t, _t*log(-1107296256*_t**3/310800559
+ 438857984*_t/310800559 + x)))
```

3.118.7 Maxima [F]

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \int \frac{(5\,x^6+3\,x^4+x^2+4)x^8}{(x^4+2\,x^2+3)^3}\,dx$$

input
```
integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

output
```
5/3*x^3 - 27*x - 1/64*(835*x^7 + 3138*x^5 + 4941*x^3 + 4104*x)/(x^8 + 4*x^
6 + 10*x^4 + 12*x^2 + 9) + 21/64*integrate((137*x^2 + 312)/(x^4 + 2*x^2 +
3), x)
```

3.118.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 585 vs. 2(187) = 374.

Time = 0.75 (sec) , antiderivative size = 585, normalized size of antiderivative = 2.42

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \frac{5}{3}\,x^3$$

$$-\frac{7}{55296}\,\sqrt{2}\left(137\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\,\sqrt{3}+18\right)^{\frac{3}{2}}+2466\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\,\sqrt{3}+18}\left(\sqrt{3}-3\right)-2466\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\right.$$

$$-\frac{7}{55296}\,\sqrt{2}\left(137\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\,\sqrt{3}+18\right)^{\frac{3}{2}}+2466\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\,\sqrt{3}+18}\left(\sqrt{3}-3\right)-2466\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\right.$$

$$-\frac{7}{110592}\,\sqrt{2}\left(2466\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\,\sqrt{3}+18}-137\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\,\sqrt{3}+18\right)^{\frac{3}{2}}+137\cdot 3^{\frac{3}{4}}\left(6\,\sqrt{3}+18\right)\right.$$

$$\left.+2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\,\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)$$

$$+\frac{7}{110592}\,\sqrt{2}\left(2466\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\,\sqrt{3}+18}-137\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\,\sqrt{3}+18\right)^{\frac{3}{2}}+137\cdot 3^{\frac{3}{4}}\left(6\,\sqrt{3}+18\right)\right.$$

$$\left.-2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\,\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)-27\,x-\frac{835\,x^7+3138\,x^5+4941\,x^3+4104\,x}{64\,(x^4+2\,x^2+3)^2}$$

input
```
integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

3.118. $\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

output
```
5/3*x^3 - 7/55296*sqrt(2)*(137*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 24
66*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 2466*3^(3/4)*(sqrt
(3) + 3)*sqrt(-6*sqrt(3) + 18) + 137*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 112
32*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 11232*3^(1/4)*sqrt(-6*sqrt(3) +
18))*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sq
rt(3) + 1/2)) - 7/55296*sqrt(2)*(137*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2
) + 2466*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 2466*3^(3/4)
*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 137*3^(3/4)*(-6*sqrt(3) + 18)^(3/2)
 - 11232*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 11232*3^(1/4)*sqrt(-6*sqrt
(3) + 18))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(
1/6*sqrt(3) + 1/2)) - 7/110592*sqrt(2)*(2466*3^(3/4)*sqrt(2)*(sqrt(3) + 3)
*sqrt(-6*sqrt(3) + 18) - 137*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 137
*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 2466*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(
3) - 3) - 11232*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 11232*3^(1/4)*sqrt
(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)
) + 7/110592*sqrt(2)*(2466*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) +
 18) - 137*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 137*3^(3/4)*(6*sqrt(3
) + 18)^(3/2) + 2466*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 11232*3^
(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 11232*3^(1/4)*sqrt(6*sqrt(3) + 18))*
log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 27*x - 1/64...
```

3.118.9 Mupad [B] (verification not implemented)

Time = 8.90 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.75

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \frac{5\,x^3}{3} - \frac{\frac{835\,x^7}{64}+\frac{1569\,x^5}{32}+\frac{4941\,x^3}{64}+\frac{513\,x}{8}}{x^8+4\,x^6+10\,x^4+12\,x^2+9} - 27\,x$$
$$+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-68542-\sqrt{2}\,27358\mathrm{i}}\,126681219\mathrm{i}}{131072\,\left(\frac{12541440681}{131072}+\frac{\sqrt{2}\,4940567541\mathrm{i}}{16384}\right)}-\frac{126681219\,\sqrt{2}\,x\,\sqrt{-68542-\sqrt{2}\,27358\mathrm{i}}}{262144\,\left(\frac{12541440681}{131072}+\frac{\sqrt{2}\,4940567541\mathrm{i}}{16384}\right)}\right)\,\sqrt{-68542-\sqrt{2}\,27358\mathrm{i}}\,21\mathrm{i}}{256}$$
$$-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-68542+\sqrt{2}\,27358\mathrm{i}}\,126681219\mathrm{i}}{131072\,\left(-\frac{12541440681}{131072}+\frac{\sqrt{2}\,4940567541\mathrm{i}}{16384}\right)}+\frac{126681219\,\sqrt{2}\,x\,\sqrt{-68542+\sqrt{2}\,27358\mathrm{i}}}{262144\,\left(-\frac{12541440681}{131072}+\frac{\sqrt{2}\,4940567541\mathrm{i}}{16384}\right)}\right)\,\sqrt{-68542+\sqrt{2}\,27358\mathrm{i}}\,21\mathrm{i}}{256}$$

input
```
int((x^8*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^3,x)
```

output
```
(atan((x*(- 2^(1/2)*27358i - 68542)^(1/2)*126681219i)/(131072*((2^(1/2)*49
40567541i)/16384 + 12541440681/131072)) - (126681219*2^(1/2)*x*(- 2^(1/2)*
27358i - 68542)^(1/2))/(262144*((2^(1/2)*4940567541i)/16384 + 12541440681/
131072)))*(- 2^(1/2)*27358i - 68542)^(1/2)*21i)/256 - ((513*x)/8 + (4941*x
^3)/64 + (1569*x^5)/32 + (835*x^7)/64)/(12*x^2 + 10*x^4 + 4*x^6 + x^8 + 9)
 - 27*x - (atan((x*(2^(1/2)*27358i - 68542)^(1/2)*126681219i)/(131072*((2^
(1/2)*4940567541i)/16384 - 12541440681/131072)) + (126681219*2^(1/2)*x*(2^
(1/2)*27358i - 68542)^(1/2))/(262144*((2^(1/2)*4940567541i)/16384 - 125414
40681/131072)))*(2^(1/2)*27358i - 68542)^(1/2)*21i)/256 + (5*x^3)/3
```

3.118. $\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

3.119 $\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

3.119.1 Optimal result

Integrand size = 31, antiderivative size = 235

$$\begin{aligned}
&\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx \\
&= 5x+\frac{25x(3-x^2)}{16\left(3+2x^2+x^4\right)^2}+\frac{7x(11+58x^2)}{64\left(3+2x^2+x^4\right)} \\
&\quad+\frac{1}{256}\sqrt{827621+1176531\sqrt{3}}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad-\frac{1}{256}\sqrt{827621+1176531\sqrt{3}}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad-\frac{1}{512}\sqrt{-827621+1176531\sqrt{3}}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) \\
&\quad+\frac{1}{512}\sqrt{-827621+1176531\sqrt{3}}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)
\end{aligned}$$

output
```
5*x+25/16*x*(-x^2+3)/(x^4+2*x^2+3)^2+7/64*x*(58*x^2+11)/(x^4+2*x^2+3)-1/51
2*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-827621+1176531*3^(1/2))^(1/2)+1
/512*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-827621+1176531*3^(1/2))^(1/2
)+1/256*arctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(827621+11
76531*3^(1/2))^(1/2)-1/256*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))
^(1/2))*(827621+1176531*3^(1/2))^(1/2)
```

3.119.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.59

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \frac{1}{256}\left(\frac{4x(3411+5112x^2+4089x^4+1686x^6+320x^8)}{(3+2x^2+x^4)^2} - \frac{i(-2644i+185\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}} + \frac{i(2644i+185\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

input
```
Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

output
```
((4*x*(3411 + 5112*x^2 + 4089*x^4 + 1686*x^6 + 320*x^8))/(3 + 2*x^2 + x^4)
^2 - (I*(-2644*I + 185*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*
Sqrt[2]] + (I*(2644*I + 185*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1
 + I*Sqrt[2]])/256
```

3.119.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2197, 27, 2206, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(5x^6+3x^4+x^2+4)}{(x^4+2x^2+3)^3}\,dx$$

$$\Big\downarrow \text{2197}$$

$$\frac{1}{96}\int -\frac{6(-80x^8+112x^6-275x^2+75)}{(x^4+2x^2+3)^2}dx + \frac{25x(3-x^2)}{16(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{25x(3-x^2)}{16(x^4+2x^2+3)^2}-\frac{1}{16}\int\frac{-80x^8+112x^6-275x^2+75}{(x^4+2x^2+3)^2}dx$$

$$\Big\downarrow \text{2206}$$

$$\frac{1}{16}\left(\frac{7x(58x^2+11)}{4(x^4+2x^2+3)}-\frac{1}{48}\int\frac{12(-320x^4+682x^2+177)}{x^4+2x^2+3}dx\right)+\frac{25x(3-x^2)}{16(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\left(\frac{7x(58x^2+11)}{4(x^4+2x^2+3)}-\frac{1}{4}\int\frac{-320x^4+682x^2+177}{x^4+2x^2+3}dx\right)+\frac{25x(3-x^2)}{16(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{2205}$$

$$\frac{1}{16}\left(\frac{7x(58x^2+11)}{4(x^4+2x^2+3)}-\frac{1}{4}\int\left(\frac{1322x^2+1137}{x^4+2x^2+3}-320\right)dx\right)+\frac{25x(3-x^2)}{16(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{16}\left(\frac{1}{4}\left(\frac{1}{4}\sqrt{827621+1176531\sqrt{3}}\arctan\left(\frac{\sqrt{2(\sqrt{3}-1)}-2x}{\sqrt{2(1+\sqrt{3})}}\right)-\frac{1}{4}\sqrt{827621+1176531\sqrt{3}}\arctan\left(\frac{2x+\sqrt{2(\sqrt{}}}{\sqrt{2(1+}}\right.\right.\right.$$
$$\frac{25x(3-x^2)}{16(x^4+2x^2+3)^2}$$

input
```
Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

output
```
(25*x*(3 - x^2))/(16*(3 + 2*x^2 + x^4)^2) + ((7*x*(11 + 58*x^2))/(4*(3 + 2
*x^2 + x^4)) + (320*x + (Sqrt[827621 + 1176531*Sqrt[3]]*ArcTan[(Sqrt[2*(-1
 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/4 - (Sqrt[827621 + 1176531*Sqr
t[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/4 - (S
qrt[-827621 + 1176531*Sqrt[3]]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^
2])/8 + (Sqrt[-827621 + 1176531*Sqrt[3]]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3
])]*x + x^2])/8)/4)/16
```

3.119. $\int\frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

3.119.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

3.119. $\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

3.119.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.31

method	result
risch	$5x+\frac{\frac{203}{32}x^7+\frac{889}{64}x^5+\frac{159}{8}x^3+\frac{531}{64}x}{(x^4+2x^2+3)^2}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^4+2\_Z^2+3\right)}\frac{\left(-1322\_R^2-1137\right)\ln\left(x-\_R\right)}{\_R^3+\_R}\right)}{256}$
default	$5x-\frac{-\frac{203}{32}x^7-\frac{889}{64}x^5-\frac{159}{8}x^3-\frac{531}{64}x}{(x^4+2x^2+3)^2}-\frac{\left(943\sqrt{-2+2\sqrt{3}}\,\sqrt{3}+185\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{1024}-\frac{\left(1516\sqrt{3}+\frac{\left(943\sqrt{\right.}}{}\right.}{}$

input
```
int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)
```

output
```
5*x+(203/32*x^7+889/64*x^5+159/8*x^3+531/64*x)/(x^4+2*x^2+3)^2+1/256*sum((
-1322*_R^2-1137)/(_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^2+3))
```

3.119.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.16

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$$
$$=\frac{2560\,x^9+13488\,x^7+32712\,x^5+40896\,x^3-\sqrt{2}(x^8+4\,x^6+10\,x^4+12\,x^2+9)\sqrt{1316761i\,\sqrt{2}-827621}\,\mathrm{l}}{}$$

input
```
integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

output
```
1/512*(2560*x^9 + 13488*x^7 + 32712*x^5 + 40896*x^3 - sqrt(2)*(x^8 + 4*x^6
 + 10*x^4 + 12*x^2 + 9)*sqrt(1316761*I*sqrt(2) - 827621)*log((379*sqrt(2)
+ 943*I)*sqrt(1316761*I*sqrt(2) - 827621) + 1176531*x) + sqrt(2)*(x^8 + 4*
x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(1316761*I*sqrt(2) - 827621)*log(-(379*sqrt
(2) + 943*I)*sqrt(1316761*I*sqrt(2) - 827621) + 1176531*x) - sqrt(2)*(x^8
+ 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-1316761*I*sqrt(2) - 827621)*log((379*
sqrt(2) - 943*I)*sqrt(-1316761*I*sqrt(2) - 827621) + 1176531*x) + sqrt(2)*
(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-1316761*I*sqrt(2) - 827621)*log(
-(379*sqrt(2) - 943*I)*sqrt(-1316761*I*sqrt(2) - 827621) + 1176531*x) + 27
288*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

3.119.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.30

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = 5x + \frac{406x^7+889x^5+1272x^3+531x}{64x^8+256x^6+640x^4+768x^2+576}$$
$$+\mathrm{RootSum}\left(17179869184t^4+216955879424t^2+4152675581883, \left(t \mapsto t\log\left(-\frac{31641829376t^3}{1549210136091} - \frac{4553}{1549}\right.\right.\right.$$

input
```
integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

output
```
5*x + (406*x**7 + 889*x**5 + 1272*x**3 + 531*x)/(64*x**8 + 256*x**6 + 640*
x**4 + 768*x**2 + 576) + RootSum(17179869184*_t**4 + 216955879424*_t**2 +
4152675581883, Lambda(_t, _t*log(-31641829376*_t**3/1549210136091 - 455309
168896*_t/1549210136091 + x)))
```

3.119.7 Maxima [F]

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \int \frac{(5\,x^6+3\,x^4+x^2+4)x^6}{(x^4+2\,x^2+3)^3}\,dx$$

input
```
integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

output
```
5*x + 1/64*(406*x^7 + 889*x^5 + 1272*x^3 + 531*x)/(x^8 + 4*x^6 + 10*x^4 +
12*x^2 + 9) - 1/64*integrate((1322*x^2 + 1137)/(x^4 + 2*x^2 + 3), x)
```

3.119.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(180) = 360$.

Time = 0.72 (sec) , antiderivative size = 580, normalized size of antiderivative = 2.47

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$$

$$= \frac{1}{82944}\sqrt{2}\left(661\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}} + 11898\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right) - 11898\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\cdots}\right.$$

$$+\frac{1}{82944}\sqrt{2}\left(661\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}} + 11898\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right) - 11898\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-\cdots}\right.$$

$$+\frac{1}{165888}\sqrt{2}\left(11898\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18} - 661\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}} + 661\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\cdots\right.\right.$$

$$\left. + 2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)$$

$$-\frac{1}{165888}\sqrt{2}\left(11898\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18} - 661\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}} + 661\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\cdots\right.\right.$$

$$\left. - 2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right) + 5\,x + \frac{406\,x^7+889\,x^5+1272\,x^3+531\,x}{64\left(x^4+2\,x^2+3\right)^2}$$

input
```
integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

output
```
1/82944*sqrt(2)*(661*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 11898*3^(3/4
)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 11898*3^(3/4)*(sqrt(3) + 3)
*sqrt(-6*sqrt(3) + 18) + 661*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 20466*3^(1/
4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 20466*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arc
tan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) +
1/2)) + 1/82944*sqrt(2)*(661*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 1189
8*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 11898*3^(3/4)*(sqrt
(3) + 3)*sqrt(-6*sqrt(3) + 18) + 661*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 204
66*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 20466*3^(1/4)*sqrt(-6*sqrt(3) +
18))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sq
rt(3) + 1/2)) + 1/165888*sqrt(2)*(11898*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt
(-6*sqrt(3) + 18) - 661*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 661*3^(3
/4)*(6*sqrt(3) + 18)^(3/2) + 11898*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) -
 3) - 20466*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 20466*3^(1/4)*sqrt(6*s
qrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) -
1/165888*sqrt(2)*(11898*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18
) - 661*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 661*3^(3/4)*(6*sqrt(3) +
 18)^(3/2) + 11898*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 20466*3^(1
/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 20466*3^(1/4)*sqrt(6*sqrt(3) + 18))*lo
g(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 5*x + 1/64*(4...
```

3.119.9 Mupad [B] (verification not implemented)

Time = 8.85 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.75

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = 5\,x+\frac{\frac{203\,x^7}{32}+\frac{889\,x^5}{64}+\frac{159\,x^3}{8}+\frac{531\,x}{64}}{x^8+4\,x^6+10\,x^4+12\,x^2+9}$$
$$+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-1655242-\sqrt{2}\,2633522\mathrm{i}}\,1316761\mathrm{i}}{131072\,\left(-\frac{3725116869}{131072}+\frac{\sqrt{2}\,14971157257\mathrm{i}}{131072}\right)}+\frac{1316761\,\sqrt{2}\,x\,\sqrt{-1655242-\sqrt{2}\,2633522\mathrm{i}}}{262144\,\left(-\frac{3725116869}{131072}+\frac{\sqrt{2}\,14971157257\mathrm{i}}{131072}\right)}\right)\,\sqrt{-1655242-\sqrt{2}\,2633522\mathrm{i}}\,1\mathrm{i}}{256}$$
$$-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-1655242+\sqrt{2}\,2633522\mathrm{i}}\,1316761\mathrm{i}}{131072\,\left(\frac{3725116869}{131072}+\frac{\sqrt{2}\,14971157257\mathrm{i}}{131072}\right)}-\frac{1316761\,\sqrt{2}\,x\,\sqrt{-1655242+\sqrt{2}\,2633522\mathrm{i}}}{262144\,\left(\frac{3725116869}{131072}+\frac{\sqrt{2}\,14971157257\mathrm{i}}{131072}\right)}\right)\,\sqrt{-1655242+\sqrt{2}\,2633522\mathrm{i}}\,1\mathrm{i}}{256}$$

input
```
int((x^6*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^3,x)
```

output
```
5*x + (atan((x*(- 2^(1/2)*2633522i - 1655242)^(1/2)*1316761i)/(131072*((2^
(1/2)*1497157257i)/131072 - 3725116869/131072)) + (1316761*2^(1/2)*x*(- 2^
(1/2)*2633522i - 1655242)^(1/2))/(262144*((2^(1/2)*1497157257i)/131072 - 3
725116869/131072)))*(- 2^(1/2)*2633522i - 1655242)^(1/2)*1i)/256 - (atan((
x*(2^(1/2)*2633522i - 1655242)^(1/2)*1316761i)/(131072*((2^(1/2)*149715725
7i)/131072 + 3725116869/131072)) - (1316761*2^(1/2)*x*(2^(1/2)*2633522i -
1655242)^(1/2))/(262144*((2^(1/2)*1497157257i)/131072 + 3725116869/131072)
))*(2^(1/2)*2633522i - 1655242)^(1/2)*1i)/256 + ((531*x)/64 + (159*x^3)/8
+ (889*x^5)/64 + (203*x^7)/32)/(12*x^2 + 10*x^4 + 4*x^6 + x^8 + 9)
```

3.119. $\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

3.120 $\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

3.120.1 Optimal result

Integrand size = 31, antiderivative size = 238

$$\begin{aligned}
&\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx \\
&= -\frac{25x(3+x^2)}{16(3+2x^2+x^4)^2}+\frac{x(238-59x^2)}{64(3+2x^2+x^4)} \\
&\quad -\frac{1}{256}\sqrt{3\left(-48835+32827\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad +\frac{1}{256}\sqrt{3\left(-48835+32827\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad +\frac{1}{512}\sqrt{3\left(48835+32827\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) \\
&\quad -\frac{1}{512}\sqrt{3\left(48835+32827\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)
\end{aligned}$$

output
```
-25/16*x*(x^2+3)/(x^4+2*x^2+3)^2+1/64*x*(-59*x^2+238)/(x^4+2*x^2+3)-1/256*
arctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-146505+98481*3^(
1/2))^(1/2)+1/256*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(
-146505+98481*3^(1/2))^(1/2)+1/512*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*
(146505+98481*3^(1/2))^(1/2)-1/512*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*
(146505+98481*3^(1/2))^(1/2)
```

3.120.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.54

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \frac{1}{256}\left(\frac{4x(414+199x^2+120x^4-59x^6)}{(3+2x^2+x^4)^2} + \frac{3\left(174+133i\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}} + \frac{3\left(174-133i\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

input
```
Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

output
```
((4*x*(414 + 199*x^2 + 120*x^4 - 59*x^6))/(3 + 2*x^2 + x^4)^2 + (3*(174 +
(133*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + (3*(
174 - (133*I)*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])
/256
```

3.120.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.23, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {2197, 27, 2206, 27, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(5x^6+3x^4+x^2+4\right)}{\left(x^4+2x^2+3\right)^3}dx$$

$$\Big\downarrow \text{2197}$$

$$\frac{1}{96}\int \frac{6\left(80x^6-112x^4-125x^2+75\right)}{\left(x^4+2x^2+3\right)^2}dx-\frac{25x\left(x^2+3\right)}{16\left(x^4+2x^2+3\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\int \frac{80x^6-112x^4-125x^2+75}{(x^4+2x^2+3)^2}dx-\frac{25x(x^2+3)}{16(x^4+2x^2+3)^2}$$

$$\Big\downarrow \ 2206$$

$$\frac{1}{16}\left(\frac{1}{48}\int -\frac{36(46-87x^2)}{x^4+2x^2+3}dx+\frac{x(238-59x^2)}{4(x^4+2x^2+3)}\right)-\frac{25x(x^2+3)}{16(x^4+2x^2+3)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{16}\left(\frac{x(238-59x^2)}{4(x^4+2x^2+3)}-\frac{3}{4}\int \frac{46-87x^2}{x^4+2x^2+3}dx\right)-\frac{25x(x^2+3)}{16(x^4+2x^2+3)^2}$$

$$\Big\downarrow \ 1483$$

$$\frac{1}{16}\left(\frac{x(238-59x^2)}{4(x^4+2x^2+3)}-\frac{3}{4}\left(\frac{\int \frac{46\sqrt{2\left(-1+\sqrt{3}\right)}-\left(46+87\sqrt{3}\right)x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx}{2\sqrt{6\left(\sqrt{3}-1\right)}}+\frac{\int \frac{\left(46+87\sqrt{3}\right)x+46\sqrt{2\left(-1+\sqrt{3}\right)}}{x^2+\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx}{2\sqrt{6\left(\sqrt{3}-1\right)}}\right)\right)-\frac{25x(x^2+3)}{16(x^4+2x^2+3)^2}$$

$$\Big\downarrow \ 1142$$

$$\frac{1}{16}\left(\frac{x(238-59x^2)}{4(x^4+2x^2+3)}-\frac{3}{4}\left(\frac{-\frac{1}{2}\sqrt{65654\sqrt{3}-97670}\int \frac{1}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx-\frac{1}{2}\left(46+87\sqrt{3}\right)\int -\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x}}{2\sqrt{6\left(\sqrt{3}-1\right)}}\right.\right.$$

$$\frac{25x(x^2+3)}{16(x^4+2x^2+3)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{16}\left(\frac{x(238-59x^2)}{4(x^4+2x^2+3)}-\frac{3}{4}\left(\frac{\frac{1}{2}\left(46+87\sqrt{3}\right)\int \frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx-\frac{1}{2}\sqrt{65654\sqrt{3}-97670}\int \frac{1}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}}{2\sqrt{6\left(\sqrt{3}-1\right)}}\right.\right.$$

$$\frac{25x(x^2+3)}{16(x^4+2x^2+3)^2}$$

3.120. $\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

$\Big\downarrow$ 1083

$$\frac{1}{16}\left(\frac{x(238-59x^2)}{4(x^4+2x^2+3)}-\frac{3}{4}\left(\frac{\frac{1}{2}(46+87\sqrt{3})\int\frac{\sqrt{2(-1+\sqrt{3})}-2x}{x^2-\sqrt{2(-1+\sqrt{3})}x+\sqrt{3}}dx+\sqrt{65654\sqrt{3}-97670}\int\frac{1}{-\left(2x-\sqrt{2(-1+\sqrt{3})}\right)^2-}}{2\sqrt{6(\sqrt{3}-1)}}\right.\right.$$

$$\frac{25x(x^2+3)}{16(x^4+2x^2+3)^2}$$

$\Big\downarrow$ 217

$$\frac{1}{16}\left(\frac{x(238-59x^2)}{4(x^4+2x^2+3)}-\frac{3}{4}\left(\frac{\frac{1}{2}(46+87\sqrt{3})\int\frac{\sqrt{2(-1+\sqrt{3})}-2x}{x^2-\sqrt{2(-1+\sqrt{3})}x+\sqrt{3}}dx-\sqrt{\frac{65654\sqrt{3}-97670}{2(1+\sqrt{3})}}\arctan\left(\frac{2x-\sqrt{2(\sqrt{3}-1)}}{\sqrt{2(1+\sqrt{3})}}\right)}{2\sqrt{6(\sqrt{3}-1)}}+\right.\right.$$

$$\frac{25x(x^2+3)}{16(x^4+2x^2+3)^2}$$

$\Big\downarrow$ 1103

$$\frac{1}{16}\left(\frac{x(238-59x^2)}{4(x^4+2x^2+3)}-\frac{3}{4}\left(\frac{-\sqrt{\frac{65654\sqrt{3}-97670}{2(1+\sqrt{3})}}\arctan\left(\frac{2x-\sqrt{2(\sqrt{3}-1)}}{\sqrt{2(1+\sqrt{3})}}\right)-\frac{1}{2}(46+87\sqrt{3})\log\left(x^2-\sqrt{2(\sqrt{3}-1)}x\right.}{2\sqrt{6(\sqrt{3}-1)}}\right.\right.$$

$$\frac{25x(x^2+3)}{16(x^4+2x^2+3)^2}$$

input
```
Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

3.120. $\int\frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

output
```
(-25*x*(3 + x^2))/(16*(3 + 2*x^2 + x^4)^2) + ((x*(238 - 59*x^2))/(4*(3 + 2
*x^2 + x^4)) - (3*((-(Sqrt[(-97670 + 65654*Sqrt[3])/(2*(1 + Sqrt[3]))]*Arc
Tan[(-Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]]) - ((46 + 87*Sq
rt[3])*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/2)/(2*Sqrt[6*(-1 + S
qrt[3])]) + (-(Sqrt[(-97670 + 65654*Sqrt[3])/(2*(1 + Sqrt[3]))]*ArcTan[(Sq
rt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]]) + ((46 + 87*Sqrt[3])*L
og[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/2)/(2*Sqrt[6*(-1 + Sqrt[3])]
)))/4)/16
```

3.120.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

3.120.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.30

method	result
risch	$\frac{-\frac{59}{64}x^7+\frac{15}{8}x^5+\frac{199}{64}x^3+\frac{207}{32}x}{(x^4+2x^2+3)^2}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+2\_Z^2+3)}\frac{(87\_R^2-46)\ln(x-\_R)}{\_R^3+\_R}\right)}{256}$
default	$\frac{-\frac{59}{64}x^7+\frac{15}{8}x^5+\frac{199}{64}x^3+\frac{207}{32}x}{(x^4+2x^2+3)^2}+\frac{\left(307\sqrt{-2+2\sqrt{3}}\sqrt{3}+399\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{1024}+\frac{\left(-184\sqrt{3}+\frac{(307\sqrt{-2+2\sqrt{3}}\sqrt{}\right.}{}$

input
```
int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)
```

output
```
(-59/64*x^7+15/8*x^5+199/64*x^3+207/32*x)/(x^4+2*x^2+3)^2+3/256*sum((87*_R
^2-46)/(_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^2+3))
```

3.120.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.12

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx =$$
$$-\frac{472\,x^7-960\,x^5-1592\,x^3+\sqrt{2}(x^8+4\,x^6+10\,x^4+12\,x^2+9)\sqrt{61773i\,\sqrt{2}+146505}\log\left(\left(46\,\sqrt{2}-30\right.\right.}{}$$

input
```
integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

output
```
-1/512*(472*x^7 - 960*x^5 - 1592*x^3 + sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*
x^2 + 9)*sqrt(61773*I*sqrt(2) + 146505)*log((46*sqrt(2) - 307*I)*sqrt(6177
3*I*sqrt(2) + 146505) + 98481*x) - sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2
+ 9)*sqrt(61773*I*sqrt(2) + 146505)*log(-(46*sqrt(2) - 307*I)*sqrt(61773*I
*sqrt(2) + 146505) + 98481*x) + sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9
)*sqrt(-61773*I*sqrt(2) + 146505)*log((46*sqrt(2) + 307*I)*sqrt(-61773*I*s
qrt(2) + 146505) + 98481*x) - sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*
sqrt(-61773*I*sqrt(2) + 146505)*log(-(46*sqrt(2) + 307*I)*sqrt(-61773*I*sq
rt(2) + 146505) + 98481*x) - 3312*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

3.120.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1198 vs. 2(201) = 402.

Time = 0.70 (sec) , antiderivative size = 1198, normalized size of antiderivative = 5.03

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

output
```
(-59*x**7 + 120*x**5 + 199*x**3 + 414*x)/(64*x**8 + 256*x**6 + 640*x**4 +
768*x**2 + 576) - sqrt(146505/262144 + 98481*sqrt(3)/262144)*log(x**2 + x*
(-307*sqrt(6)*sqrt(48835 + 32827*sqrt(3))*sqrt(1603106545*sqrt(3) + 280884
6506)/675940757 + 10626354*sqrt(3)*sqrt(48835 + 32827*sqrt(3))/675940757 +
 1228*sqrt(48835 + 32827*sqrt(3))/20591) - 941929306825573*sqrt(2)*sqrt(16
03106545*sqrt(3) + 2808846506)/456895906973733049 - 47771215762*sqrt(6)*sq
rt(1603106545*sqrt(3) + 2808846506)/41754888382161 + 9747794966679088235 3/
456895906973733049 + 5200450130596150*sqrt(3)/41754888382161) + sqrt(14650
5/262144 + 98481*sqrt(3)/262144)*log(x**2 + x*(-1228*sqrt(48835 + 32827*sq
rt(3))/20591 - 10626354*sqrt(3)*sqrt(48835 + 32827*sqrt(3))/675940757 + 30
7*sqrt(6)*sqrt(48835 + 32827*sqrt(3))*sqrt(1603106545*sqrt(3) + 2808846506
)/675940757) - 941929306825573*sqrt(2)*sqrt(1603106545*sqrt(3) + 280884650
6)/456895906973733049 - 47771215762*sqrt(6)*sqrt(1603106545*sqrt(3) + 2808
846506)/41754888382161 + 97477949666790882353/456895906973733049 + 5200450
130596150*sqrt(3)/41754888382161) + 2*sqrt(-3*sqrt(2)*sqrt(1603106545*sqrt
(3) + 2808846506)/131072 + 146505/262144 + 295443*sqrt(3)/262144)*atan(135
1881514*sqrt(3)*x/(-1894372*sqrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808
846506) + 48835 + 98481*sqrt(3)) + 307*sqrt(2)*sqrt(1603106545*sqrt(3) + 2
808846506)*sqrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 +
 98481*sqrt(3))) - 40311556*sqrt(3)*sqrt(48835 + 32827*sqrt(3))/(-18943...
```

3.120.7 Maxima [F]

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \int \frac{(5\,x^6+3\,x^4+x^2+4)x^4}{(x^4+2\,x^2+3)^3}\,dx$$

input
```
integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

output
```
-1/64*(59*x^7 - 120*x^5 - 199*x^3 - 414*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2
+ 9) + 3/64*integrate((87*x^2 - 46)/(x^4 + 2*x^2 + 3), x)
```

3.120.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(177) = 354$.

Time = 0.87 (sec) , antiderivative size = 577, normalized size of antiderivative = 2.42

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx =$$

$$-\frac{1}{18432}\sqrt{2}\left(29\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+522\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-522\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}\right.$$

$$-\frac{1}{18432}\sqrt{2}\left(29\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+522\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-522\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}\right.$$

$$-\frac{1}{36864}\sqrt{2}\left(522\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-29\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+29\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+\right.$$

$$\left.+2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)$$

$$+\frac{1}{36864}\sqrt{2}\left(522\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-29\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+29\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+\right.$$

$$\left.-2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)-\frac{59\,x^7-120\,x^5-199\,x^3-414\,x}{64\,(x^4+2\,x^2+3)^2}$$

input
```
integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

output
```
-1/18432*sqrt(2)*(29*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 522*3^(3/4)*
sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 522*3^(3/4)*(sqrt(3) + 3)*sqr
t(-6*sqrt(3) + 18) + 29*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 552*3^(1/4)*sqrt
(2)*sqrt(6*sqrt(3) + 18) - 552*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3
^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1
/18432*sqrt(2)*(29*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 522*3^(3/4)*sq
rt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 522*3^(3/4)*(sqrt(3) + 3)*sqrt(
-6*sqrt(3) + 18) + 29*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 552*3^(1/4)*sqrt(2
)*sqrt(6*sqrt(3) + 18) - 552*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(
3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/3
6864*sqrt(2)*(522*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 29
*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 29*3^(3/4)*(6*sqrt(3) + 18)^(3/
2) + 522*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 552*3^(1/4)*sqrt(2)*
sqrt(-6*sqrt(3) + 18) + 552*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1
/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/36864*sqrt(2)*(522*3^(3/4)*s
qrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 29*3^(3/4)*sqrt(2)*(-6*sqrt(3
) + 18)^(3/2) + 29*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 522*3^(3/4)*sqrt(6*sqr
t(3) + 18)*(sqrt(3) - 3) + 552*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 552
*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1
/2) + sqrt(3)) - 1/64*(59*x^7 - 120*x^5 - 199*x^3 - 414*x)/(x^4 + 2*x^2...
```

3.120.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.73

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{\left(3+2x^2+x^4\right)^3}\,dx = \frac{-\frac{59\,x^7}{64}+\frac{15\,x^5}{8}+\frac{199\,x^3}{64}+\frac{207\,x}{32}}{x^8+4\,x^6+10\,x^4+12\,x^2+9}$$
$$+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{293010-\sqrt{2}\,123546\mathrm{i}}\,61773\mathrm{i}}{131072\,\left(\frac{56892933}{131072}+\frac{\sqrt{2}\,4262337\mathrm{i}}{65536}\right)}+\frac{61773\,\sqrt{2}\,x\,\sqrt{293010-\sqrt{2}\,123546\mathrm{i}}}{262144\,\left(\frac{56892933}{131072}+\frac{\sqrt{2}\,4262337\mathrm{i}}{65536}\right)}\right)\,\sqrt{293010-\sqrt{2}\,123546\mathrm{i}}\,1\mathrm{i}}{256}$$
$$-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{293010+\sqrt{2}\,123546\mathrm{i}}\,61773\mathrm{i}}{131072\,\left(-\frac{56892933}{131072}+\frac{\sqrt{2}\,4262337\mathrm{i}}{65536}\right)}-\frac{61773\,\sqrt{2}\,x\,\sqrt{293010+\sqrt{2}\,123546\mathrm{i}}}{262144\,\left(-\frac{56892933}{131072}+\frac{\sqrt{2}\,4262337\mathrm{i}}{65536}\right)}\right)\,\sqrt{293010+\sqrt{2}\,123546\mathrm{i}}\,1\mathrm{i}}{256}$$

input
```
int((x^4*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^3,x)
```

output
```
((207*x)/32 + (199*x^3)/64 + (15*x^5)/8 - (59*x^7)/64)/(12*x^2 + 10*x^4 +
4*x^6 + x^8 + 9) + (atan((x*(293010 - 2^(1/2)*123546i)^(1/2)*61773i)/(1310
72*((2^(1/2)*4262337i)/65536 + 56892933/131072)) + (61773*2^(1/2)*x*(29301
0 - 2^(1/2)*123546i)^(1/2))/(262144*((2^(1/2)*4262337i)/65536 + 56892933/1
31072)))*(293010 - 2^(1/2)*123546i)^(1/2)*1i)/256 - (atan((x*(2^(1/2)*1235
46i + 293010)^(1/2)*61773i)/(131072*((2^(1/2)*4262337i)/65536 - 56892933/1
31072)) - (61773*2^(1/2)*x*(2^(1/2)*123546i + 293010)^(1/2))/(262144*((2^(
1/2)*4262337i)/65536 - 56892933/131072)))*(2^(1/2)*123546i + 293010)^(1/2)
*1i)/256
```

3.120. $\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

3.121 $\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

3.121.1 Optimal result

Integrand size = 31, antiderivative size = 246

$$\begin{aligned}
&\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx \\
&= \frac{25x(1+x^2)}{16\,(3+2x^2+x^4)^2} - \frac{x(353+88x^2)}{192\,(3+2x^2+x^4)} \\
&\quad - \frac{11}{768}\sqrt{\frac{1}{3}\left(-1825+1089\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad + \frac{11}{768}\sqrt{\frac{1}{3}\left(-1825+1089\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad - \frac{11\sqrt{\frac{1}{3}\left(1825+1089\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)}{1536} \\
&\quad + \frac{11\sqrt{\frac{1}{3}\left(1825+1089\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)}{1536}
\end{aligned}$$

output
```
25/16*x*(x^2+1)/(x^4+2*x^2+3)^2-1/192*x*(88*x^2+353)/(x^4+2*x^2+3)-11/2304
*arctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-5475+3267*3^(1/
2))^(1/2)+11/2304*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(
-5475+3267*3^(1/2))^(1/2)-11/4608*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(
5475+3267*3^(1/2))^(1/2)+11/4608*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(5
475+3267*3^(1/2))^(1/2)
```

3.121.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.54

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \frac{1}{768}\left(-\frac{4x(759+670x^2+529x^4+88x^6)}{(3+2x^2+x^4)^2} - \frac{11i(-16i+31\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}} + \frac{11i(16i+31\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

input
```
Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

output
```
((-4*x*(759 + 670*x^2 + 529*x^4 + 88*x^6))/(3 + 2*x^2 + x^4)^2 - ((11*I)*(
-16*I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + (
(11*I)*(16*I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[
2]])/768
```

3.121.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.18, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {2197, 27, 2206, 27, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(5x^6+3x^4+x^2+4\right)}{\left(x^4+2x^2+3\right)^3}dx$$

$$\Big\downarrow \text{2197}$$

$$\frac{1}{96}\int -\frac{6\left(-80x^4-13x^2+25\right)}{\left(x^4+2x^2+3\right)^2}dx+\frac{25x\left(x^2+1\right)}{16\left(x^4+2x^2+3\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{25x\left(x^2+1\right)}{16\left(x^4+2x^2+3\right)^2}-\frac{1}{16}\int \frac{-80x^4-13x^2+25}{\left(x^4+2x^2+3\right)^2}dx$$

$$\Big\downarrow \text{2206}$$

$$\frac{1}{16}\left(-\frac{1}{48}\int -\frac{44\left(23-8x^2\right)}{x^4+2x^2+3}dx-\frac{x\left(88x^2+353\right)}{12\left(x^4+2x^2+3\right)}\right)+\frac{25x\left(x^2+1\right)}{16\left(x^4+2x^2+3\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\left(\frac{11}{12}\int \frac{23-8x^2}{x^4+2x^2+3}dx-\frac{x\left(88x^2+353\right)}{12\left(x^4+2x^2+3\right)}\right)+\frac{25x\left(x^2+1\right)}{16\left(x^4+2x^2+3\right)^2}$$

$$\Big\downarrow \text{1483}$$

$$\frac{1}{16}\left(\frac{11}{12}\left(\frac{\int \frac{23\sqrt{2\left(-1+\sqrt{3}\right)}-\left(23+8\sqrt{3}\right)x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx}{2\sqrt{6\left(\sqrt{3}-1\right)}}+\frac{\int \frac{\left(23+8\sqrt{3}\right)x+23\sqrt{2\left(-1+\sqrt{3}\right)}}{x^2+\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx}{2\sqrt{6\left(\sqrt{3}-1\right)}}\right)-\frac{x\left(88x^2+353\right)}{12\left(x^4+2x^2+3\right)}\right)+$$
$$\frac{25x\left(x^2+1\right)}{16\left(x^4+2x^2+3\right)^2}$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{16}\left(\frac{11}{12}\left(\frac{\frac{1}{2}\sqrt{2178\sqrt{3}-3650}\int \frac{1}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx-\frac{1}{2}\left(23+8\sqrt{3}\right)\int -\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx}{2\sqrt{6\left(\sqrt{3}-1\right)}}+\frac{\frac{1}{2}\sqrt{2178\sqrt{3}-}\cdots}{\cdots}\right.\right.$$
$$\frac{25x\left(x^2+1\right)}{16\left(x^4+2x^2+3\right)^2}$$

$$\Big\downarrow \text{25}$$

3.121. $\int \frac{x^2\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^3}dx$

$$\frac{1}{16}\left(\frac{11}{12}\left(\frac{\frac{1}{2}\sqrt{2178\sqrt{3}-3650}\int\frac{1}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx+\frac{1}{2}\left(23+8\sqrt{3}\right)\int\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx}{2\sqrt{6\left(\sqrt{3}-1\right)}}+\frac{\frac{1}{2}\sqrt{2178\sqrt{3}-3\cdots}}{\cdots}\right.\right.$$

$$\frac{25x\left(x^2+1\right)}{16\left(x^4+2x^2+3\right)^2}$$

$\Big\downarrow$ 1083

$$\frac{1}{16}\left(\frac{11}{12}\left(\frac{\frac{1}{2}\left(23+8\sqrt{3}\right)\int\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx-\sqrt{2178\sqrt{3}-3650}\int\frac{1}{-\left(2x-\sqrt{2\left(-1+\sqrt{3}\right)}\right)^2-2\left(1+\sqrt{3}\right)}d\left(2x-\sqrt{2\left(\cdots\right.}\right.}{2\sqrt{6\left(\sqrt{3}-1\right)}}\right.\right.$$

$$\frac{25x\left(x^2+1\right)}{16\left(x^4+2x^2+3\right)^2}$$

$\Big\downarrow$ 217

$$\frac{1}{16}\left(\frac{11}{12}\left(\frac{\frac{1}{2}\left(23+8\sqrt{3}\right)\int\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx+\sqrt{\frac{2178\sqrt{3}-3650}{2\left(1+\sqrt{3}\right)}}\arctan\left(\frac{2x-\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{2\sqrt{6\left(\sqrt{3}-1\right)}}+\frac{\frac{1}{2}\left(23+8\sqrt{3}\right)\int\frac{2x\cdots}{x^2+\sqrt{\cdots}}}{\cdots}\right.\right.$$

$$\frac{25x\left(x^2+1\right)}{16\left(x^4+2x^2+3\right)^2}$$

$\Big\downarrow$ 1103

$$\frac{1}{16}\left(\frac{11}{12}\left(\frac{\sqrt{\frac{2178\sqrt{3}-3650}{2\left(1+\sqrt{3}\right)}}\arctan\left(\frac{2x-\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{2}\left(23+8\sqrt{3}\right)\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{2\sqrt{6\left(\sqrt{3}-1\right)}}+\frac{\sqrt{\frac{2178\sqrt{3}-36\cdots}{2\left(1+\sqrt{3}\right)}}\cdots}{\cdots}\right.\right.$$

$$\frac{25x\left(x^2+1\right)}{16\left(x^4+2x^2+3\right)^2}$$

3.121. $\int\frac{x^2\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^3}dx$

input
```
Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

output
```
(25*x*(1 + x^2))/(16*(3 + 2*x^2 + x^4)^2) + (-1/12*(x*(353 + 88*x^2))/(3 +
 2*x^2 + x^4) + (11*((Sqrt[(-3650 + 2178*Sqrt[3])/(2*(1 + Sqrt[3]))]*ArcTa
n[(-Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]] - ((23 + 8*Sqrt[3
])*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/2)/(2*Sqrt[6*(-1 + Sqrt[
3])]) + (Sqrt[(-3650 + 2178*Sqrt[3])/(2*(1 + Sqrt[3]))]*ArcTan[(Sqrt[2*(-1
 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]] + ((23 + 8*Sqrt[3])*Log[Sqrt[3]
 + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/2)/(2*Sqrt[6*(-1 + Sqrt[3])])))/12)/16
```

3.121.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.121. $\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

3.121.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.29

method	result
risch	$\frac{-\frac{11}{24}x^7-\frac{529}{192}x^5-\frac{335}{96}x^3-\frac{253}{64}x}{(x^4+2x^2+3)^2}+\frac{11\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+2\_Z^2+3)}\frac{(-8\_R^2+23)\ln(x-\_R)}{\_R^3+\_R}\right)}{768}$
default	$\frac{-\frac{11}{24}x^7-\frac{529}{192}x^5-\frac{335}{96}x^3-\frac{253}{64}x}{(x^4+2x^2+3)^2}+\frac{11\left(-47\sqrt{-2+2\sqrt{3}}\sqrt{3}-93\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{9216}+\frac{11\left(92\sqrt{3}+\frac{(-47\sqrt{-2+2\sqrt{}}}{}\right.}{}$

input
```
int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)
```

output
```
(-11/24*x^7-529/192*x^5-335/96*x^3-253/64*x)/(x^4+2*x^2+3)^2+11/768*sum((-
8*_R^2+23)/(_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^2+3))
```

3.121.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.13

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx =$$
$$-\frac{2112\,x^7+12696\,x^5+16080\,x^3-\sqrt{6}(x^8+4\,x^6+10\,x^4+12\,x^2+9)\sqrt{40777i\,\sqrt{2}+220825}\log\left(\sqrt{6}\sqrt{4}\right.}{}$$

input
```
integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

output
```
-1/4608*(2112*x^7 + 12696*x^5 + 16080*x^3 - sqrt(6)*(x^8 + 4*x^6 + 10*x^4
+ 12*x^2 + 9)*sqrt(40777*I*sqrt(2) + 220825)*log(sqrt(6)*sqrt(40777*I*sqrt
(2) + 220825)*(47*I*sqrt(2) + 46) + 71874*x) + sqrt(6)*(x^8 + 4*x^6 + 10*x
^4 + 12*x^2 + 9)*sqrt(40777*I*sqrt(2) + 220825)*log(sqrt(6)*sqrt(40777*I*s
qrt(2) + 220825)*(-47*I*sqrt(2) - 46) + 71874*x) + sqrt(6)*(x^8 + 4*x^6 +
10*x^4 + 12*x^2 + 9)*sqrt(-40777*I*sqrt(2) + 220825)*log(sqrt(6)*(47*I*sqr
t(2) - 46)*sqrt(-40777*I*sqrt(2) + 220825) + 71874*x) - sqrt(6)*(x^8 + 4*x
^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-40777*I*sqrt(2) + 220825)*log(sqrt(6)*(-47
*I*sqrt(2) + 46)*sqrt(-40777*I*sqrt(2) + 220825) + 71874*x) + 18216*x)/(x^
8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

3.121.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1200 vs. $2(207) = 414$.

Time = 0.72 (sec) , antiderivative size = 1200, normalized size of antiderivative = 4.88

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

output
```
(-88*x**7 - 529*x**5 - 670*x**3 - 759*x)/(192*x**8 + 768*x**6 + 1920*x**4
+ 2304*x**2 + 1728) - sqrt(220825/7077888 + 14641*sqrt(3)/786432)*log(x**2
 + x*(-47*sqrt(6)*sqrt(1825 + 1089*sqrt(3))*sqrt(1987425*sqrt(3) + 3444194
)/366993 + 52016*sqrt(3)*sqrt(1825 + 1089*sqrt(3))/366993 + 188*sqrt(1825
+ 1089*sqrt(3))/337) - 24765218375*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194)
/134683862049 - 38128468*sqrt(6)*sqrt(1987425*sqrt(3) + 3444194)/371029923
 + 90413874433403/134683862049 + 144251139148*sqrt(3)/371029923) + sqrt(22
0825/7077888 + 14641*sqrt(3)/786432)*log(x**2 + x*(-188*sqrt(1825 + 1089*s
qrt(3))/337 - 52016*sqrt(3)*sqrt(1825 + 1089*sqrt(3))/366993 + 47*sqrt(6)*
sqrt(1825 + 1089*sqrt(3))*sqrt(1987425*sqrt(3) + 3444194)/366993) - 247652
18375*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194)/134683862049 - 38128468*sqrt
(6)*sqrt(1987425*sqrt(3) + 3444194)/371029923 + 90413874433403/13468386204
9 + 144251139148*sqrt(3)/371029923) + 2*sqrt(-121*sqrt(2)*sqrt(1987425*sqr
t(3) + 3444194)/3538944 + 220825/7077888 + 14641*sqrt(3)/262144)*atan(7339
86*sqrt(3)*x/(15502*sqrt(-2*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194) + 1825
 + 3267*sqrt(3)) + 47*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194)*sqrt(-2*sqrt
(2)*sqrt(1987425*sqrt(3) + 3444194) + 1825 + 3267*sqrt(3))) - 204732*sqrt(
3)*sqrt(1825 + 1089*sqrt(3))/(15502*sqrt(-2*sqrt(2)*sqrt(1987425*sqrt(3) +
 3444194) + 1825 + 3267*sqrt(3)) + 47*sqrt(2)*sqrt(1987425*sqrt(3) + 34441
94)*sqrt(-2*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194) + 1825 + 3267*sqrt(...
```

3.121.7 Maxima [F]

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \int \frac{(5\,x^6+3\,x^4+x^2+4)x^2}{(x^4+2\,x^2+3)^3}\,dx$$

input
```
integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

output
```
-1/192*(88*x^7 + 529*x^5 + 670*x^3 + 759*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2
 + 9) - 11/192*integrate((8*x^2 - 23)/(x^4 + 2*x^2 + 3), x)
```

3.121.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(177) = 354$.

Time = 0.75 (sec) , antiderivative size = 577, normalized size of antiderivative = 2.35

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$$

$$= \frac{11}{124416}\sqrt{2}\left(2\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+36\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-36\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}\right.$$

$$+\frac{11}{124416}\sqrt{2}\left(2\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+36\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-36\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+}\right.$$

$$+\frac{11}{248832}\sqrt{2}\left(36\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-2\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+2\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+36\right.$$

$$\left.+2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)$$

$$-\frac{11}{248832}\sqrt{2}\left(36\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-2\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+2\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+36\right.$$

$$\left.-2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)-\frac{88\,x^7+529\,x^5+670\,x^3+759\,x}{192\,(x^4+2\,x^2+3)^2}$$

input
```
integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

output
```
11/124416*sqrt(2)*(2*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 36*3^(3/4)*s
qrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 36*3^(3/4)*(sqrt(3) + 3)*sqrt(
-6*sqrt(3) + 18) + 2*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 207*3^(1/4)*sqrt(2)
*sqrt(6*sqrt(3) + 18) - 207*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3
/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 11/1
24416*sqrt(2)*(2*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 36*3^(3/4)*sqrt(
2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 36*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*s
qrt(3) + 18) + 2*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 207*3^(1/4)*sqrt(2)*sqr
t(6*sqrt(3) + 18) - 207*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*
(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 11/24883
2*sqrt(2)*(36*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 2*3^(3
/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 2*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 3
6*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 207*3^(1/4)*sqrt(2)*sqrt(-6
*sqrt(3) + 18) + 207*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*s
qrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 11/248832*sqrt(2)*(36*3^(3/4)*sqrt(2)
*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 2*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)
^(3/2) + 2*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 36*3^(3/4)*sqrt(6*sqrt(3) + 18
)*(sqrt(3) - 3) + 207*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 207*3^(1/4)*
sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqr
t(3)) - 1/192*(88*x^7 + 529*x^5 + 670*x^3 + 759*x)/(x^4 + 2*x^2 + 3)^2
```

3.121.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.71

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = -\frac{\frac{11\,x^7}{24}+\frac{529\,x^5}{192}+\frac{335\,x^3}{96}+\frac{253\,x}{64}}{x^8+4\,x^6+10\,x^4+12\,x^2+9} + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{10950-\sqrt{2}\,2022\mathrm{i}}\,448547\mathrm{i}}{31850496\,\left(-\frac{21081709}{10616832}+\frac{\sqrt{2}\,10316581\mathrm{i}}{10616832}\right)}-\frac{448547\,\sqrt{2}\,x\,\sqrt{10950-\sqrt{2}\,2022\mathrm{i}}}{63700992\,\left(-\frac{21081709}{10616832}+\frac{\sqrt{2}\,10316581\mathrm{i}}{10616832}\right)}\right)\,\sqrt{10950-\sqrt{2}\,2022\mathrm{i}}\,11\mathrm{i}}{2304} - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{10950+\sqrt{2}\,2022\mathrm{i}}\,448547\mathrm{i}}{31850496\,\left(\frac{21081709}{10616832}+\frac{\sqrt{2}\,10316581\mathrm{i}}{10616832}\right)}+\frac{448547\,\sqrt{2}\,x\,\sqrt{10950+\sqrt{2}\,2022\mathrm{i}}}{63700992\,\left(\frac{21081709}{10616832}+\frac{\sqrt{2}\,10316581\mathrm{i}}{10616832}\right)}\right)\,\sqrt{10950+\sqrt{2}\,2022\mathrm{i}}\,11\mathrm{i}}{2304}$$

input
```
int((x^2*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^3,x)
```

output
```
(atan((x*(10950 - 2^(1/2)*2022i)^(1/2)*448547i)/(31850496*((2^(1/2)*103165
81i)/10616832 - 21081709/10616832)) - (448547*2^(1/2)*x*(10950 - 2^(1/2)*2
022i)^(1/2))/(63700992*((2^(1/2)*10316581i)/10616832 - 21081709/10616832))
)*(10950 - 2^(1/2)*2022i)^(1/2)*11i)/2304 - ((253*x)/64 + (335*x^3)/96 + (
529*x^5)/192 + (11*x^7)/24)/(12*x^2 + 10*x^4 + 4*x^6 + x^8 + 9) - (atan((x
*(2^(1/2)*2022i + 10950)^(1/2)*448547i)/(31850496*((2^(1/2)*10316581i)/106
16832 + 21081709/10616832)) + (448547*2^(1/2)*x*(2^(1/2)*2022i + 10950)^(1
/2))/(63700992*((2^(1/2)*10316581i)/10616832 + 21081709/10616832)))*(2^(1/
2)*2022i + 10950)^(1/2)*11i)/2304
```

3.121. $\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

3.122 $\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^3}\,dx$

3.122.1 Optimal result

Integrand size = 28, antiderivative size = 248

$$\begin{aligned}\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^3}\,dx &= \frac{25x(1-x^2)}{48\,(3+2x^2+x^4)^2}+\frac{x(64+51x^2)}{192\,(3+2x^2+x^4)}\\
&-\frac{1}{256}\sqrt{\frac{1}{3}\left(-1291+1019\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)\\
&+\frac{1}{256}\sqrt{\frac{1}{3}\left(-1291+1019\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)\\
&+\frac{1}{512}\sqrt{\frac{1}{3}\left(1291+1019\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)\\
&-\frac{1}{512}\sqrt{\frac{1}{3}\left(1291+1019\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)\end{aligned}$$

output
```
25/48*x*(-x^2+1)/(x^4+2*x^2+3)^2+1/192*x*(51*x^2+64)/(x^4+2*x^2+3)-1/768*a
rctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-3873+3057*3^(1/2)
)^(1/2)+1/768*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-387
3+3057*3^(1/2))^(1/2)+1/1536*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(3873+
3057*3^(1/2))^(1/2)-1/1536*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(3873+30
57*3^(1/2))^(1/2)
```

3.122.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.52

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^3}\,dx = \frac{1}{768}\left(\frac{4x(292+181x^2+166x^4+51x^6)}{(3+2x^2+x^4)^2} + \frac{3(34+21i\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}} + \frac{3(34-21i\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^3,x]
```

output
```
((4*x*(292 + 181*x^2 + 166*x^4 + 51*x^6))/(3 + 2*x^2 + x^4)^2 + (3*(34 + (
21*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + (3*(34
 - (21*I)*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/768
```

3.122.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {2206, 27, 1492, 27, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{(x^4+2x^2+3)^3}\,dx$$

$$\Big\downarrow \text{2206}$$

$$\frac{1}{96}\int \frac{2(115x^2+39)}{(x^4+2x^2+3)^2}dx + \frac{25x(1-x^2)}{48(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{48}\int \frac{115x^2+39}{(x^4+2x^2+3)^2}dx + \frac{25x(1-x^2)}{48(x^4+2x^2+3)^2}$$

$\downarrow$ 1492

$$\frac{1}{48}\left(\frac{1}{48}\int -\frac{36\left(4-17x^2\right)}{x^4+2x^2+3}dx+\frac{x\left(51x^2+64\right)}{4\left(x^4+2x^2+3\right)}\right)+\frac{25x\left(1-x^2\right)}{48\left(x^4+2x^2+3\right)^2}$$

$\downarrow$ 27

$$\frac{1}{48}\left(\frac{x\left(51x^2+64\right)}{4\left(x^4+2x^2+3\right)}-\frac{3}{4}\int \frac{4-17x^2}{x^4+2x^2+3}dx\right)+\frac{25x\left(1-x^2\right)}{48\left(x^4+2x^2+3\right)^2}$$

$\downarrow$ 1483

$$\frac{1}{48}\left(\frac{x\left(51x^2+64\right)}{4\left(x^4+2x^2+3\right)}-\frac{3}{4}\left(\frac{\int \frac{4\sqrt{2\left(-1+\sqrt{3}\right)}-\left(4+17\sqrt{3}\right)x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx}{2\sqrt{6\left(\sqrt{3}-1\right)}}+\frac{\int \frac{\left(4+17\sqrt{3}\right)x+4\sqrt{2\left(-1+\sqrt{3}\right)}}{x^2+\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx}{2\sqrt{6\left(\sqrt{3}-1\right)}}\right)\right)+$$
$$\frac{25x\left(1-x^2\right)}{48\left(x^4+2x^2+3\right)^2}$$

$\downarrow$ 1142

$$\frac{1}{48}\left(\frac{x\left(51x^2+64\right)}{4\left(x^4+2x^2+3\right)}-\frac{3}{4}\left(\frac{-\frac{1}{2}\sqrt{2038\sqrt{3}-2582}\int \frac{1}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx-\frac{1}{2}\left(4+17\sqrt{3}\right)\int -\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}}{2\sqrt{6\left(\sqrt{3}-1\right)}}\right.\right.$$
$$\frac{25x\left(1-x^2\right)}{48\left(x^4+2x^2+3\right)^2}$$

$\downarrow$ 25

$$\frac{1}{48}\left(\frac{x\left(51x^2+64\right)}{4\left(x^4+2x^2+3\right)}-\frac{3}{4}\left(\frac{\frac{1}{2}\left(4+17\sqrt{3}\right)\int \frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx-\frac{1}{2}\sqrt{2038\sqrt{3}-2582}\int \frac{1}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx}{2\sqrt{6\left(\sqrt{3}-1\right)}}\right.\right.$$
$$\frac{25x\left(1-x^2\right)}{48\left(x^4+2x^2+3\right)^2}$$

$\downarrow$ 1083

3.122. $\int \frac{4+x^2+3x^4+5x^6}{\left(3+2x^2+x^4\right)^3}\,dx$

$$\frac{1}{48}\left(\frac{x(51x^2+64)}{4(x^4+2x^2+3)}-\frac{3}{4}\left(\frac{\frac{1}{2}(4+17\sqrt{3})\int\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx+\sqrt{2038\sqrt{3}-2582}\int\frac{1}{-\left(2x-\sqrt{2\left(-1+\sqrt{3}\right)}\right)^2-2(1}}{2\sqrt{6\left(\sqrt{3}-1\right)}}\right.\right.$$

$$\frac{25x(1-x^2)}{48(x^4+2x^2+3)^2}$$

$\Big\downarrow$ 217

$$\frac{1}{48}\left(\frac{x(51x^2+64)}{4(x^4+2x^2+3)}-\frac{3}{4}\left(\frac{\frac{1}{2}(4+17\sqrt{3})\int\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx-\sqrt{\frac{2038\sqrt{3}-2582}{2\left(1+\sqrt{3}\right)}}\arctan\left(\frac{2x-\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{2\sqrt{6\left(\sqrt{3}-1\right)}}+\frac{\frac{1}{2}(}{}\right.\right.$$

$$\frac{25x(1-x^2)}{48(x^4+2x^2+3)^2}$$

$\Big\downarrow$ 1103

$$\frac{1}{48}\left(\frac{x(51x^2+64)}{4(x^4+2x^2+3)}-\frac{3}{4}\left(\frac{-\sqrt{\frac{2038\sqrt{3}-2582}{2\left(1+\sqrt{3}\right)}}\arctan\left(\frac{2x-\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{2}(4+17\sqrt{3})\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\right.}{2\sqrt{6\left(\sqrt{3}-1\right)}}\right.\right.$$

$$\frac{25x(1-x^2)}{48(x^4+2x^2+3)^2}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^3,x]
```

output
```
(25*x*(1 - x^2))/(48*(3 + 2*x^2 + x^4)^2) + ((x*(64 + 51*x^2))/(4*(3 + 2*x
^2 + x^4)) - (3*((-(Sqrt[(-2582 + 2038*Sqrt[3])/(2*(1 + Sqrt[3]))]*ArcTan[
(-Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]]) - ((4 + 17*Sqrt[3]
)*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/2)/(2*Sqrt[6*(-1 + Sqrt[3
])]) + (-(Sqrt[(-2582 + 2038*Sqrt[3])/(2*(1 + Sqrt[3]))]*ArcTan[(Sqrt[2*(-
1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]]) + ((4 + 17*Sqrt[3])*Log[Sqrt[
3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/2)/(2*Sqrt[6*(-1 + Sqrt[3])])))/4)/4
8
```

3.122. $\int\frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^3}\,dx$

3.122.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

3.122. $\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^3}\,dx$

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

3.122.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.29

method	result
risch	$\frac{\frac{17}{64}x^7+\frac{83}{96}x^5+\frac{181}{192}x^3+\frac{73}{48}x}{(x^4+2x^2+3)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+2\_Z^2+3)}\frac{(17\_R^2-4)\ln(x-\_R)}{\_R^3+\_R}\right)}{256}$
default	$\frac{\frac{17}{64}x^7+\frac{83}{96}x^5+\frac{181}{192}x^3+\frac{73}{48}x}{(x^4+2x^2+3)^2}+\frac{\left(55\sqrt{-2+2\sqrt{3}}\sqrt{3}+63\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{3072}+\frac{\left(-16\sqrt{3}+\frac{(55\sqrt{-2+2\sqrt{3}}\sqrt{3}+63\sqrt{-}}{2}\right.}{7}$

input
```
int((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)
```

output
```
(17/64*x^7+83/96*x^5+181/192*x^3+73/48*x)/(x^4+2*x^2+3)^2+1/256*sum((17*_R
^2-4)/(_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^2+3))
```

3.122.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.12

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^3}\,dx$$
$$=\frac{408\,x^7+1328\,x^5+1448\,x^3+\sqrt{6}(x^8+4\,x^6+10\,x^4+12\,x^2+9)\sqrt{851i\,\sqrt{2}+1291}\log\left(\sqrt{6}\sqrt{851i\,\sqrt{2}+1\right.}{}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

output
```
1/1536*(408*x^7 + 1328*x^5 + 1448*x^3 + sqrt(6)*(x^8 + 4*x^6 + 10*x^4 + 12
*x^2 + 9)*sqrt(851*I*sqrt(2) + 1291)*log(sqrt(6)*sqrt(851*I*sqrt(2) + 1291
)*(55*I*sqrt(2) - 8) + 6114*x) - sqrt(6)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 +
9)*sqrt(851*I*sqrt(2) + 1291)*log(sqrt(6)*sqrt(851*I*sqrt(2) + 1291)*(-55*
I*sqrt(2) + 8) + 6114*x) - sqrt(6)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqr
t(-851*I*sqrt(2) + 1291)*log(sqrt(6)*(55*I*sqrt(2) + 8)*sqrt(-851*I*sqrt(2
) + 1291) + 6114*x) + sqrt(6)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-85
1*I*sqrt(2) + 1291)*log(sqrt(6)*(-55*I*sqrt(2) - 8)*sqrt(-851*I*sqrt(2) +
1291) + 6114*x) + 2336*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

3.122.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1195 vs. $2(201) = 402$.

Time = 0.70 (sec) , antiderivative size = 1195, normalized size of antiderivative = 4.82

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

output
```
(51*x**7 + 166*x**5 + 181*x**3 + 292*x)/(192*x**8 + 768*x**6 + 1920*x**4 +
 2304*x**2 + 1728) - sqrt(1291/786432 + 1019*sqrt(3)/786432)*log(x**2 + x*
(-55*sqrt(6)*sqrt(1291 + 1019*sqrt(3))*sqrt(1315529*sqrt(3) + 2390882)/867
169 + 49606*sqrt(3)*sqrt(1291 + 1019*sqrt(3))/867169 + 220*sqrt(1291 + 101
9*sqrt(3))/851) - 26628761029*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882)/7519
82074561 - 40176070*sqrt(6)*sqrt(1315529*sqrt(3) + 2390882)/2213882457 + 7
6094994709709/751982074561 + 133967471914*sqrt(3)/2213882457) + sqrt(1291/
786432 + 1019*sqrt(3)/786432)*log(x**2 + x*(-220*sqrt(1291 + 1019*sqrt(3))
/851 - 49606*sqrt(3)*sqrt(1291 + 1019*sqrt(3))/867169 + 55*sqrt(6)*sqrt(12
91 + 1019*sqrt(3))*sqrt(1315529*sqrt(3) + 2390882)/867169) - 26628761029*s
qrt(2)*sqrt(1315529*sqrt(3) + 2390882)/751982074561 - 40176070*sqrt(6)*sqr
t(1315529*sqrt(3) + 2390882)/2213882457 + 76094994709709/751982074561 + 13
3967471914*sqrt(3)/2213882457) + 2*sqrt(-sqrt(2)*sqrt(1315529*sqrt(3) + 23
90882)/393216 + 1291/786432 + 1019*sqrt(3)/262144)*atan(1734338*sqrt(3)*x/
(-6808*sqrt(-2*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882) + 1291 + 3057*sqrt(
3)) + 55*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882)*sqrt(-2*sqrt(2)*sqrt(1315
529*sqrt(3) + 2390882) + 1291 + 3057*sqrt(3))) - 224180*sqrt(3)*sqrt(1291
+ 1019*sqrt(3))/(-6808*sqrt(-2*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882) + 1
291 + 3057*sqrt(3)) + 55*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882)*sqrt(-2*s
qrt(2)*sqrt(1315529*sqrt(3) + 2390882) + 1291 + 3057*sqrt(3))) - 148818...
```

3.122.7 Maxima [F]

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^3}\,dx = \int \frac{5\,x^6+3\,x^4+x^2+4}{(x^4+2\,x^2+3)^3}\,dx$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

output
```
1/192*(51*x^7 + 166*x^5 + 181*x^3 + 292*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2
+ 9) + 1/64*integrate((17*x^2 - 4)/(x^4 + 2*x^2 + 3), x)
```

3.122.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. 2(177) = 354.

Time = 0.72 (sec) , antiderivative size = 577, normalized size of antiderivative = 2.33

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^3}\,dx =$$

$$-\frac{1}{165888}\sqrt{2}\Big(17\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+306\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-306\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}}$$

$$-\frac{1}{165888}\sqrt{2}\Big(17\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+306\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-306\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}}$$

$$-\frac{1}{331776}\sqrt{2}\Big(306\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-17\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+17\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+$$

$$+2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\Big)$$

$$+\frac{1}{331776}\sqrt{2}\Big(306\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-17\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+17\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+$$

$$-2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\Big)+\frac{51\,x^7+166\,x^5+181\,x^3+292\,x}{192\,(x^4+2\,x^2+3)^2}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

output
```
-1/165888*sqrt(2)*(17*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 306*3^(3/4)
*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 306*3^(3/4)*(sqrt(3) + 3)*sq
rt(-6*sqrt(3) + 18) + 17*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 144*3^(1/4)*sqr
t(2)*sqrt(6*sqrt(3) + 18) - 144*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*
3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) -
1/165888*sqrt(2)*(17*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 306*3^(3/4)*
sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 306*3^(3/4)*(sqrt(3) + 3)*sqr
t(-6*sqrt(3) + 18) + 17*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 144*3^(1/4)*sqrt
(2)*sqrt(6*sqrt(3) + 18) - 144*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3
^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1
/331776*sqrt(2)*(306*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) -
 17*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 17*3^(3/4)*(6*sqrt(3) + 18)^
(3/2) + 306*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 144*3^(1/4)*sqrt(
2)*sqrt(-6*sqrt(3) + 18) + 144*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3
^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/331776*sqrt(2)*(306*3^(3/
4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 17*3^(3/4)*sqrt(2)*(-6*sq
rt(3) + 18)^(3/2) + 17*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 306*3^(3/4)*sqrt(6
*sqrt(3) + 18)*(sqrt(3) - 3) + 144*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) +
 144*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3)
 + 1/2) + sqrt(3)) + 1/192*(51*x^7 + 166*x^5 + 181*x^3 + 292*x)/(x^4 + ...
```

3.122.9 Mupad [B] (verification not implemented)

Time = 8.70 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.70

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^3}\,dx$$
$$= \frac{\frac{17\,x^7}{64}+\frac{83\,x^5}{96}+\frac{181\,x^3}{192}+\frac{73\,x}{48}}{x^8+4\,x^6+10\,x^4+12\,x^2+9} + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{7746-\sqrt{2}\,5106\mathrm{i}}\,851\mathrm{i}}{1179648\left(\frac{46805}{393216}+\frac{\sqrt{2}\,851\mathrm{i}}{98304}\right)}+\frac{851\,\sqrt{2}\,x\,\sqrt{7746-\sqrt{2}\,5106\mathrm{i}}}{2359296\left(\frac{46805}{393216}+\frac{\sqrt{2}\,851\mathrm{i}}{98304}\right)}\right)\sqrt{7746-\sqrt{2}\,5106\mathrm{i}}\,1\mathrm{i}}{768} - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{7746+\sqrt{2}\,5106\mathrm{i}}\,851\mathrm{i}}{1179648\left(-\frac{46805}{393216}+\frac{\sqrt{2}\,851\mathrm{i}}{98304}\right)}-\frac{851\,\sqrt{2}\,x\,\sqrt{7746+\sqrt{2}\,5106\mathrm{i}}}{2359296\left(-\frac{46805}{393216}+\frac{\sqrt{2}\,851\mathrm{i}}{98304}\right)}\right)\sqrt{7746+\sqrt{2}\,5106\mathrm{i}}\,1\mathrm{i}}{768}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(2*x^2 + x^4 + 3)^3,x)
```

output
```
((73*x)/48 + (181*x^3)/192 + (83*x^5)/96 + (17*x^7)/64)/(12*x^2 + 10*x^4 +
 4*x^6 + x^8 + 9) + (atan((x*(7746 - 2^(1/2)*5106i)^(1/2)*851i)/(1179648*(
(2^(1/2)*851i)/98304 + 46805/393216)) + (851*2^(1/2)*x*(7746 - 2^(1/2)*510
6i)^(1/2))/(2359296*((2^(1/2)*851i)/98304 + 46805/393216)))*(7746 - 2^(1/2
)*5106i)^(1/2)*1i)/768 - (atan((x*(2^(1/2)*5106i + 7746)^(1/2)*851i)/(1179
648*((2^(1/2)*851i)/98304 - 46805/393216)) - (851*2^(1/2)*x*(2^(1/2)*5106i
 + 7746)^(1/2))/(2359296*((2^(1/2)*851i)/98304 - 46805/393216)))*(2^(1/2)*
5106i + 7746)^(1/2)*1i)/768
```

3.122. $\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^3}\,dx$

3.123 $\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^3}\,dx$

3.123.1 Optimal result

Integrand size = 31, antiderivative size = 253

$$\begin{aligned}\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^3}\,dx = &-\frac{4}{27x}-\frac{25x(5+x^2)}{144\left(3+2x^2+x^4\right)^2}-\frac{x(325+242x^2)}{1728\left(3+2x^2+x^4\right)}\\ &+\frac{\sqrt{\frac{1}{3}\left(59711+55161\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{2304}\\ &-\frac{\sqrt{\frac{1}{3}\left(59711+55161\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{2304}\\ &-\frac{\sqrt{\frac{1}{3}\left(-59711+55161\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)}{4608}\\ &+\frac{\sqrt{\frac{1}{3}\left(-59711+55161\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)}{4608}\end{aligned}$$

output
```
-4/27/x-25/144*x*(x^2+5)/(x^4+2*x^2+3)^2-1/1728*x*(242*x^2+325)/(x^4+2*x^2
+3)-1/13824*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-179133+165483*3^(1/2)
)^(1/2)+1/13824*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-179133+165483*3^(
1/2))^(1/2)+1/6912*arctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))
*(179133+165483*3^(1/2))^(1/2)-1/6912*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2
+2*3^(1/2))^(1/2))*(179133+165483*3^(1/2))^(1/2)
```

3.123.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.55

$$\int \frac{4+x^2+3x^4+5x^6}{x^2(3+2x^2+x^4)^3}\,dx = \frac{-\frac{12(768+1849x^2+1412x^4+611x^6+166x^8)}{x(3+2x^2+x^4)^2}+\frac{3i(332i+7\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}-\frac{3i(-332i+7\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}}{6912}$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^3),x]
```

output
```
((-12*(768 + 1849*x^2 + 1412*x^4 + 611*x^6 + 166*x^8))/(x*(3 + 2*x^2 + x^4
)^2) + ((3*I)*(332*I + 7*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 -
I*Sqrt[2]] - ((3*I)*(-332*I + 7*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sq
rt[1 + I*Sqrt[2]])/6912
```

3.123.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2198, 27, 2198, 27, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^2(x^4+2x^2+3)^3}\,dx$$

$$\Big\downarrow \text{2198}$$

$$\frac{1}{96}\int \frac{2(-125x^4+45x^2+192)}{3x^2(x^4+2x^2+3)^2}dx-\frac{25x(x^2+5)}{144(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{144}\int \frac{-125x^4+45x^2+192}{x^2(x^4+2x^2+3)^2}dx-\frac{25x(x^2+5)}{144(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{2198}$$

$$\frac{1}{144}\left(\frac{1}{48}\int \frac{4\left(-242x^4-7x^2+768\right)}{x^2\left(x^4+2x^2+3\right)}dx-\frac{x\left(242x^2+325\right)}{12\left(x^4+2x^2+3\right)}\right)-\frac{25x\left(x^2+5\right)}{144\left(x^4+2x^2+3\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{144}\left(\frac{1}{12}\int \frac{-242x^4-7x^2+768}{x^2\left(x^4+2x^2+3\right)}dx-\frac{x\left(242x^2+325\right)}{12\left(x^4+2x^2+3\right)}\right)-\frac{25x\left(x^2+5\right)}{144\left(x^4+2x^2+3\right)^2}$$

$$\Big\downarrow \text{2195}$$

$$\frac{1}{144}\left(\frac{1}{12}\int \left(\frac{256}{x^2}-\frac{3\left(166x^2+173\right)}{x^4+2x^2+3}\right)dx-\frac{x\left(242x^2+325\right)}{12\left(x^4+2x^2+3\right)}\right)-\frac{25x\left(x^2+5\right)}{144\left(x^4+2x^2+3\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{144}\left(\frac{1}{12}\left(\frac{1}{4}\sqrt{3\left(59711+55161\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{4}\sqrt{3\left(59711+55161\sqrt{3}\right)}\arctan\left(\frac{2x+\sqrt{}}{\sqrt{2}}\right.\right.\right.$$
$$\frac{25x\left(x^2+5\right)}{144\left(x^4+2x^2+3\right)^2}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^3),x]
```

output
```
(-25*x*(5 + x^2))/(144*(3 + 2*x^2 + x^4)^2) + (-1/12*(x*(325 + 242*x^2))/(
3 + 2*x^2 + x^4) + (-256/x + (Sqrt[3*(59711 + 55161*Sqrt[3])]*ArcTan[(Sqrt
[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/4 - (Sqrt[3*(59711 + 551
61*Sqrt[3])]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])
/4 - (Sqrt[3*(-59711 + 55161*Sqrt[3])]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])
]*x + x^2])/8 + (Sqrt[3*(-59711 + 55161*Sqrt[3])]*Log[Sqrt[3] + Sqrt[2*(-1
 + Sqrt[3])]*x + x^2])/8)/12)/144
```

3.123.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.123. $\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^3}\,dx$

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

3.123.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.29

method	result
risch	$\frac{-\frac{83}{288}x^8-\frac{611}{576}x^6-\frac{353}{144}x^4-\frac{1849}{576}x^2-\frac{4}{3}}{x(x^4+2x^2+3)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(12\_Z^4+238844\_Z^2+3042735921\right)}\_R\ln\left(-1950\_R^3-37653769\_R+2909\right.\right.}{2304}$
default	$-\frac{4}{27x}-\frac{\frac{121}{32}x^7+\frac{809}{64}x^5+\frac{419}{16}x^3+\frac{2475}{64}x}{27(x^4+2x^2+3)^2}-\frac{\left(325\sqrt{-2+2\sqrt{3}}\sqrt{3}-21\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{27648}-\frac{\left(692\sqrt{3}+\frac{(325\sqrt{}}{}\right.}{}$

input
```
int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)
```

output
```
(-83/288*x^8-611/576*x^6-353/144*x^4-1849/576*x^2-4/3)/x/(x^4+2*x^2+3)^2+1
/2304*sum(_R*ln(-1950*_R^3-37653769*_R+2909135979*x),_R=RootOf(12*_Z^4+238
844*_Z^2+3042735921))
```

3.123.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.15

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^3}\,dx =$$
$$-\frac{3984\,x^8+14664\,x^6+33888\,x^4+\sqrt{6}(x^9+4\,x^7+10\,x^5+12\,x^3+9\,x)\sqrt{52739i\,\sqrt{2}-59711}\log\left(\sqrt{6}\sqrt{5}\right.}{}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

output
```
-1/13824*(3984*x^8 + 14664*x^6 + 33888*x^4 + sqrt(6)*(x^9 + 4*x^7 + 10*x^5
 + 12*x^3 + 9*x)*sqrt(52739*I*sqrt(2) - 59711)*log(sqrt(6)*sqrt(52739*I*sq
rt(2) - 59711)*(325*I*sqrt(2) + 346) + 330966*x) - sqrt(6)*(x^9 + 4*x^7 +
10*x^5 + 12*x^3 + 9*x)*sqrt(52739*I*sqrt(2) - 59711)*log(sqrt(6)*sqrt(5273
9*I*sqrt(2) - 59711)*(-325*I*sqrt(2) - 346) + 330966*x) - sqrt(6)*(x^9 + 4
*x^7 + 10*x^5 + 12*x^3 + 9*x)*sqrt(-52739*I*sqrt(2) - 59711)*log(sqrt(6)*(
325*I*sqrt(2) - 346)*sqrt(-52739*I*sqrt(2) - 59711) + 330966*x) + sqrt(6)*
(x^9 + 4*x^7 + 10*x^5 + 12*x^3 + 9*x)*sqrt(-52739*I*sqrt(2) - 59711)*log(s
qrt(6)*(-325*I*sqrt(2) + 346)*sqrt(-52739*I*sqrt(2) - 59711) + 330966*x) +
 44376*x^2 + 18432)/(x^9 + 4*x^7 + 10*x^5 + 12*x^3 + 9*x)
```

3.123.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.30

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^3}\,dx = \frac{-166x^8-611x^6-1412x^4-1849x^2-768}{576x^9+2304x^7+5760x^5+6912x^3+5184x}$$
$$+\operatorname{RootSum}\left(4174708211712t^4+15652880384t^2+37564641,\left(t\mapsto t\log\left(-\frac{98146713600t^3}{11971753}-\frac{96393648}{3232373}\right.\right.\right.$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+2*x**2+3)**3,x)
```

output
```
(-166*x**8 - 611*x**6 - 1412*x**4 - 1849*x**2 - 768)/(576*x**9 + 2304*x**7
 + 5760*x**5 + 6912*x**3 + 5184*x) + RootSum(4174708211712*_t**4 + 1565288
0384*_t**2 + 37564641, Lambda(_t, _t*log(-98146713600*_t**3/11971753 - 963
9364864*_t/323237331 + x)))
```

3.123.7 Maxima [F]

$$\int \frac{4+x^2+3x^4+5x^6}{x^2(3+2x^2+x^4)^3}\,dx=\int \frac{5\,x^6+3\,x^4+x^2+4}{(x^4+2\,x^2+3)^3x^2}\,dx$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

output
```
-1/576*(166*x^8 + 611*x^6 + 1412*x^4 + 1849*x^2 + 768)/(x^9 + 4*x^7 + 10*x
^5 + 12*x^3 + 9*x) - 1/576*integrate((166*x^2 + 173)/(x^4 + 2*x^2 + 3), x)
```

3.123.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 582 vs. $2(182) = 364$.

Time = 0.74 (sec) , antiderivative size = 582, normalized size of antiderivative = 2.30

$$\int \frac{4+x^2+3x^4+5x^6}{x^2(3+2x^2+x^4)^3}\,dx$$

$$=\frac{1}{746496}\sqrt{2}\left(83\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+1494\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-1494\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}\cdots}\right.$$

$$+\frac{1}{746496}\sqrt{2}\left(83\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+1494\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-1494\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\cdots}\right.$$

$$+\frac{1}{1492992}\sqrt{2}\left(1494\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-83\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+83\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}\cdots\right.$$
$$\left.+2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)$$

$$-\frac{1}{1492992}\sqrt{2}\left(1494\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-83\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+83\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}\cdots\right.$$
$$\left.-2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)-\frac{242\,x^7+809\,x^5+1676\,x^3+2475\,x}{1728\,(x^4+2\,x^2+3)^2}-\frac{4}{27\,x}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

output
```
1/746496*sqrt(2)*(83*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 1494*3^(3/4)
*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1494*3^(3/4)*(sqrt(3) + 3)*s
qrt(-6*sqrt(3) + 18) + 83*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 3114*3^(1/4)*s
qrt(2)*sqrt(6*sqrt(3) + 18) + 3114*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1
/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2))
 + 1/746496*sqrt(2)*(83*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 1494*3^(3
/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1494*3^(3/4)*(sqrt(3) + 3
)*sqrt(-6*sqrt(3) + 18) + 83*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 3114*3^(1/4
)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 3114*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arcta
n(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/
2)) + 1/1492992*sqrt(2)*(1494*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3
) + 18) - 83*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 83*3^(3/4)*(6*sqrt(
3) + 18)^(3/2) + 1494*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 3114*3^
(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 3114*3^(1/4)*sqrt(6*sqrt(3) + 18))*l
og(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/1492992*sqrt(
2)*(1494*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 83*3^(3/4)*
sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 83*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 1494
*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 3114*3^(1/4)*sqrt(2)*sqrt(-6
*sqrt(3) + 18) - 3114*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*
sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/1728*(242*x^7 + 809*x^5 + 1676*...
```

3.123.9 Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.71

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^3}\,dx = -\frac{\frac{83\,x^8}{288}+\frac{611\,x^6}{576}+\frac{353\,x^4}{144}+\frac{1849\,x^2}{576}+\frac{4}{3}}{x^9+4\,x^7+10\,x^5+12\,x^3+9\,x} + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-358266-\sqrt{2}\,316434\mathrm{i}}\,52739\mathrm{i}}{859963392\,\left(-\frac{17140175}{286654464}+\frac{\sqrt{2}\,9123847\mathrm{i}}{286654464}\right)}+\frac{52739\,\sqrt{2}\,x\,\sqrt{-358266-\sqrt{2}\,316434\mathrm{i}}}{1719926784\,\left(-\frac{17140175}{286654464}+\frac{\sqrt{2}\,9123847\mathrm{i}}{286654464}\right)}\right)\,\sqrt{-358266-\sqrt{2}\,316434\mathrm{i}}\,1\mathrm{i}}{6912} - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-358266+\sqrt{2}\,316434\mathrm{i}}\,52739\mathrm{i}}{859963392\,\left(\frac{17140175}{286654464}+\frac{\sqrt{2}\,9123847\mathrm{i}}{286654464}\right)}-\frac{52739\,\sqrt{2}\,x\,\sqrt{-358266+\sqrt{2}\,316434\mathrm{i}}}{1719926784\,\left(\frac{17140175}{286654464}+\frac{\sqrt{2}\,9123847\mathrm{i}}{286654464}\right)}\right)\,\sqrt{-358266+\sqrt{2}\,316434\mathrm{i}}\,1\mathrm{i}}{6912}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^2*(2*x^2 + x^4 + 3)^3),x)
```

output

```
(atan((x*(- 2^(1/2)*316434i - 358266)^(1/2)*52739i)/(859963392*((2^(1/2)*9
123847i)/286654464 - 17140175/286654464)) + (52739*2^(1/2)*x*(- 2^(1/2)*31
6434i - 358266)^(1/2))/(1719926784*((2^(1/2)*9123847i)/286654464 - 1714017
5/286654464)))*(- 2^(1/2)*316434i - 358266)^(1/2)*1i)/6912 - (atan((x*(2^(
1/2)*316434i - 358266)^(1/2)*52739i)/(859963392*((2^(1/2)*9123847i)/286654
464 + 17140175/286654464)) - (52739*2^(1/2)*x*(2^(1/2)*316434i - 358266)^(
1/2))/(1719926784*((2^(1/2)*9123847i)/286654464 + 17140175/286654464)))*(2
^(1/2)*316434i - 358266)^(1/2)*1i)/6912 - ((1849*x^2)/576 + (353*x^4)/144
+ (611*x^6)/576 + (83*x^8)/288 + 4/3)/(9*x + 12*x^3 + 10*x^5 + 4*x^7 + x^9
)
```

3.123. $\int \frac{4+x^2+3x^4+5x^6}{x^2(3+2x^2+x^4)^3}\,dx$

3.124 $\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^3}\,dx$

3.124.1 Optimal result

Integrand size = 31, antiderivative size = 262

$$\begin{aligned}
&\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^3}\,dx \\
&= -\frac{4}{81x^3}+\frac{7}{27x}+\frac{25x(7+5x^2)}{432(3+2x^2+x^4)^2}+\frac{x(1474+1025x^2)}{5184(3+2x^2+x^4)} \\
&\quad-\frac{\sqrt{\frac{1}{3}\left(10004741+11240451\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{20736} \\
&\quad+\frac{\sqrt{\frac{1}{3}\left(10004741+11240451\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{20736} \\
&\quad+\frac{\sqrt{\frac{1}{3}\left(-10004741+11240451\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)}{41472} \\
&\quad-\frac{\sqrt{\frac{1}{3}\left(-10004741+11240451\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)}{41472}
\end{aligned}$$

output
```
-4/81/x^3+7/27/x+25/432*x*(5*x^2+7)/(x^4+2*x^2+3)^2+1/5184*x*(1025*x^2+147
4)/(x^4+2*x^2+3)+1/124416*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-3001422
3+33721353*3^(1/2))^(1/2)-1/124416*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*
(-30014223+33721353*3^(1/2))^(1/2)-1/62208*arctan((-2*x+(-2+2*3^(1/2))^(1/
2))/(2+2*3^(1/2))^(1/2))*(30014223+33721353*3^(1/2))^(1/2)+1/62208*arctan(
(2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(30014223+33721353*3^(1/2)
)^(1/2)
```

3.124.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.53

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^3}\,dx = \frac{\frac{4(-2304+9024x^2+20090x^4+19939x^6+8644x^8+2369x^{10})}{x^3(3+2x^2+x^4)^2}+\frac{\left(4738+127i\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{\left(4738-127i\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}}{20736}$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^3),x]
```

output
```
((4*(-2304 + 9024*x^2 + 20090*x^4 + 19939*x^6 + 8644*x^8 + 2369*x^10))/(x^
3*(3 + 2*x^2 + x^4)^2) + ((4738 + (127*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqr
t[2]]])/Sqrt[1 - I*Sqrt[2]] + ((4738 - (127*I)*Sqrt[2])*ArcTan[x/Sqrt[1 +
I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/20736
```

3.124.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2198, 27, 2198, 27, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^4(x^4+2x^2+3)^3}\,dx$$
$$\Big\downarrow \text{ 2198}$$

$$\frac{1}{96}\int \frac{2\left(625x^6+225x^4-240x^2+576\right)}{9x^4\left(x^4+2x^2+3\right)^2}dx+\frac{25x\left(5x^2+7\right)}{432\left(x^4+2x^2+3\right)^2}$$

$\downarrow$ 27

$$\frac{1}{432}\int \frac{625x^6+225x^4-240x^2+576}{x^4\left(x^4+2x^2+3\right)^2}dx+\frac{25x\left(5x^2+7\right)}{432\left(x^4+2x^2+3\right)^2}$$

$\downarrow$ 2198

$$\frac{1}{432}\left(\frac{1}{48}\int \frac{4\left(1025x^6+322x^4-2496x^2+2304\right)}{x^4\left(x^4+2x^2+3\right)}dx+\frac{x\left(1025x^2+1474\right)}{12\left(x^4+2x^2+3\right)}\right)+\frac{25x\left(5x^2+7\right)}{432\left(x^4+2x^2+3\right)^2}$$

$\downarrow$ 27

$$\frac{1}{432}\left(\frac{1}{12}\int \frac{1025x^6+322x^4-2496x^2+2304}{x^4\left(x^4+2x^2+3\right)}dx+\frac{x\left(1025x^2+1474\right)}{12\left(x^4+2x^2+3\right)}\right)+\frac{25x\left(5x^2+7\right)}{432\left(x^4+2x^2+3\right)^2}$$

$\downarrow$ 2195

$$\frac{1}{432}\left(\frac{1}{12}\int \left(\frac{2369x^2+2242}{x^4+2x^2+3}-\frac{1344}{x^2}+\frac{768}{x^4}\right)dx+\frac{x\left(1025x^2+1474\right)}{12\left(x^4+2x^2+3\right)}\right)+\frac{25x\left(5x^2+7\right)}{432\left(x^4+2x^2+3\right)^2}$$

$\downarrow$ 2009

$$\frac{1}{432}\left(\frac{1}{12}\left(-\frac{1}{4}\sqrt{\frac{1}{3}\left(10004741+11240451\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(10004741+11240451\sqrt{3}\right)}\right.\right.$$
$$\frac{25x\left(5x^2+7\right)}{432\left(x^4+2x^2+3\right)^2}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^3),x]
```

output
```
(25*x*(7 + 5*x^2))/(432*(3 + 2*x^2 + x^4)^2) + ((x*(1474 + 1025*x^2))/(12*
(3 + 2*x^2 + x^4)) + (-256/x^3 + 1344/x - (Sqrt[(10004741 + 11240451*Sqrt[
3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/4 + (
Sqrt[(10004741 + 11240451*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x
)/Sqrt[2*(1 + Sqrt[3])]])/4 + (Sqrt[(-10004741 + 11240451*Sqrt[3])/3]*Log[
Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/8 - (Sqrt[(-10004741 + 11240451
*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/8)/12)/432
```

3.124. $\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^3}\,dx$

3.124.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

3.124.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.30

method	result
risch	$\frac{\frac{2369}{5184}x^{10}+\frac{2161}{1296}x^8+\frac{19939}{5184}x^6+\frac{10045}{2592}x^4+\frac{47}{27}x^2-\frac{4}{9}}{x^3(x^4+2x^2+3)^2}+\frac{\left(\sum_{\_R=\text{RootOf}\left(12\_Z^4+40018964\_Z^2+126347738683401\right)}\_R\ln\left(29190\_R^3+101\right.\right.}{20736}$
default	$-\frac{4}{81x^3}+\frac{7}{27x}+\frac{\frac{1025}{192}x^7+\frac{881}{48}x^5+\frac{7523}{192}x^3+\frac{1087}{32}x}{27(x^4+2x^2+3)^2}+\frac{\left(4865\sqrt{-2+2\sqrt{3}}\,\sqrt{3}+381\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{248832}+\frac{(8}{}$

input
```
int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)
```

output
```
(2369/5184*x^10+2161/1296*x^8+19939/5184*x^6+10045/2592*x^4+47/27*x^2-4/9)
/x^3/(x^4+2*x^2+3)^2+1/20736*sum(_R*ln(29190*_R^3+101628741761*_R+13274881
5833469*x),_R=RootOf(12*_Z^4+40018964*_Z^2+126347738683401))
```

3.124.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.16

$$\int \frac{4+x^2+3x^4+5x^6}{x^4 (3+2x^2+x^4)^3} \, dx$$
$$= \frac{56856\,x^{10}+207456\,x^8+478536\,x^6+482160\,x^4+\sqrt{6}(x^{11}+4\,x^9+10\,x^7+12\,x^5+9\,x^3)\sqrt{11809919i\,\sqrt{2}}}{}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

output
```
1/124416*(56856*x^10 + 207456*x^8 + 478536*x^6 + 482160*x^4 + sqrt(6)*(x^1
1 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3)*sqrt(11809919*I*sqrt(2) - 10004741)*l
og(sqrt(6)*sqrt(11809919*I*sqrt(2) - 10004741)*(4865*I*sqrt(2) + 4484) + 6
7442706*x) - sqrt(6)*(x^11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3)*sqrt(1180991
9*I*sqrt(2) - 10004741)*log(sqrt(6)*sqrt(11809919*I*sqrt(2) - 10004741)*(-
4865*I*sqrt(2) - 4484) + 67442706*x) - sqrt(6)*(x^11 + 4*x^9 + 10*x^7 + 12
*x^5 + 9*x^3)*sqrt(-11809919*I*sqrt(2) - 10004741)*log(sqrt(6)*(4865*I*sqr
t(2) - 4484)*sqrt(-11809919*I*sqrt(2) - 10004741) + 67442706*x) + sqrt(6)*
(x^11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3)*sqrt(-11809919*I*sqrt(2) - 100047
41)*log(sqrt(6)*(-4865*I*sqrt(2) + 4484)*sqrt(-11809919*I*sqrt(2) - 100047
41) + 67442706*x) + 216576*x^2 - 55296)/(x^11 + 4*x^9 + 10*x^7 + 12*x^5 +
9*x^3)
```

3.124.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.31

$$\int \frac{4+x^2+3x^4+5x^6}{x^4 (3+2x^2+x^4)^3} \, dx$$
$$= \operatorname{RootSum}\left(338151365148672t^4+2622682824704t^2+19257390441, \left(t \mapsto t\log\left(\frac{357010935644160t^3}{182097141061}+\right.\right.\right.$$
$$+ \frac{2369x^{10}+8644x^8+19939x^6+20090x^4+9024x^2-2304}{5184x^{11}+20736x^9+51840x^7+62208x^5+46656x^3}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+2*x**2+3)**3,x)
```

output
```
RootSum(338151365148672*_t**4 + 2622682824704*_t**2 + 19257390441, Lambda(
_t, _t*log(357010935644160*_t**3/182097141061 + 26016957890816*_t/16388742
69549 + x))) + (2369*x**10 + 8644*x**8 + 19939*x**6 + 20090*x**4 + 9024*x*
*2 - 2304)/(5184*x**11 + 20736*x**9 + 51840*x**7 + 62208*x**5 + 46656*x**3
)
```

3.124.7 Maxima [F]

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^3}\,dx = \int \frac{5\,x^6+3\,x^4+x^2+4}{(x^4+2\,x^2+3)^3x^4}\,dx$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

output
```
1/5184*(2369*x^10 + 8644*x^8 + 19939*x^6 + 20090*x^4 + 9024*x^2 - 2304)/(x
^11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3) + 1/5184*integrate((2369*x^2 + 2242
)/(x^4 + 2*x^2 + 3), x)
```

3.124.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 589 vs. $2(189) = 378$.

Time = 0.77 (sec) , antiderivative size = 589, normalized size of antiderivative = 2.25

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^3}\,dx =$$

$$-\frac{1}{13436928}\sqrt{2}\left(2369\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+42642\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-42642\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right.\right.$$

$$-\frac{1}{13436928}\sqrt{2}\left(2369\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+42642\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-42642\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right.\right.$$

$$-\frac{1}{26873856}\sqrt{2}\left(42642\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-2369\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+2369\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}\right.\right.$$

$$\left.+2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)$$

$$+\frac{1}{26873856}\sqrt{2}\left(42642\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-2369\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+2369\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}\right.\right.$$

$$\left.-2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)+\frac{1025\,x^7+3524\,x^5+7523\,x^3+6522\,x}{5184\,\left(x^4+2\,x^2+3\right)^2}+\frac{21\,x^2-4}{81\,x^3}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

3.124. $\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^3}\,dx$

output
```
-1/13436928*sqrt(2)*(2369*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 42642*3
^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 42642*3^(3/4)*(sqrt(3)
 + 3)*sqrt(-6*sqrt(3) + 18) + 2369*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 80712
*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 80712*3^(1/4)*sqrt(-6*sqrt(3) + 18
))*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt
(3) + 1/2)) - 1/13436928*sqrt(2)*(2369*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3
/2) + 42642*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 42642*3^(
3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 2369*3^(3/4)*(-6*sqrt(3) + 18)^
(3/2) - 80712*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 80712*3^(1/4)*sqrt(-6
*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/
sqrt(1/6*sqrt(3) + 1/2)) - 1/26873856*sqrt(2)*(42642*3^(3/4)*sqrt(2)*(sqrt
(3) + 3)*sqrt(-6*sqrt(3) + 18) - 2369*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3
/2) + 2369*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 42642*3^(3/4)*sqrt(6*sqrt(3) +
 18)*(sqrt(3) - 3) - 80712*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 80712*3
^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2
) + sqrt(3)) + 1/26873856*sqrt(2)*(42642*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqr
t(-6*sqrt(3) + 18) - 2369*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 2369*3
^(3/4)*(6*sqrt(3) + 18)^(3/2) + 42642*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3
) - 3) - 80712*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 80712*3^(1/4)*sqrt(
6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(...
```

3.124.9 Mupad [B] (verification not implemented)

Time = 8.75 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.71

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^3}\,dx = \frac{\frac{2369\,x^{10}}{5184}+\frac{2161\,x^8}{1296}+\frac{19939\,x^6}{5184}+\frac{10045\,x^4}{2592}+\frac{47\,x^2}{27}-\frac{4}{9}}{x^{11}+4\,x^9+10\,x^7+12\,x^5+9\,x^3}$$
$$-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-60028446-\sqrt{2}\,70859514\mathrm{i}}\,11809919\mathrm{i}}{626913312768\left(-\frac{57455255935}{208971104256}+\frac{\sqrt{2}\,132389191 99\mathrm{i}}{104485552128}\right)}+\frac{11809919\,\sqrt{2}\,x\,\sqrt{-60028446-\sqrt{2}\,70859514\mathrm{i}}}{1253826625536\left(-\frac{57455255935}{208971104256}+\frac{\sqrt{2}\,13238919199\mathrm{i}}{104485552128}\right)}\right)\,\sqrt{-60028446-\sqrt{}}}{62208}$$
$$+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-60028446+\sqrt{2}\,70859514\mathrm{i}}\,11809919\mathrm{i}}{626913312768\left(\frac{57455255935}{208971104256}+\frac{\sqrt{2}\,13238919199\mathrm{i}}{104485552128}\right)}-\frac{11809919\,\sqrt{2}\,x\,\sqrt{-60028446+\sqrt{2}\,70859514\mathrm{i}}}{1253826625536\left(\frac{57455255935}{208971104256}+\frac{\sqrt{2}\,13238919199\mathrm{i}}{104485552128}\right)}\right)\,\sqrt{-60028446+\sqrt{2}}}{62208}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^4*(2*x^2 + x^4 + 3)^3),x)
```

output
```
((47*x^2)/27 + (10045*x^4)/2592 + (19939*x^6)/5184 + (2161*x^8)/1296 + (23
69*x^10)/5184 - 4/9)/(9*x^3 + 12*x^5 + 10*x^7 + 4*x^9 + x^11) - (atan((x*(
- 2^(1/2)*70859514i - 60028446)^(1/2)*11809919i)/(626913312768*((2^(1/2)*1
3238919199i)/104485552128 - 57455255935/208971104256)) + (11809919*2^(1/2)
*x*(- 2^(1/2)*70859514i - 60028446)^(1/2))/(1253826625536*((2^(1/2)*132389
19199i)/104485552128 - 57455255935/208971104256)))*(- 2^(1/2)*70859514i -
60028446)^(1/2)*1i)/62208 + (atan((x*(2^(1/2)*70859514i - 60028446)^(1/2)*
11809919i)/(626913312768*((2^(1/2)*13238919199i)/104485552128 + 5745525593
5/208971104256)) - (11809919*2^(1/2)*x*(2^(1/2)*70859514i - 60028446)^(1/2
))/(1253826625536*((2^(1/2)*13238919199i)/104485552128 + 57455255935/20897
1104256)))*(2^(1/2)*70859514i - 60028446)^(1/2)*1i)/62208
```

3.124. $\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^3}\,dx$

3.125 $\int \frac{x(d+ex^2+fx^4+gx^6)}{a+bx^2+cx^4}\,dx$

3.125.1 Optimal result

Integrand size = 33, antiderivative size = 149

$$\int \frac{x(d+ex^2+fx^4+gx^6)}{a+bx^2+cx^4}\,dx = \frac{(cf-bg)x^2}{2c^2}+\frac{gx^4}{4c}-\frac{(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag))\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}} + \frac{(c^2e+b^2g-c(bf+ag))\log(a+bx^2+cx^4)}{4c^3}$$

output
```
1/2*(-b*g+c*f)*x^2/c^2+1/4*g*x^4/c+1/4*(c^2*e+b^2*g-c*(a*g+b*f))*ln(c*x^4+
b*x^2+a)/c^3-1/2*(2*c^3*d-c^2*(2*a*f+b*e)-b^3*g+b*c*(3*a*g+b*f))*arctanh((
2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)
```

3.125.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

$$\int \frac{x(d+ex^2+fx^4+gx^6)}{a+bx^2+cx^4}\,dx = \frac{2c(cf-bg)x^2+c^2gx^4+\frac{2(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag))\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(c^2e+b^2g-c(bf+ag))\log(a-}{4c^3}$$

input
```
Integrate[(x*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4),x]
```

output
```
(2*c*(c*f - b*g)*x^2 + c^2*g*x^4 + (2*(2*c^3*d - c^2*(b*e + 2*a*f) - b^3*g
 + b*c*(b*f + 3*a*g))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2
+ 4*a*c] + (c^2*e + b^2*g - c*(b*f + a*g))*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

3.125.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2194, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(d+ex^2+fx^4+gx^6\right)}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \text{ 2194}$$

$$\frac{1}{2}\int \frac{gx^6+fx^4+ex^2+d}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{ 2188}$$

$$\frac{1}{2}\int \left(\frac{gx^2}{c}+\frac{cf-bg}{c^2}+\frac{dc^2-afc+\left(gb^2+c^2e-c(bf+ag)\right)x^2+abg}{c^2\left(cx^4+bx^2+a\right)}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)}{c^3\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)\left(-c(ag+bf)+\right.}{2c^3}\right.$$

input
```
Int[(x*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4),x]
```

output
```
(((c*f - b*g)*x^2)/c^2 + (g*x^4)/(2*c) - ((2*c^3*d - c^2*(b*e + 2*a*f) - b
^3*g + b*c*(b*f + 3*a*g))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^3*S
qrt[b^2 - 4*a*c]) + ((c^2*e + b^2*g - c*(b*f + a*g))*Log[a + b*x^2 + c*x^4
])/(2*c^3))/2
```

3.125.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.125.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.01

method	result
default	$-\frac{-\frac{1}{2}cg\,x^{4}+bg\,x^{2}-cf\,x^{2}}{2c^{2}}+\frac{\frac{\left(-acg+b^{2}g-fbc+e\,c^{2}\right)\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2c}+\frac{2\left(abg-acf+c^{2}d-\frac{\left(-acg+b^{2}g-fbc+e\,c^{2}\right)b}{2c}\right)\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2c^{2}}$
risch	Expression too large to display

input
```
int(x*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/c^2*(-1/2*c*g*x^4+b*g*x^2-c*f*x^2)+1/2/c^2*(1/2*(-a*c*g+b^2*g-b*c*f+c
^2*e)/c*ln(c*x^4+b*x^2+a)+2*(a*b*g-a*c*f+c^2*d-1/2*(-a*c*g+b^2*g-b*c*f+c^2
*e)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

3.125.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 486, normalized size of antiderivative = 3.26

$$\int \frac{x(d+ex^2+fx^4+gx^6)}{a+bx^2+cx^4}\,dx$$
$$= \left[\frac{(b^2c^2-4\,ac^3)gx^4+2\,((b^2c^2-4\,ac^3)f-(b^3c-4\,abc^2)g)x^2+(2\,c^3d-bc^2e+(b^2c-2\,ac^2)f-(b^3-3\,ab\right.$$

input
```
integrate(x*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*((b^2*c^2 - 4*a*c^3)*g*x^4 + 2*((b^2*c^2 - 4*a*c^3)*f - (b^3*c - 4*a*
b*c^2)*g)*x^2 + (2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f - (b^3 - 3*a*b*c)
*g)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2
+ b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((b^2*c^2 - 4*a*c^3)*e - (b
^3*c - 4*a*b*c^2)*f + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*g)*log(c*x^4 + b*x^2 +
 a))/(b^2*c^3 - 4*a*c^4), 1/4*((b^2*c^2 - 4*a*c^3)*g*x^4 + 2*((b^2*c^2 - 4
*a*c^3)*f - (b^3*c - 4*a*b*c^2)*g)*x^2 - 2*(2*c^3*d - b*c^2*e + (b^2*c - 2
*a*c^2)*f - (b^3 - 3*a*b*c)*g)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sq
rt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + ((b^2*c^2 - 4*a*c^3)*e - (b^3*c - 4*a*b*
c^2)*f + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*g)*log(c*x^4 + b*x^2 + a))/(b^2*c^3
 - 4*a*c^4)]
```

3.125.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(d+ex^2+fx^4+gx^6)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x*(g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.125.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(d+ex^2+fx^4+gx^6)}{a+bx^2+cx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.125.8 Giac [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.97

$$\begin{aligned}&\int \frac{x(d+ex^2+fx^4+gx^6)}{a+bx^2+cx^4}\,dx\\&=\frac{cgx^4+2\,cfx^2-2\,bgx^2}{4\,c^2}+\frac{(c^2e-bcf+b^2g-acg)\log\left(cx^4+bx^2+a\right)}{4\,c^3}\\&+\frac{(2\,c^3d-bc^2e+b^2cf-2\,ac^2f-b^3g+3\,abcg)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^3}\end{aligned}$$

input
```
integrate(x*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/4*(c*g*x^4 + 2*c*f*x^2 - 2*b*g*x^2)/c^2 + 1/4*(c^2*e - b*c*f + b^2*g - a
*c*g)*log(c*x^4 + b*x^2 + a)/c^3 + 1/2*(2*c^3*d - b*c^2*e + b^2*c*f - 2*a*
c^2*f - b^3*g + 3*a*b*c*g)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(
-b^2 + 4*a*c)*c^3)
```

3.125.9 Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 1834, normalized size of antiderivative = 12.31

$$\int \frac{x(d+ex^2+fx^4+gx^6)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((x*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4),x)
```

output
```
x^2*(f/(2*c) - (b*g)/(2*c^2)) + (g*x^4)/(4*c) - (log(a + b*x^2 + c*x^4)*(2
*b^4*g + 2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e - 2*b^3*c*f + 8*a*b*c^2*f -
 10*a*b^2*c*g))/(2*(16*a*c^4 - 4*b^2*c^3)) + (atan((2*c^4*(4*a*c - b^2)*(x
^2*(((((4*c^6*d + 6*b^2*c^4*f - 6*b^3*c^3*g - 4*a*c^5*f - 6*b*c^5*e + 10*a
*b*c^4*g)/c^4 - (4*b*c^2*(2*b^4*g + 2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e
- 2*b^3*c*f + 8*a*b*c^2*f - 10*a*b^2*c*g))/(16*a*c^4 - 4*b^2*c^3))*(2*c^3*
d - b^3*g - 2*a*c^2*f - b*c^2*e + b^2*c*f + 3*a*b*c*g))/(8*c^3*(4*a*c - b^
2)^(1/2)) - (b*(2*c^3*d - b^3*g - 2*a*c^2*f - b*c^2*e + b^2*c*f + 3*a*b*c*
g)*(2*b^4*g + 2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e - 2*b^3*c*f + 8*a*b*c^
2*f - 10*a*b^2*c*g))/(2*c*(4*a*c - b^2)^(1/2)*(16*a*c^4 - 4*b^2*c^3)))/a +
 (b*((((4*c^6*d + 6*b^2*c^4*f - 6*b^3*c^3*g - 4*a*c^5*f - 6*b*c^5*e + 10*a
*b*c^4*g)/c^4 - (4*b*c^2*(2*b^4*g + 2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e
- 2*b^3*c*f + 8*a*b*c^2*f - 10*a*b^2*c*g))/(16*a*c^4 - 4*b^2*c^3))*(2*b^4*
g + 2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e - 2*b^3*c*f + 8*a*b*c^2*f - 10*a
*b^2*c*g))/(2*(16*a*c^4 - 4*b^2*c^3)) - (b^5*g^2 + b*c^4*e^2 + b^3*c^2*f^2
 - c^5*d*e + 2*a^2*b*c^2*g^2 + a*c^4*d*g + a*c^4*e*f + b*c^4*d*f - 2*b^4*c
*f*g - a*b*c^3*f^2 - 3*a*b^3*c*g^2 - b^2*c^3*d*g - 2*b^2*c^3*e*f - a^2*c^3
*f*g + 2*b^3*c^2*e*g + 4*a*b^2*c^2*f*g - 3*a*b*c^3*e*g)/c^4 + (b*(2*c^3*d
- b^3*g - 2*a*c^2*f - b*c^2*e + b^2*c*f + 3*a*b*c*g)^2)/(2*c^4*(4*a*c - b^
2))))/(2*a*(4*a*c - b^2)^(1/2))) + ((((8*a^2*c^4*g - 8*a*c^5*e + 8*a*b*...
```

3.126 $\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx$

3.126.1 Optimal result

Integrand size = 35, antiderivative size = 594

$$\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \frac{(cf-2bg)x}{c^3}+\frac{gx^3}{3c^2}$$
$$+\frac{x(a(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag))+(b^3cf+bc^2(cd-3af)-b^4g-b^2c(ce-4ag)+2ac^2(}{2c^3(b^2-4ac)(a+bx^2+cx^4)}$$
$$-\frac{\left(3b^3cf-bc^2(cd+13af)-5b^4g-b^2c(ce-24ag)+2ac^2(3ce-7ag)-\frac{3b^4cf-4ac^3(cd-5af)-b^2c^2(cd+19af)-5}{\sqrt{b^2-4}}\right.}{2\sqrt{2}c^{7/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\left(3b^3cf-bc^2(cd+13af)-5b^4g-b^2c(ce-24ag)+2ac^2(3ce-7ag)+\frac{3b^4cf-4ac^3(cd-5af)-b^2c^2(cd+19af)-5}{\sqrt{b^2-4}}\right.}{2\sqrt{2}c^{7/2}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
(-2*b*g+c*f)*x/c^3+1/3*g*x^3/c^2+1/2*x*(a*(2*c^3*d-c^2*(2*a*f+b*e)-b^3*g+b
*c*(3*a*g+b*f))+(b^3*c*f+b*c^2*(-3*a*f+c*d)-b^4*g-b^2*c*(-4*a*g+c*e)+2*a*c
^2*(-a*g+c*e))*x^2)/c^3/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*
c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(3*b^3*c*f-b*c^2*(13*a*f+c*d)-5*b^4*
g-b^2*c*(-24*a*g+c*e)+2*a*c^2*(-7*a*g+3*c*e)+(-3*b^4*c*f+4*a*c^3*(-5*a*f+c
*d)+b^2*c^2*(19*a*f+c*d)+5*b^5*g+b^3*c*(-34*a*g+c*e)-4*a*b*c^2*(-13*a*g+2*
c*e))/(-4*a*c+b^2)^(1/2))/c^(7/2)/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/
2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(3*b^
3*c*f-b*c^2*(13*a*f+c*d)-5*b^4*g-b^2*c*(-24*a*g+c*e)+2*a*c^2*(-7*a*g+3*c*e
)+(3*b^4*c*f-4*a*c^3*(-5*a*f+c*d)-b^2*c^2*(19*a*f+c*d)-5*b^5*g-b^3*c*(-34*
a*g+c*e)+4*a*b*c^2*(-13*a*g+2*c*e))/(-4*a*c+b^2)^(1/2))/c^(7/2)/(-4*a*c+b^
2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.126.2 Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 721, normalized size of antiderivative = 1.21

$$\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{12\sqrt{c}(cf-2bg)x+4c^{3/2}gx^3+\frac{6\sqrt{c}x\left(b\left(c^3d-bc^2e+b^2cf-b^3g\right)x^2+a^2c\left(3bg-2c\left(f+gx^2\right)\right)+a\left(-b^3g+2c^3\left(d+ex^2\right)-bc^2\left(e+3fx^2\right)+b^2c\right.\right.}{(b^2-4ac)(a+bx^2+cx^4)}}{}$$

input
```
Integrate[(x^4*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(12*Sqrt[c]*(c*f - 2*b*g)*x + 4*c^(3/2)*g*x^3 + (6*Sqrt[c]*x*(b*(c^3*d - b
*c^2*e + b^2*c*f - b^3*g)*x^2 + a^2*c*(3*b*g - 2*c*(f + g*x^2)) + a*(-(b^3
*g) + 2*c^3*(d + e*x^2) - b*c^2*(e + 3*f*x^2) + b^2*c*(f + 4*g*x^2))))/((b
^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*(-5*b^5*g - b^3*c*(c*e + 3*S
qrt[b^2 - 4*a*c]*f - 34*a*g) + b^4*(3*c*f + 5*Sqrt[b^2 - 4*a*c]*g) + 2*a*c
^2*(-2*c^2*d - 3*c*Sqrt[b^2 - 4*a*c]*e + 10*a*c*f + 7*a*Sqrt[b^2 - 4*a*c]*
g) - b^2*c*(c^2*d - c*Sqrt[b^2 - 4*a*c]*e + 19*a*c*f + 24*a*Sqrt[b^2 - 4*a
*c]*g) + b*c^2*(c*(Sqrt[b^2 - 4*a*c]*d + 8*a*e) + 13*a*(Sqrt[b^2 - 4*a*c]*
f - 4*a*g)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^
2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*(5*b^5*g + b^3*
c*(c*e - 3*Sqrt[b^2 - 4*a*c]*f - 34*a*g) + b^4*(-3*c*f + 5*Sqrt[b^2 - 4*a*
c]*g) + b^2*c*(c^2*d + c*Sqrt[b^2 - 4*a*c]*e + 19*a*c*f - 24*a*Sqrt[b^2 -
4*a*c]*g) + 2*a*c^2*(2*c^2*d - 3*c*Sqrt[b^2 - 4*a*c]*e - 10*a*c*f + 7*a*Sq
rt[b^2 - 4*a*c]*g) + b*c^2*(c*(Sqrt[b^2 - 4*a*c]*d - 8*a*e) + 13*a*(Sqrt[b
^2 - 4*a*c]*f + 4*a*g)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*
a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*c^(7/2))
```

3.126.3 Rubi [A] (verified)

Time = 8.04 (sec) , antiderivative size = 607, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2197, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \text{ 2197}$$

$$\frac{x\left(a\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)+x^2\left(-b^2c(ce-4ag)+bc^2(cd-3af)+2ac^2(ce-ag)+b^4(\right.\right.}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\frac{\int \frac{2a\left(4a-\frac{b^2}{c}\right)gx^6-\frac{2a\left(b^2-4ac\right)(cf-bg)x^4}{c^2}+\frac{a\left(-gb^4+cfb^3-c(ce-6ag)b^2-c^2(cd+5af)b+6ac^2(ce-ag)\right)x^2}{c^3}+\frac{a^2\left(-gb^3+c(bf+3ag)b+2c^3d-c^2(be+2af)\right)}{c^3}}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{ 2205}$$

$$\frac{x\left(a\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)+x^2\left(-b^2c(ce-4ag)+bc^2(cd-3af)+2ac^2(ce-ag)+b^4(\right.\right.}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\frac{\int\left(-\frac{2a(b^2-4ac)gx^2}{c^2}-\frac{2a(b^2-4ac)(cf-2bg)}{c^3}+\frac{(-5gb^3+c(3bf+19ag)b+2c^3d-c^2(be+10af))a^2+(-5gb^4+3cfb^3-c(ce-24ag)b^2-c^2(cd+13a}{c^3(cx^4+bx^2+a)}\right.}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x\left(a\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)+x^2\left(-b^2c(ce-4ag)+bc^2(cd-3af)+2ac^2(ce-ag)+b^4(\right.\right.}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\frac{a\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-b^2c(ce-24ag)-\frac{-b^3c(ce-34ag)-b^2c^2(19af+cd)+4abc^2(2ce-13ag)-4ac^3(cd-5af)-5b^5g+3b^4cf}{\sqrt{b^2-4ac}}-bc^2(13af+cd)+2ac^2(3ce-\right.}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

input

```
Int[(x^4*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x]
```

output

```
(x*(a*(2*c^3*d - c^2*(b*e + 2*a*f) - b^3*g + b*c*(b*f + 3*a*g)) + (b^3*c*f
 + b*c^2*(c*d - 3*a*f) - b^4*g - b^2*c*(c*e - 4*a*g) + 2*a*c^2*(c*e - a*g)
)*x^2))/(2*c^3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((-2*a*(b^2 - 4*a*c)*(
c*f - 2*b*g)*x)/c^3 - (2*a*(b^2 - 4*a*c)*g*x^3)/(3*c^2) + (a*(3*b^3*c*f -
b*c^2*(c*d + 13*a*f) - 5*b^4*g - b^2*c*(c*e - 24*a*g) + 2*a*c^2*(3*c*e - 7
*a*g) - (3*b^4*c*f - 4*a*c^3*(c*d - 5*a*f) - b^2*c^2*(c*d + 19*a*f) - 5*b^
5*g - b^3*c*(c*e - 34*a*g) + 4*a*b*c^2*(2*c*e - 13*a*g))/Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2
)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (a*(3*b^3*c*f - b*c^2*(c*d + 13*a*f) - 5*
b^4*g - b^2*c*(c*e - 24*a*g) + 2*a*c^2*(3*c*e - 7*a*g) + (3*b^4*c*f - 4*a*
c^3*(c*d - 5*a*f) - b^2*c^2*(c*d + 19*a*f) - 5*b^5*g - b^3*c*(c*e - 34*a*g
) + 4*a*b*c^2*(2*c*e - 13*a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b + Sqrt[b^2 - 4*a
*c]]))/(2*a*(b^2 - 4*a*c))
```

3.126. $\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx$

3.126.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

3.126.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.59

method	result
risch	$\frac{g\,x^3}{3c^2}-\frac{2bgx}{c^3}+\frac{fx}{c^2}+\frac{\frac{\left(2g\,a^2c^2-4a\,b^2cg+3ab\,c^2f-2a\,c^3e+b^4g-b^3cf+b^2c^2e-b\,c^3d\right)x^3}{8ac-2b^2}-\frac{a\left(3abgc-2a\,c^2f-b^3g+b^2cf-b\,c^2e+2c^3d\right)x}{2\left(4ac-b^2\right)}}{c^3(c\,x^4+b\,x^2+a)}+$
default	$-\frac{-\frac{1}{3}g\,x^3c+2bgx-cfx}{c^3}+\frac{\frac{\left(2g\,a^2c^2-4a\,b^2cg+3ab\,c^2f-2a\,c^3e+b^4g-b^3cf+b^2c^2e-b\,c^3d\right)x^3}{8ac-2b^2}-\frac{a\left(3abgc-2a\,c^2f-b^3g+b^2cf-b\,c^2e+2c^3d\right)x}{2\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+$

input
```
int(x^4*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*g*x^3/c^2-2/c^3*b*g*x+f*x/c^2+(1/2*(2*a^2*c^2*g-4*a*b^2*c*g+3*a*b*c^2*
f-2*a*c^3*e+b^4*g-b^3*c*f+b^2*c^2*e-b*c^3*d)/(4*a*c-b^2)*x^3-1/2*a*(3*a*b*
c*g-2*a*c^2*f-b^3*g+b^2*c*f-b*c^2*e+2*c^3*d)/(4*a*c-b^2)*x)/c^3/(c*x^4+b*x
^2+a)+1/4/c^3*sum((-(14*a^2*c^2*g-24*a*b^2*c*g+13*a*b*c^2*f-6*a*c^3*e+5*b^
4*g-3*b^3*c*f+b^2*c^2*e+b*c^3*d)/(4*a*c-b^2)*_R^2+a*(19*a*b*c*g-10*a*c^2*f
-5*b^3*g+3*b^2*c*f-b*c^2*e+2*c^3*d)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),
_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.126.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^4*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

3.126.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.126.7 Maxima [F]

$$\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(gx^6+fx^4+ex^2+d)x^4}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^4*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima
")
```

output
```
1/2*((b*c^3*d - (b^2*c^2 - 2*a*c^3)*e + (b^3*c - 3*a*b*c^2)*f - (b^4 - 4*a
*b^2*c + 2*a^2*c^2)*g)*x^3 + (2*a*c^3*d - a*b*c^2*e + (a*b^2*c - 2*a^2*c^2
)*f - (a*b^3 - 3*a^2*b*c)*g)*x)/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^
5)*x^4 + (b^3*c^3 - 4*a*b*c^4)*x^2) + 1/2*integrate(-(2*a*c^3*d - a*b*c^2*
e - (b*c^3*d + (b^2*c^2 - 6*a*c^3)*e - (3*b^3*c - 13*a*b*c^2)*f + (5*b^4 -
 24*a*b^2*c + 14*a^2*c^2)*g)*x^2 + (3*a*b^2*c - 10*a^2*c^2)*f - (5*a*b^3 -
 19*a^2*b*c)*g)/(c*x^4 + b*x^2 + a), x)/(b^2*c^3 - 4*a*c^4) + 1/3*(c*g*x^3
 + 3*(c*f - 2*b*g)*x)/c^3
```

3.126.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10752 vs. $2(550) = 1100$.

Time $= 2.34$ (sec) , antiderivative size $= 10752$, normalized size of antiderivative $= 18.10$

$$\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b*c^3*d*x^3 - b^2*c^2*e*x^3 + 2*a*c^3*e*x^3 + b^3*c*f*x^3 - 3*a*b*c^2
*f*x^3 - b^4*g*x^3 + 4*a*b^2*c*g*x^3 - 2*a^2*c^2*g*x^3 + 2*a*c^3*d*x - a*b
*c^2*e*x + a*b^2*c*f*x - 2*a^2*c^2*f*x - a*b^3*g*x + 3*a^2*b*c*g*x)/((b^2*
c^3 - 4*a*c^4)*(c*x^4 + b*x^2 + a)) + 1/16*((2*b^3*c^5 - 8*a*b*c^6 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 2*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^5 - 2*(b^2 - 4*a*c)*b*c^5)*(b^2
*c^3 - 4*a*c^4)^2*d + (2*b^4*c^4 - 20*a*b^2*c^5 + 48*a^2*c^6 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 10*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 24*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 12*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*c^5 - 2*(b^2 - 4*a*c)*b^2*c^4 + 12*(b^2 - 4*a*
c)*a*c^5)*(b^2*c^3 - 4*a*c^4)^2*e - (6*b^5*c^3 - 50*a*b^3*c^4 + 104*a^2*b*
c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c +
25*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 6
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 52...
```

3.126.9 Mupad [B] (verification not implemented)

Time = 10.71 (sec) , antiderivative size = 47339, normalized size of antiderivative = 79.70

$$\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^4*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
((x^3*(b^4*g + b^2*c^2*e + 2*a^2*c^2*g - 2*a*c^3*e - b*c^3*d - b^3*c*f + 3
*a*b*c^2*f - 4*a*b^2*c*g))/(2*(4*a*c - b^2)) + (x*(2*a^2*c^2*f - 2*a*c^3*d
 + a*b^3*g + a*b*c^2*e - a*b^2*c*f - 3*a^2*b*c*g))/(2*(4*a*c - b^2)))/(a*c
^3 + c^4*x^4 + b*c^3*x^2) + x*(f/c^2 - (2*b*g)/c^3) + atan(((((2048*a^4*c^
10*d - 10240*a^5*c^9*f + 384*a^2*b^4*c^8*d - 1536*a^3*b^2*c^9*d - 192*a^2*
b^5*c^7*e + 768*a^3*b^3*c^8*e + 736*a^2*b^6*c^6*f - 4224*a^3*b^4*c^7*f + 1
0752*a^4*b^2*c^8*f - 1264*a^2*b^7*c^5*g + 7488*a^3*b^5*c^6*g - 19712*a^4*b
^3*c^7*g - 32*a*b^6*c^7*d + 16*a*b^7*c^6*e - 1024*a^4*b*c^9*e - 48*a*b^8*c
^5*f + 80*a*b^9*c^4*g + 19456*a^5*b*c^8*g)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a
*b^4*c^6 - 48*a^2*b^2*c^7)) - (x*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-
(4*a*c - b^2)^9)^(1/2) + b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a
*c - b^2)^9)^(1/2) - 768*a^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^
9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*f^2 + 26880*
a^6*b*c^8*f^2 - 80640*a^7*b*c^7*g^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 +
 512*a^3*b^3*c^9*d^2 + 288*a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a
^4*b^3*c^8*e^2 + 2077*a^2*b^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*
b^5*c^6*f^2 - 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1
/2) + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*g^2 - 35767
*a^3*b^9*c^3*g^2 + 116928*a^4*b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 21504
0*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2...
```

3.126. $\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx$

3.127 $\int \frac{x^2(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx$

3.127.1 Optimal result

Integrand size = 35, antiderivative size = 471

$$\int \frac{x^2(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \frac{gx}{c^2} - \frac{x(bc(cd+af)-ab^2g-2ac(ce-ag)+(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag))x^2)}{2c^2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2c^3d-c^2(be-6af)+3b^3g-bc(bf+13ag)+\frac{b^3cf-4bc^2(cd+2af)-3b^4g+4ac^2(ce-5ag)+b^2c(ce+19ag)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2c^3d-c^2(be-6af)+3b^3g-bc(bf+13ag)-\frac{b^3cf-4bc^2(cd+2af)-3b^4g+4ac^2(ce-5ag)+b^2c(ce+19ag)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
g*x/c^2-1/2*x*(b*c*(a*f+c*d)-a*b^2*g-2*a*c*(-a*g+c*e)+(2*c^3*d-c^2*(2*a*f+
b*e)-b^3*g+b*c*(3*a*g+b*f))*x^2)/c^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*arct
an(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(2*c^3*d-c^2*(-6*a*f+b*
e)+3*b^3*g-b*c*(13*a*g+b*f)+(b^3*c*f-4*b*c^2*(2*a*f+c*d)-3*b^4*g+4*a*c^2*(
-5*a*g+c*e)+b^2*c*(19*a*g+c*e))/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2
^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*
c+b^2)^(1/2))^(1/2))*(2*c^3*d-c^2*(-6*a*f+b*e)+3*b^3*g-b*c*(13*a*g+b*f)+(-
b^3*c*f+4*b*c^2*(2*a*f+c*d)+3*b^4*g-4*a*c^2*(-5*a*g+c*e)-b^2*c*(19*a*g+c*e
))/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))
^(1/2)
```

3.127.2 Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.22

$$\int \frac{x^2(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{4\sqrt{c}gx - \frac{2\sqrt{c}x(-b^3gx^2+b^2(-ag+cfx^2)+2c(a^2g+c^2dx^2-ac(e+fx^2))+bc(c(d-ex^2)+a(f+3gx^2)))}{(b^2-4ac)(a+bx^2+cx^4)} - \frac{\sqrt{2}\left(-3b^4g+b^2c\left(ce-\sqrt{b^2-4ac}f+\right.\right.}{}}{}$$

input
```
Integrate[(x^2*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(4*Sqrt[c]*g*x - (2*Sqrt[c]*x*(-(b^3*g*x^2) + b^2*(-(a*g) + c*f*x^2) + 2*c
*(a^2*g + c^2*d*x^2 - a*c*(e + f*x^2)) + b*c*(c*(d - e*x^2) + a*(f + 3*g*x
^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[2]*(-3*b^4*g + b^2*c*(c
*e - Sqrt[b^2 - 4*a*c]*f + 19*a*g) + 2*c^2*(c*Sqrt[b^2 - 4*a*c]*d + 2*a*c*
e + 3*a*Sqrt[b^2 - 4*a*c]*f - 10*a^2*g) + b^3*(c*f + 3*Sqrt[b^2 - 4*a*c]*g
) - b*c*(4*c^2*d + c*Sqrt[b^2 - 4*a*c]*e + 8*a*c*f + 13*a*Sqrt[b^2 - 4*a*c
]*g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a
*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(3*b^4*g - b^2*c*(c*e +
Sqrt[b^2 - 4*a*c]*f + 19*a*g) + 2*c^2*(c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e + 3
*a*Sqrt[b^2 - 4*a*c]*f + 10*a^2*g) + b^3*(-(c*f) + 3*Sqrt[b^2 - 4*a*c]*g)
+ b*c*(4*c^2*d - c*Sqrt[b^2 - 4*a*c]*e + 8*a*c*f - 13*a*Sqrt[b^2 - 4*a*c]*
g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c
)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*c^(5/2))
```

3.127.3 Rubi [A] (verified)

Time = 3.93 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2197, 25, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \; 2197$$

$$-\frac{\int -\frac{-2a\left(4a-\frac{b^2}{c}\right)gx^4-\frac{a\left(gb^3-c(bf+5ag)b+2c^3d-c^2(be-6af)\right)x^2}{c^2}+\frac{a\left(-agb^2+c(cd+af)b-2ac(ce-ag)\right)}{c^2}}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}-\frac{x\left(x^2\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)-ab^2g+bc(af+cd)-2ac(ce-ag)\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{-2a\left(4a-\frac{b^2}{c}\right)gx^4-\frac{a\left(gb^3-c(bf+5ag)b+2c^3d-c^2(be-6af)\right)x^2}{c^2}+\frac{a\left(-agb^2+c(cd+af)b-2ac(ce-ag)\right)}{c^2}}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}-\frac{x\left(x^2\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)-ab^2g+bc(af+cd)-2ac(ce-ag)\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 2205$$

$$\frac{\int \left(\frac{2a\left(b^2-4ac\right)g}{c^2}+\frac{a\left(-3agb^2+c(cd+af)b-2ac(ce-5ag)\right)-a\left(3gb^3-c(bf+13ag)b+2c^3d-c^2(be-6af)\right)x^2}{c^2\left(cx^4+bx^2+a\right)}\right)dx}{2a\left(b^2-4ac\right)}-\frac{x\left(x^2\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)-ab^2g+bc(af+cd)-2ac(ce-ag)\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{a\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2c(19ag+ce)-4bc^2(2af+cd)+4ac^2(ce-5ag)-3b^4g+b^3cf}{\sqrt{b^2-4ac}}-c^2(be-6af)-bc(13ag+bf)+3b^3g+2c^3d\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{a\arctan\left(\frac{\sqrt{2}\cdots}{\sqrt{\sqrt{b^2\cdots}}}\right)\cdots}{\cdots}}{2a\left(b^2-4ac\right)}-\frac{x\left(x^2\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)-ab^2g+bc(af+cd)-2ac(ce-ag)\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input

```
Int[(x^2*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x]
```

output

```
-1/2*(x*(b*c*(c*d + a*f) - a*b^2*g - 2*a*c*(c*e - a*g) + (2*c^3*d - c^2*(b
*e + 2*a*f) - b^3*g + b*c*(b*f + 3*a*g))*x^2))/(c^2*(b^2 - 4*a*c)*(a + b*x
^2 + c*x^4)) + ((2*a*(b^2 - 4*a*c)*g*x)/c^2 - (a*(2*c^3*d - c^2*(b*e - 6*a
*f) + 3*b^3*g - b*c*(b*f + 13*a*g) + (b^3*c*f - 4*b*c^2*(c*d + 2*a*f) - 3*
b^4*g + 4*a*c^2*(c*e - 5*a*g) + b^2*c*(c*e + 19*a*g))/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*S
qrt[b - Sqrt[b^2 - 4*a*c]]) - (a*(2*c^3*d - c^2*(b*e - 6*a*f) + 3*b^3*g -
b*c*(b*f + 13*a*g) - (b^3*c*f - 4*b*c^2*(c*d + 2*a*f) - 3*b^4*g + 4*a*c^2*
(c*e - 5*a*g) + b^2*c*(c*e + 19*a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2
 - 4*a*c]]))/(2*a*(b^2 - 4*a*c))
```

3.127. $\int \frac{x^2\left(d+ex^2+fx^4+gx^6\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

3.127.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

3.127.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.58

method	result
risch	$\frac{gx}{c^2}+\frac{\frac{\left(3abgc-2a\,c^2f-b^3g+b^2cf-b\,c^2e+2c^3d\right)x^3}{8ac-2b^2}+\frac{\left(2a^2cg-a\,b^2g+abcf-2a\,c^2e+b\,c^2d\right)x}{8ac-2b^2}}{c^2\left(c\,x^4+b\,x^2+a\right)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\left(-\frac{\left(13abgc-6a\right.}{}\right.}{}$
default	$\frac{gx}{c^2}-\frac{-\frac{\left(3abgc-2a\,c^2f-b^3g+b^2cf-b\,c^2e+2c^3d\right)x^3}{2\left(4ac-b^2\right)}-\frac{\left(2a^2cg-a\,b^2g+abcf-2a\,c^2e+b\,c^2d\right)x}{2\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{\left(13\sqrt{-4ac+b^2}\,abcg-6a\,c^2f\sqrt{-4ac+b^2}-3b^3\right.}{}\right.}{}$

3.127. $\int \frac{x^2\left(d+ex^2+fx^4+gx^6\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

input
```
int(x^2*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
g*x/c^2+(1/2*(3*a*b*c*g-2*a*c^2*f-b^3*g+b^2*c*f-b*c^2*e+2*c^3*d)/(4*a*c-b^
2)*x^3+1/2*(2*a^2*c*g-a*b^2*g+a*b*c*f-2*a*c^2*e+b*c^2*d)/(4*a*c-b^2)*x)/c^
2/(c*x^4+b*x^2+a)+1/4/c^2*sum((-(13*a*b*c*g-6*a*c^2*f-3*b^3*g+b^2*c*f+b*c^
2*e-2*c^3*d)/(4*a*c-b^2)*_R^2-(10*a^2*c*g-3*a*b^2*g+a*b*c*f-2*a*c^2*e+b*c^
2*d)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.127.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23774 vs. $2(430) = 860$.

Time = 181.70 (sec) , antiderivative size = 23774, normalized size of antiderivative = 50.48

$$\int \frac{x^2(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas
")
```

output
```
Too large to include
```

3.127.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.127.7 Maxima [F]

$$\int \frac{x^2(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(gx^6+fx^4+ex^2+d)x^2}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^2*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima
")
```

output
```
-1/2*((2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f - (b^3 - 3*a*b*c)*g)*x^3 +
(b*c^2*d - 2*a*c^2*e + a*b*c*f - (a*b^2 - 2*a^2*c)*g)*x)/(a*b^2*c^2 - 4*a^
2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) + g*x/c^2 + 1
/2*integrate((b*c^2*d - 2*a*c^2*e + a*b*c*f - (2*c^3*d - b*c^2*e - (b^2*c
- 6*a*c^2)*f + (3*b^3 - 13*a*b*c)*g)*x^2 - (3*a*b^2 - 10*a^2*c)*g)/(c*x^4
+ b*x^2 + a), x)/(b^2*c^2 - 4*a*c^3)
```

3.127.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9152 vs. 2(430) = 860.

Time = 2.04 (sec) , antiderivative size = 9152, normalized size of antiderivative = 19.43

$$\int \frac{x^2(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
g*x/c^2 - 1/2*(2*c^3*d*x^3 - b*c^2*e*x^3 + b^2*c*f*x^3 - 2*a*c^2*f*x^3 - b
^3*g*x^3 + 3*a*b*c*g*x^3 + b*c^2*d*x - 2*a*c^2*e*x + a*b*c*f*x - a*b^2*g*x
 + 2*a^2*c*g*x)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) - 1/16*(2*(2*b^2
*c^5 - 8*a*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c
^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^4 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^5 - 2*(b^2 - 4*
a*c)*c^5)*(b^2*c^2 - 4*a*c^3)^2*d - (2*b^3*c^4 - 8*a*b*c^5 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^4 - 2*(b^2 - 4*a*c)*b*c^4)*(b^2*c^2 - 4
*a*c^3)^2*e - (2*b^4*c^3 - 20*a*b^2*c^4 + 48*a^2*c^5 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 12*(b^2 - 4*a*c)*a*c^...
```

3.127.9 Mupad [B] (verification not implemented)

Time = 10.39 (sec) , antiderivative size = 36589, normalized size of antiderivative = 77.68

$$\int \frac{x^2(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^2*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
((x^3*(2*c^3*d - b^3*g - 2*a*c^2*f - b*c^2*e + b^2*c*f + 3*a*b*c*g))/(2*(4
*a*c - b^2)) + (x*(b*c^2*d - 2*a*c^2*e - a*b^2*g + 2*a^2*c*g + a*b*c*f))/(
2*(4*a*c - b^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) - atan(((((10240*a^5*c^7*g
 - 16*b^7*c^5*d - 2048*a^4*c^8*e - 768*a^2*b^3*c^7*d - 384*a^2*b^4*c^6*e +
 1536*a^3*b^2*c^7*e + 192*a^2*b^5*c^5*f - 768*a^3*b^3*c^6*f - 736*a^2*b^6*
c^4*g + 4224*a^3*b^4*c^5*g - 10752*a^4*b^2*c^6*g + 192*a*b^5*c^6*d + 1024*
a^3*b*c^8*d + 32*a*b^6*c^5*e - 16*a*b^7*c^4*f + 1024*a^4*b*c^7*f + 48*a*b^
8*c^3*g)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*(
(c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^5*d^2 - 9*a*b^13*g^2 + 768*a^4*b
*c^9*d^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 - a*c^4*e^2*(-(4*a*c - b^2)^9
)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 - 9*a*b^4*g^2*(-(4*a*c - b^2
)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2
 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2
*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c
^6*f^2 + 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 1
0656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 - 25*
a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g
 - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d
*f + 3584*a^5*b*c^8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10
*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 1...
```

3.127. $\int \frac{x^2(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx$

3.128 $\int \frac{d+ex^2+fx^4+gx^6}{(a+bx^2+cx^4)^2}\,dx$

3.128.1 Optimal result

Integrand size = 32, antiderivative size = 449

$$\int \frac{d+ex^2+fx^4+gx^6}{(a+bx^2+cx^4)^2}\,dx = \frac{x\left(c\left(b^2d-2a(cd-af)-\frac{ab(ce+ag)}{c}\right)+\left(bc(cd+af)-ab^2g-2ac(ce-ag)\right)x^2\right)}{2ac\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{\left(b(cd+af)+\frac{ab^2g}{c}-2a(ce+3ag)+\frac{b^2c(cd-af)-4ac^2(3cd+af)-ab^3g+4abc(ce+2ag)}{c\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b(cd+af)+\frac{ab^2g}{c}-2a(ce+3ag)-\frac{b^2c(cd-af)-4ac^2(3cd+af)-ab^3g+4abc(ce+2ag)}{c\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{c}\,(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
1/2*x*(c*(b^2*d-2*a*(-a*f+c*d)-a*b*(a*g+c*e)/c)+(b*c*(a*f+c*d)-a*b^2*g-2*a
*c*(-a*g+c*e))*x^2)/a/c/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*
c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*(a*f+c*d)+a*b^2*g/c-2*a*(3*a*g+c*
e)+(b^2*c*(-a*f+c*d)-4*a*c^2*(a*f+3*c*d)-a*b^3*g+4*a*b*c*(2*a*g+c*e))/c/(-
4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1
/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*(a*f+c*d
)+a*b^2*g/c-2*a*(3*a*g+c*e)+(-b^2*c*(-a*f+c*d)+4*a*c^2*(a*f+3*c*d)+a*b^3*g
-4*a*b*c*(2*a*g+c*e))/c/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/c^(1/2)
/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.128.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.14

$$\int \frac{d+ex^2+fx^4+gx^6}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{\frac{2\sqrt{c}x(b(-ace-a^2g+c^2dx^2+acfx^2)+b^2(cd-agx^2)+2ac(-c(d+ex^2)+a(f+gx^2)))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(-ab^3g+bc\left(c\sqrt{b^2-4ac}d+4ace+a\sqrt{b^2-4ac}f+8a^2g\right)\right.}{}}{}$$

input
```
Integrate[(d + e*x^2 + f*x^4 + g*x^6)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((2*Sqrt[c]*x*(b*(-(a*c*e) - a^2*g + c^2*d*x^2 + a*c*f*x^2) + b^2*(c*d - a
*g*x^2) + 2*a*c*(-(c*(d + e*x^2)) + a*(f + g*x^2))))/((b^2 - 4*a*c)*(a + b
*x^2 + c*x^4)) + (Sqrt[2]*(-(a*b^3*g) + b*c*(c*Sqrt[b^2 - 4*a*c]*d + 4*a*c
*e + a*Sqrt[b^2 - 4*a*c]*f + 8*a^2*g) + b^2*(c^2*d - a*c*f + a*Sqrt[b^2 -
4*a*c]*g) - 2*a*c*(6*c^2*d + c*Sqrt[b^2 - 4*a*c]*e + 2*a*c*f + 3*a*Sqrt[b^
2 - 4*a*c]*g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((
b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(a*b^3*g + b*c*
(c*Sqrt[b^2 - 4*a*c]*d - 4*a*c*e + a*Sqrt[b^2 - 4*a*c]*f - 8*a^2*g) + 2*a*
c*(6*c^2*d - c*Sqrt[b^2 - 4*a*c]*e + 2*a*c*f - 3*a*Sqrt[b^2 - 4*a*c]*g) +
b^2*(-(c^2*d) + a*c*f + a*Sqrt[b^2 - 4*a*c]*g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*
a*c]]))/(4*a*c^(3/2))
```

3.128.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 435, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2206, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4+gx^6}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \; 2206$$

$$\frac{x\left(x^2\left(-ab^2g+bc(af+cd)-2ac(ce-ag)\right)+c\left(-\frac{ab(ag+ce)}{c}-2a(cd-af)+b^2d\right)\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{db^2+\frac{a(ce+ag)b}{c}+\left(\frac{agb^2}{c}+(cd+af)b-2a(ce+3ag)\right)x^2-2a(3cd+af)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{db^2+\frac{a(ce+ag)b}{c}+\left(\frac{agb^2}{c}+(cd+af)b-2a(ce+3ag)\right)x^2-2a(3cd+af)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{x\left(x^2\left(-ab^2g+bc(af+cd)-2ac(ce-ag)\right)+c\left(-\frac{ab(ag+ce)}{c}-2a(cd-af)+b^2d\right)\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$\Big\downarrow$ 1480

$$\frac{\frac{1}{2}\left(\frac{ab^2g}{c}+\frac{-ab^3g+b^2c(cd-af)+4abc(2ag+ce)-4ac^2(af+3cd)}{c\sqrt{b^2-4ac}}+b(af+cd)-2a(3ag+ce)\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{ab^2g}{c}\right.}{2a\left(b^2-4ac\right)}$$

$$\frac{x\left(x^2\left(-ab^2g+bc(af+cd)-2ac(ce-ag)\right)+c\left(-\frac{ab(ag+ce)}{c}-2a(cd-af)+b^2d\right)\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$\Big\downarrow$ 218

$$\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{ab^2g}{c}+\frac{-ab^3g+b^2c(cd-af)+4abc(2ag+ce)-4ac^2(af+3cd)}{c\sqrt{b^2-4ac}}+b(af+cd)-2a(3ag+ce)\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{ab^2g}{c}-\frac{-a}{}\right.}{}}{2a\left(b^2-4ac\right)}$$

$$\frac{x\left(x^2\left(-ab^2g+bc(af+cd)-2ac(ce-ag)\right)+c\left(-\frac{ab(ag+ce)}{c}-2a(cd-af)+b^2d\right)\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input

```
Int[(d + e*x^2 + f*x^4 + g*x^6)/(a + b*x^2 + c*x^4)^2,x]
```

3.128. $\int \frac{d+ex^2+fx^4+gx^6}{(a+bx^2+cx^4)^2}\,dx$

output
```
(x*(c*(b^2*d - 2*a*(c*d - a*f) - (a*b*(c*e + a*g))/c) + (b*c*(c*d + a*f) -
 a*b^2*g - 2*a*c*(c*e - a*g))*x^2))/(2*a*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) + (((b*(c*d + a*f) + (a*b^2*g)/c - 2*a*(c*e + 3*a*g) + (b^2*c*(c*d - a
*f) - 4*a*c^2*(3*c*d + a*f) - a*b^3*g + 4*a*b*c*(c*e + 2*a*g))/(c*Sqrt[b^2
 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*(c*d + a*f) + (a*b^2*g)/c -
 2*a*(c*e + 3*a*g) - (b^2*c*(c*d - a*f) - 4*a*c^2*(3*c*d + a*f) - a*b^3*g
+ 4*a*b*c*(c*e + 2*a*g))/(c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]
]))/(2*a*(b^2 - 4*a*c))
```

3.128.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

3.128.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.60

method	result
risch	$\frac{-\frac{\left(2a^2cg-a\,b^2g+abcf-2a\,c^2e+b\,c^2d\right)x^3}{2a\left(4ac-b^2\right)c}+\frac{\left(a^2bg-2a^2cf+abce+2a\,c^2d-b^2cd\right)x}{2ac\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\frac{\left(6a^2cg-a\,b^2g-abcf+2a\right.}{4ac-b^2}\right.}{}}{}$
default	$\frac{-\frac{\left(2a^2cg-a\,b^2g+abcf-2a\,c^2e+b\,c^2d\right)x^3}{2a\left(4ac-b^2\right)c}+\frac{\left(a^2bg-2a^2cf+abce+2a\,c^2d-b^2cd\right)x}{2ac\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{\left(6a^2cg\sqrt{-4ac+b^2}-a\,b^2g\sqrt{-4ac+b^2}-\sqrt{-4ac+b^2}\,abcf+2\sqrt{}\right.}{}$

input
```
int((g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2/a*(2*a^2*c*g-a*b^2*g+a*b*c*f-2*a*c^2*e+b*c^2*d)/(4*a*c-b^2)/c*x^3+1/
2*(a^2*b*g-2*a^2*c*f+a*b*c*e+2*a*c^2*d-b^2*c*d)/a/c/(4*a*c-b^2)*x)/(c*x^4+
b*x^2+a)+1/4/a/c*sum(((6*a^2*c*g-a*b^2*g-a*b*c*f+2*a*c^2*e-b*c^2*d)/(4*a*c
-b^2)*_R^2-(a^2*b*g-2*a^2*c*f+a*b*c*e-6*a*c^2*d+b^2*c*d)/(4*a*c-b^2))/(2*_
R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.128.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19375 vs. 2(408) = 816.

Time = 151.26 (sec) , antiderivative size = 19375, normalized size of antiderivative = 43.15

$$\int \frac{d+ex^2+fx^4+gx^6}{\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.128.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d + ex^2 + fx^4 + gx^6}{(a + bx^2 + cx^4)^2} dx = \text{Timed out}$$

input
```
integrate((g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.128.7 Maxima [F]

$$\int \frac{d + ex^2 + fx^4 + gx^6}{(a + bx^2 + cx^4)^2} dx = \int \frac{gx^6 + fx^4 + ex^2 + d}{(cx^4 + bx^2 + a)^2} dx$$

input
```
integrate((g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*((b*c^2*d - 2*a*c^2*e + a*b*c*f - (a*b^2 - 2*a^2*c)*g)*x^3 - (a*b*c*e
- 2*a^2*c*f + a^2*b*g - (b^2*c - 2*a*c^2)*d)*x)/(a^2*b^2*c - 4*a^3*c^2 + (
a*b^2*c^2 - 4*a^2*c^3)*x^4 + (a*b^3*c - 4*a^2*b*c^2)*x^2) - 1/2*integrate(
-(a*b*c*e - 2*a^2*c*f + a^2*b*g + (b*c^2*d - 2*a*c^2*e + a*b*c*f + (a*b^2
- 6*a^2*c)*g)*x^2 + (b^2*c - 6*a*c^2)*d)/(c*x^4 + b*x^2 + a), x)/(a*b^2*c
- 4*a^2*c^2)
```

3.128.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8905 vs. $2(408) = 816$.

Time = 1.89 (sec) , antiderivative size = 8905, normalized size of antiderivative = 19.83

$$\int \frac{d + ex^2 + fx^4 + gx^6}{(a + bx^2 + cx^4)^2} dx = \text{Too large to display}$$

input
```
integrate((g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b*c^2*d*x^3 - 2*a*c^2*e*x^3 + a*b*c*f*x^3 - a*b^2*g*x^3 + 2*a^2*c*g*x
^3 + b^2*c*d*x - 2*a*c^2*d*x - a*b*c*e*x + 2*a^2*c*f*x - a^2*b*g*x)/((c*x^
4 + b*x^2 + a)*(a*b^2*c - 4*a^2*c^2)) + 1/16*((2*b^3*c^4 - 8*a*b*c^5 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^4 - 2*(b^2 - 4*a*c)*b*c^4)*(a
*b^2*c - 4*a^2*c^2)^2*d - 2*(2*a*b^2*c^4 - 8*a^2*c^5 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*a*c^4)*(a*b^2*c - 4*a^2
*c^2)^2*e + (2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*(a*b^2*c - 4*a^2*c^2)^2*
f + (2*a*b^4*c^2 - 20*a^2*b^2*c^3 + 48*a^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr...
```

3.128.9 Mupad [B] (verification not implemented)

Time = 11.16 (sec) , antiderivative size = 32587, normalized size of antiderivative = 72.58

$$\int \frac{d+ex^2+fx^4+gx^6}{\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4 + g*x^6)/(a + b*x^2 + c*x^4)^2,x)
```

output
```
((x*(2*a*c^2*d - b^2*c*d + a^2*b*g - 2*a^2*c*f + a*b*c*e))/(2*a*c*(4*a*c -
 b^2)) - (x^3*(b*c^2*d - 2*a*c^2*e - a*b^2*g + 2*a^2*c*g + a*b*c*f))/(2*a*
c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) - atan(((((6144*a^5*c^7*d + 2048*a^6
*c^6*f - 288*a^2*b^6*c^4*d + 1920*a^3*b^4*c^5*d - 5632*a^4*b^2*c^6*d + 16*
a^2*b^7*c^3*e - 192*a^3*b^5*c^4*e + 768*a^4*b^3*c^5*e - 32*a^3*b^6*c^3*f +
 384*a^4*b^4*c^4*f - 1536*a^5*b^2*c^5*f + 16*a^3*b^7*c^2*g - 192*a^4*b^5*c
^3*g + 768*a^5*b^3*c^4*g + 16*a*b^8*c^3*d - 1024*a^5*b*c^6*e - 1024*a^6*b*
c^5*g)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)) - (x
*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 - 9
*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^
2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4*c*g^2*(-(4*a*c - b^2)^9)
^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2
 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 + a^2*c^3*e^
2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^
9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 - a^3*b^2*g^2*(-(4*a*
c - b^2)^9)^(1/2) - a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2
*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 92
16*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e +
3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^
4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*...
```

3.128. $\int \frac{d+ex^2+fx^4+gx^6}{(a+bx^2+cx^4)^2}\,dx$

3.129 $\int \frac{d+ex^2+fx^4+gx^6}{x^2(a+bx^2+cx^4)^2}\,dx$

3.129.1 Optimal result

Integrand size = 35, antiderivative size = 460

$$\int \frac{d+ex^2+fx^4+gx^6}{x^2(a+bx^2+cx^4)^2}\,dx = -\frac{d}{a^2x} - \frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)-2a^2g\right)+\left(b^2cd-2ac(cd-af)-ab(ce+ag)\right)x^2\right)}{2a^2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(3b^2cd-2ac(5cd-af)-ab(ce+ag)+\frac{3b^3cd-4abc(4cd+af)-ab^2(ce-ag)+4a^2c(3ce+ag)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(3b^2cd-2ac(5cd-af)-ab(ce+ag)-\frac{3b^3cd-4abc(4cd+af)-ab^2(ce-ag)+4a^2c(3ce+ag)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-d/a^2/x-1/2*x*(a*(b^3*d/a-b*(b*e+3*c*d)+a*(b*f+2*c*e)-2*a^2*g)+(b^2*c*d-2
*a*c*(-a*f+c*d)-a*b*(a*g+c*e))*x^2)/a^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*a
rctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(3*b^2*c*d-2*a*c*(-a
*f+5*c*d)-a*b*(a*g+c*e)+(3*b^3*c*d-4*a*b*c*(a*f+4*c*d)-a*b^2*(-a*g+c*e)+4*
a^2*c*(a*g+3*c*e))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b
-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1
/2))^(1/2))*(3*b^2*c*d-2*a*c*(-a*f+5*c*d)-a*b*(a*g+c*e)+(-3*b^3*c*d+4*a*b*
c*(a*f+4*c*d)+a*b^2*(-a*g+c*e)-4*a^2*c*(a*g+3*c*e))/(-4*a*c+b^2)^(1/2))/a^
2/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.129.2 Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.15

$$\int \frac{d+ex^2+fx^4+gx^6}{x^2\left(a+bx^2+cx^4\right)^2}\,dx =$$

$$-\frac{\frac{4d}{x}-\frac{2x\left(-b^3d+b^2\left(ae-cdx^2\right)+ab\left(3cd-af+cex^2+agx^2\right)+2a\left(a^2g+c^2dx^2-ac\left(e+fx^2\right)\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(3b^3cd+b^2\left(3c\sqrt{b^2-4ac}d-ace+a^2g\right)+2ac\left(\right.\right.}{}}{}$$

input
```
Integrate[(d + e*x^2 + f*x^4 + g*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

output
```
-1/4*((4*d)/x - (2*x*(-(b^3*d) + b^2*(a*e - c*d*x^2) + a*b*(3*c*d - a*f +
c*e*x^2 + a*g*x^2) + 2*a*(a^2*g + c^2*d*x^2 - a*c*(e + f*x^2))))/((b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(3*b^3*c*d + b^2*(3*c*Sqrt[b^2 - 4*a
*c]*d - a*c*e + a^2*g) + 2*a*c*(-5*c*Sqrt[b^2 - 4*a*c]*d + 6*a*c*e + a*Sqr
t[b^2 - 4*a*c]*f + 2*a^2*g) - a*b*(16*c^2*d + c*Sqrt[b^2 - 4*a*c]*e + 4*a*
c*f + a*Sqrt[b^2 - 4*a*c]*g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (
Sqrt[2]*(-3*b^3*c*d + b^2*(3*c*Sqrt[b^2 - 4*a*c]*d + a*c*e - a^2*g) - 2*a*
c*(5*c*Sqrt[b^2 - 4*a*c]*d + 6*a*c*e - a*Sqrt[b^2 - 4*a*c]*f + 2*a^2*g) +
a*b*(16*c^2*d - c*Sqrt[b^2 - 4*a*c]*e + 4*a*c*f - a*Sqrt[b^2 - 4*a*c]*g))*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4
*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a^2
```

3.129.3 Rubi [A] (verified)

Time = 2.69 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2198, 25, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4+gx^6}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$$

$$\Big\downarrow \; 2198$$

$$-\frac{\int -\frac{-\left(\left(\frac{cdb^2}{a}-c(2cd+be)+a(2cf-bg)\right)x^4\right)-\left(\frac{db^3}{a}-(5cd+be)b+a(6ce-bf)+2a^2g\right)x^2+2\left(b^2-4ac\right)d}{x^2\left(cx^4+bx^2+a\right)}dx}{2a\left(b^2-4ac\right)}-\frac{x\left(a\left(-2a^2g+\frac{b^3d}{a}+a(bf+2ce)-b(be+3cd)\right)+x^2\left(-ab(ag+ce)-2ac(cd-af)+b^2cd\right)\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{-\frac{\left(cdb^2-a(ce+ag)b-2ac(cd-af)\right)x^4}{a}-\left(\frac{db^3}{a}-eb^2-5cdb-afb+6ace+2a^2g\right)x^2+2\left(b^2-4ac\right)d}{x^2\left(cx^4+bx^2+a\right)}dx}{2a\left(b^2-4ac\right)}-\frac{x\left(a\left(-2a^2g+\frac{b^3d}{a}+a(bf+2ce)-b(be+3cd)\right)+x^2\left(-ab(ag+ce)-2ac(cd-af)+b^2cd\right)\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 2195$$

$$\frac{\int \left(\frac{-3db^3+aeb^2+a(13cd+af)b-\left(3cdb^2-a(ce+ag)b-2ac(5cd-af)\right)x^2-2a^2(3ce+ag)}{a\left(cx^4+bx^2+a\right)}-\frac{2\left(4ac-b^2\right)d}{ax^2}\right)dx}{2a\left(b^2-4ac\right)}-\frac{x\left(a\left(-2a^2g+\frac{b^3d}{a}+a(bf+2ce)-b(be+3cd)\right)+x^2\left(-ab(ag+ce)-2ac(cd-af)+b^2cd\right)\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{4a^2c(ag+3ce)-ab^2(ce-ag)-4abc(af+4cd)+3b^3cd}{\sqrt{b^2-4ac}}-ab(ag+ce)-2ac(5cd-af)+3b^2cd\right)}{\sqrt{2}a\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{4a^2 \cdots}{\cdots}\right)}{\cdots}}{2a\left(b^2-4ac\right)}$$

$$-\frac{x\left(a\left(-2a^2g+\frac{b^3d}{a}+a(bf+2ce)-b(be+3cd)\right)+x^2\left(-ab(ag+ce)-2ac(cd-af)+b^2cd\right)\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input
```
Int[(d + e*x^2 + f*x^4 + g*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

output
```
-1/2*(x*(a*((b^3*d)/a - b*(3*c*d + b*e) + a*(2*c*e + b*f) - 2*a^2*g) + (b^
2*c*d - 2*a*c*(c*d - a*f) - a*b*(c*e + a*g))*x^2))/(a^2*(b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) + ((-2*(b^2 - 4*a*c)*d)/(a*x) - ((3*b^2*c*d - 2*a*c*(5*c*
d - a*f) - a*b*(c*e + a*g) + (3*b^3*c*d - 4*a*b*c*(4*c*d + a*f) - a*b^2*(c
*e - a*g) + 4*a^2*c*(3*c*e + a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[c]*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) - ((3*b^2*c*d - 2*a*c*(5*c*d - a*f) - a*b*(c*e + a*g) - (3*b^3*
c*d - 4*a*b*c*(4*c*d + a*f) - a*b^2*(c*e - a*g) + 4*a^2*c*(3*c*e + a*g))/S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*a*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*(b^2 - 4*a*c))
```

3.129. $\int \frac{d+ex^2+fx^4+gx^6}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$

3.129.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

3.129.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.20

method	result
default	$-\frac{d}{a^{2} x}+\frac{-\frac{\left(a^{2} b g-2 a^{2} c f+a b c e+2 a\, c^{2} d-b^{2} c d\right) x^{3}}{2\left(4 a c-b^{2}\right)}-\frac{\left(2 a^{3} g-a^{2} b f-2 a^{2} c e+a\, b^{2} e+3 a b c d-b^{3} d\right) x}{2\left(4 a c-b^{2}\right)}}{c\, x^{4}+b\, x^{2}+a}+\frac{2 c \left(\frac{\left(-a^{2} b g \sqrt{-4 a c+b^{2}}+2 a^{2} c f \sqrt{-4 a c+b^{2}}-a \cdots\right.}{\cdots}\right.}{\cdots}$
risch	Expression too large to display

input
```
int((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-d/a^2/x+1/a^2*((-1/2*(a^2*b*g-2*a^2*c*f+a*b*c*e+2*a*c^2*d-b^2*c*d)/(4*a*c
-b^2)*x^3-1/2*(2*a^3*g-a^2*b*f-2*a^2*c*e+a*b^2*e+3*a*b*c*d-b^3*d)/(4*a*c-b
^2)*x)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(1/8*(-a^2*b*g*(-4*a*c+b^2)^(1/2)+2
*a^2*c*f*(-4*a*c+b^2)^(1/2)-a*b*c*e*(-4*a*c+b^2)^(1/2)-10*(-4*a*c+b^2)^(1/
2)*a*c^2*d+3*(-4*a*c+b^2)^(1/2)*b^2*c*d-4*a^3*g*c-a^2*b^2*g+4*a^2*b*c*f-12
*a^2*c^2*e+a*b^2*c*e+16*a*b*c^2*d-3*b^3*c*d)/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2))-1/8*(-a^2*b*g*(-4*a*c+b^2)^(1/2)+2*a^2*c*f*(-4*a*c+b^2)^(1/2)-
a*b*c*e*(-4*a*c+b^2)^(1/2)-10*(-4*a*c+b^2)^(1/2)*a*c^2*d+3*(-4*a*c+b^2)^(1
/2)*b^2*c*d+4*a^3*g*c+a^2*b^2*g-4*a^2*b*c*f+12*a^2*c^2*e-a*b^2*c*e-16*a*b*
c^2*d+3*b^3*c*d)/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

3.129.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23991 vs. 2(418) = 836.

Time = 144.80 (sec) , antiderivative size = 23991, normalized size of antiderivative = 52.15

$$\int \frac{d+ex^2+fx^4+gx^6}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas
")
```

output
```
Too large to include
```

3.129.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4+gx^6}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((g*x**6+f*x**4+e*x**2+d)/x**2/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.129.7 Maxima [F]

$$\int \frac{d+ex^2+fx^4+gx^6}{x^2\left(a+bx^2+cx^4\right)^2}\,dx=\int \frac{gx^6+fx^4+ex^2+d}{\left(cx^4+bx^2+a\right)^2x^2}\,dx$$

input
```
integrate((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima
")
```

output
```
1/2*((a*b*c*e - 2*a^2*c*f + a^2*b*g - (3*b^2*c - 10*a*c^2)*d)*x^4 - (a^2*b
*f - 2*a^3*g + (3*b^3 - 11*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*x^2 - 2*(a*b^2
- 4*a^2*c)*d)/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (
a^3*b^2 - 4*a^4*c)*x) + 1/2*integrate((a^2*b*f - 2*a^3*g + (a*b*c*e - 2*a^
2*c*f + a^2*b*g - (3*b^2*c - 10*a*c^2)*d)*x^2 - (3*b^3 - 13*a*b*c)*d + (a*
b^2 - 6*a^2*c)*e)/(c*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c)
```

3.129.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9167 vs. 2(418) = 836.

Time = 1.75 (sec) , antiderivative size = 9167, normalized size of antiderivative = 19.93

$$\int \frac{d+ex^2+fx^4+gx^6}{x^2\left(a+bx^2+cx^4\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(3*b^2*c*d*x^4 - 10*a*c^2*d*x^4 - a*b*c*e*x^4 + 2*a^2*c*f*x^4 - a^2*b
*g*x^4 + 3*b^3*d*x^2 - 11*a*b*c*d*x^2 - a*b^2*e*x^2 + 2*a^2*c*e*x^2 + a^2*
b*f*x^2 - 2*a^3*g*x^2 + 2*a*b^2*d - 8*a^2*c*d)/((c*x^5 + b*x^3 + a*x)*(a^2
*b^2 - 4*a^3*c)) + 1/16*((6*b^4*c^3 - 44*a*b^2*c^4 + 80*a^2*c^5 - 3*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 22*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 6*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 40*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 20*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 3*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 6*(b^2 - 4*a*c)*b^2*c^3 + 20*(b^2
- 4*a*c)*a*c^4)*(a^2*b^2 - 4*a^3*c)^2*d - (2*a*b^3*c^3 - 8*a^2*b*c^4 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b
*c^3)*(a^2*b^2 - 4*a^3*c)^2*e + 2*(2*a^2*b^2*c^3 - 8*a^3*c^4 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 4*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 -...
```

3.129.9 Mupad [B] (verification not implemented)

Time = 12.15 (sec) , antiderivative size = 40860, normalized size of antiderivative = 88.83

$$\int \frac{d+ex^2+fx^4+gx^6}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4 + g*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x)
```

output
```
atan(((((213*a*b^11*c^2*d^2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c - b^2)^9)^(1/
2) - 9*b^13*c*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*
e^2 + 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5
*f^2 + a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b
^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3
*c^6*d^2 + 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 -
288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 - 9*a^3*
c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^
2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^
8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*
a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3
584*a^8*b*c^5*e*g - 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*
g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 2
2400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^
4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*
f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 22
40*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*
b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c
^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g
+ 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e - 51*a*b^2*c^2*d^2*(-(4*a*c - b^...
```

3.129. $\int \frac{d+ex^2+fx^4+gx^6}{x^2(a+bx^2+cx^4)^2}\,dx$

3.130 $\int \frac{d+ex^2+fx^4+gx^6}{x^4(a+bx^2+cx^4)^2}\,dx$

3.130.1 Optimal result

Integrand size = 35, antiderivative size = 542

$$\int \frac{d+ex^2+fx^4+gx^6}{x^4(a+bx^2+cx^4)^2}\,dx = -\frac{d}{3a^2x^3}+\frac{2bd-ae}{a^3x}$$
$$+\frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-a(2cf+bg)\right)+c(b^3d-ab^2e-ab(3cd-af)+2a^2(ce-a$$
$$\overline{2a^3(b^2-4ac)(a+bx^2+cx^4)}$$
$$+\frac{\sqrt{c}\left(5b^3d-3ab^2e-ab(19cd-af)+2a^2(5ce-ag)+\frac{5b^4d-3ab^3e+4a^2c(7cd-3af)-ab^2(29cd-af)+4a^2b(4ce+ag)}{\sqrt{b^2-4ac}}\right)\text{ar}}{2\sqrt{2}a^3(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{c}\left(5b^3d-3ab^2e-ab(19cd-af)+2a^2(5ce-ag)-\frac{5b^4d-3ab^3e+4a^2c(7cd-3af)-ab^2(29cd-af)+4a^2b(4ce+ag)}{\sqrt{b^2-4ac}}\right)\text{ar}}{2\sqrt{2}a^3(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-1/3*d/a^2/x^3+(-a*e+2*b*d)/a^3/x+1/2*x*(a^2*(b^4*d/a^2+2*c^2*d+3*b*c*e-b^
2*(b*e+4*c*d)/a+b^2*f-a*(b*g+2*c*f))+c*(b^3*d-a*b^2*e-a*b*(-a*f+3*c*d)+2*a
^2*(-a*g+c*e))*x^2)/a^3/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*
c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(5*b^3*d-3*a*b^2*e-a*b*(-a*f
+19*c*d)+2*a^2*(-a*g+5*c*e)+(5*b^4*d-3*a*b^3*e+4*a^2*c*(-3*a*f+7*c*d)-a*b^
2*(-a*f+29*c*d)+4*a^2*b*(a*g+4*c*e))/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)*
2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a
*c+b^2)^(1/2))^(1/2))*c^(1/2)*(5*b^3*d-3*a*b^2*e-a*b*(-a*f+19*c*d)+2*a^2*(
-a*g+5*c*e)+(-5*b^4*d+3*a*b^3*e-4*a^2*c*(-3*a*f+7*c*d)+a*b^2*(-a*f+29*c*d)
-4*a^2*b*(a*g+4*c*e))/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)*2^(1/2)/(b+(-4*
a*c+b^2)^(1/2))^(1/2)
```

3.130.2 Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 612, normalized size of antiderivative = 1.13

$$\int \frac{d+ex^2+fx^4+gx^6}{x^4\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{-\frac{4ad}{x^3}+\frac{24bd-12ae}{x}+\frac{6x\left(b^4d+b^3\left(-ae+cdx^2\right)+ab^2\left(af-c\left(4d+ex^2\right)\right)+ab\left(-a^2g-3c^2dx^2+ac\left(3e+fx^2\right)\right)+2a^2c\left(c\left(d+ex^2\right)-a\left(f+gx^2\right)\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+}{}$$

input
```
Integrate[(d + e*x^2 + f*x^4 + g*x^6)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

output
```
((-4*a*d)/x^3 + (24*b*d - 12*a*e)/x + (6*x*(b^4*d + b^3*(-(a*e) + c*d*x^2)
 + a*b^2*(a*f - c*(4*d + e*x^2)) + a*b*(-(a^2*g) - 3*c^2*d*x^2 + a*c*(3*e
+ f*x^2)) + 2*a^2*c*(c*(d + e*x^2) - a*(f + g*x^2))))/((b^2 - 4*a*c)*(a +
b*x^2 + c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(5*b^4*d + b^3*(5*Sqrt[b^2 - 4*a*c]*d
 - 3*a*e) + a*b^2*(-29*c*d - 3*Sqrt[b^2 - 4*a*c]*e + a*f) + a*b*(-19*c*Sqr
t[b^2 - 4*a*c]*d + 16*a*c*e + a*Sqrt[b^2 - 4*a*c]*f + 4*a^2*g) - 2*a^2*(-1
4*c^2*d - 5*c*Sqrt[b^2 - 4*a*c]*e + 6*a*c*f + a*Sqrt[b^2 - 4*a*c]*g))*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*
Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (3*Sqrt[2]*Sqrt[c]*(5*b^4*d - b^3*(5*Sqrt[b
^2 - 4*a*c]*d + 3*a*e) + a*b^2*(-29*c*d + 3*Sqrt[b^2 - 4*a*c]*e + a*f) + a
*b*(19*c*Sqrt[b^2 - 4*a*c]*d + 16*a*c*e - a*Sqrt[b^2 - 4*a*c]*f + 4*a^2*g)
 + 2*a^2*(14*c^2*d - 5*c*Sqrt[b^2 - 4*a*c]*e - 6*a*c*f + a*Sqrt[b^2 - 4*a*
c]*g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*
a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3)
```

3.130.3 Rubi [A] (verified)

Time = 3.95 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2198, 25, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4+gx^6}{x^4\left(a+bx^2+cx^4\right)^2}\,dx$$

$$\Big\downarrow \; 2198$$

$$\frac{x\left(a^2\left(\frac{b^4d}{a^2}-\frac{b^2(be+4cd)}{a}-a(bg+2cf)+b^2f+3bce+2c^2d\right)+cx^2\left(2a^2(ce-ag)-ab^2e-ab(3cd-af)+b^3d\right)\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{c\left(\frac{db^3}{a^2}-\frac{(3cd+be)b}{a}+fb+2ce-2ag\right)x^6+\frac{\left(db^4-aeb^3-a(6cd-af)b^2+a^2(5ce+ag)b+6a^2c(cd-af)\right)x^4}{a^2}-\frac{2\left(b^2-4ac\right)(bd-ae)x^2}{a}+2\left(b^2-4ac\right)d}{x^4\left(cx^4+bx^2+a\right)}dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{c\left(\frac{db^3}{a^2}-\frac{(3cd+be)b}{a}+fb+2ce-2ag\right)x^6+\frac{\left(db^4-aeb^3-a(6cd-af)b^2+a^2(5ce+ag)b+6a^2c(cd-af)\right)x^4}{a^2}-\frac{2\left(b^2-4ac\right)(bd-ae)x^2}{a}+2\left(b^2-4ac\right)d}{x^4\left(cx^4+bx^2+a\right)}dx}{2a\left(b^2-4ac\right)}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}-\frac{b^2(be+4cd)}{a}-a(bg+2cf)+b^2f+3bce+2c^2d\right)+cx^2\left(2a^2(ce-ag)-ab^2e-ab(3cd-af)+b^3d\right)\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \ 2195$$

$$\frac{\int \left(-\frac{2\left(4ac-b^2\right)d}{ax^4}+\frac{5db^4-3aeb^3-a(24cd-af)b^2+a^2(13ce+ag)b+c\left(5db^3-3aeb^2-a(19cd-af)b+2a^2(5ce-ag)\right)x^2+2a^2c(7cd-3af)}{a^2\left(cx^4+bx^2+a\right)}-\frac{2(4ac-\cdots}{\cdots}\right.}{2a\left(b^2-4ac\right)}\cdots+\frac{x\left(a^2\left(\frac{b^4d}{a^2}-\frac{b^2(be+4cd)}{a}-a(bg+2cf)+b^2f+3bce+2c^2d\right)+cx^2\left(2a^2(ce-ag)-ab^2e-ab(3cd-af)+b^3d\right)\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{4a^2b(ag+4ce)+4a^2c(7cd-3af)-3ab^3e-ab^2(29cd-af)+5b^4d}{\sqrt{b^2-4ac}}+2a^2(5ce-ag)-3ab^2e-ab(19cd-af)+5b^3d\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\arctan\left(\cdots\right.}{\cdots}}{2\cdots}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}-\frac{b^2(be+4cd)}{a}-a(bg+2cf)+b^2f+3bce+2c^2d\right)+cx^2\left(2a^2(ce-ag)-ab^2e-ab(3cd-af)+b^3d\right)\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input
```
Int[(d + e*x^2 + f*x^4 + g*x^6)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

3.130. $\int \frac{d+ex^2+fx^4+gx^6}{x^4\left(a+bx^2+cx^4\right)^2}\,dx$

output
```
(x*(a^2*((b^4*d)/a^2 + 2*c^2*d + 3*b*c*e - (b^2*(4*c*d + b*e))/a + b^2*f -
 a*(2*c*f + b*g)) + c*(b^3*d - a*b^2*e - a*b*(3*c*d - a*f) + 2*a^2*(c*e -
a*g))*x^2))/(2*a^3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((-2*(b^2 - 4*a*c)
*d)/(3*a*x^3) + (2*(b^2 - 4*a*c)*(2*b*d - a*e))/(a^2*x) + (Sqrt[c]*(5*b^3*
d - 3*a*b^2*e - a*b*(19*c*d - a*f) + 2*a^2*(5*c*e - a*g) + (5*b^4*d - 3*a*
b^3*e + 4*a^2*c*(7*c*d - 3*a*f) - a*b^2*(29*c*d - a*f) + 4*a^2*b*(4*c*e +
a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*
a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(5*b^3*d - 3*
a*b^2*e - a*b*(19*c*d - a*f) + 2*a^2*(5*c*e - a*g) - (5*b^4*d - 3*a*b^3*e
+ 4*a^2*c*(7*c*d - 3*a*f) - a*b^2*(29*c*d - a*f) + 4*a^2*b*(4*c*e + a*g))/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*(b^2 - 4*a*c))
```

3.130.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

3.130.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.16

method	result
default	$-\frac{d}{3a^{2}x^{3}}-\frac{ae-2bd}{a^{3}x}+\frac{\frac{c\left(2a^{3}g-a^{2}bf-2a^{2}ce+a\,b^{2}e+3abcd-b^{3}d\right)x^{3}}{8ac-2b^{2}}+\frac{\left(a^{3}bg+2a^{3}cf-a^{2}b^{2}f-3a^{2}bce-2a^{2}c^{2}d+a\,b^{3}e+4a\,b^{2}cd-d\,b^{4}\right)x}{8ac-2b^{2}}}{c\,x^{4}+b\,x^{2}+a}+ \frac{2c\left(\right.}{}$
risch	Expression too large to display

input
```
int((g*x^6+f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*d/a^2/x^3-(a*e-2*b*d)/a^3/x+1/a^3*((1/2*c*(2*a^3*g-a^2*b*f-2*a^2*c*e+
a*b^2*e+3*a*b*c*d-b^3*d)/(4*a*c-b^2)*x^3+1/2*(a^3*b*g+2*a^3*c*f-a^2*b^2*f-
3*a^2*b*c*e-2*a^2*c^2*d+a*b^3*e+4*a*b^2*c*d-b^4*d)/(4*a*c-b^2)*x)/(c*x^4+b
*x^2+a)+2/(4*a*c-b^2)*c*(1/8*(2*a^3*g*(-4*a*c+b^2)^(1/2)-a^2*b*f*(-4*a*c+b
^2)^(1/2)-10*a^2*c*e*(-4*a*c+b^2)^(1/2)+3*a*b^2*e*(-4*a*c+b^2)^(1/2)+19*a*
b*c*d*(-4*a*c+b^2)^(1/2)-5*b^3*d*(-4*a*c+b^2)^(1/2)+4*a^3*b*g-12*a^3*c*f+a
^2*b^2*f+16*a^2*b*c*e+28*a^2*c^2*d-3*a*b^3*e-29*a*b^2*c*d+5*d*b^4)/(-4*a*c
+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(2*a^3*g*(-4*a*c+b^2)^(1/2)-a^2*b*f*(-
4*a*c+b^2)^(1/2)-10*a^2*c*e*(-4*a*c+b^2)^(1/2)+3*a*b^2*e*(-4*a*c+b^2)^(1/2
)+19*a*b*c*d*(-4*a*c+b^2)^(1/2)-5*b^3*d*(-4*a*c+b^2)^(1/2)-4*a^3*b*g+12*a^
3*c*f-a^2*b^2*f-16*a^2*b*c*e-28*a^2*c^2*d+3*a*b^3*e+29*a*b^2*c*d-5*d*b^4)/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

3.130.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33432 vs. 2(498) = 996.

Time = 297.37 (sec) , antiderivative size = 33432, normalized size of antiderivative = 61.68

$$\int \frac{d+ex^2+fx^4+gx^6}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((g*x^6+f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="fricas
")
```

output
```
Too large to include
```

3.130.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4+gx^6}{x^4 (a+bx^2+cx^4)^2} \, dx = \text{Timed out}$$

input
```
integrate((g*x**6+f*x**4+e*x**2+d)/x**4/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.130.7 Maxima [F]

$$\int \frac{d+ex^2+fx^4+gx^6}{x^4 (a+bx^2+cx^4)^2} \, dx = \int \frac{gx^6+fx^4+ex^2+d}{(cx^4+bx^2+a)^2 x^4} \, dx$$

input
```
integrate((g*x^6+f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="maxima
")
```

output
```
1/6*(3*(a^2*b*c*f - 2*a^3*c*g + (5*b^3*c - 19*a*b*c^2)*d - (3*a*b^2*c - 10
*a^2*c^2)*e)*x^6 - (3*a^3*b*g - (15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d + 3*(
3*a*b^3 - 11*a^2*b*c)*e - 3*(a^2*b^2 - 2*a^3*c)*f)*x^4 + 2*(5*(a*b^3 - 4*a
^2*b*c)*d - 3*(a^2*b^2 - 4*a^3*c)*e)*x^2 - 2*(a^2*b^2 - 4*a^3*c)*d)/((a^3*
b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x
^3) - 1/2*integrate(-(a^3*b*g + (a^2*b*c*f - 2*a^3*c*g + (5*b^3*c - 19*a*b
*c^2)*d - (3*a*b^2*c - 10*a^2*c^2)*e)*x^2 + (5*b^4 - 24*a*b^2*c + 14*a^2*c
^2)*d - (3*a*b^3 - 13*a^2*b*c)*e + (a^2*b^2 - 6*a^3*c)*f)/(c*x^4 + b*x^2 +
 a), x)/(a^3*b^2 - 4*a^4*c)
```

3.130.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10411 vs. 2(498) = 996.

Time = 1.80 (sec) , antiderivative size = 10411, normalized size of antiderivative = 19.21

$$\int \frac{d+ex^2+fx^4+gx^6}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((g*x^6+f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b^3*c*d*x^3 - 3*a*b*c^2*d*x^3 - a*b^2*c*e*x^3 + 2*a^2*c^2*e*x^3 + a^2
*b*c*f*x^3 - 2*a^3*c*g*x^3 + b^4*d*x - 4*a*b^2*c*d*x + 2*a^2*c^2*d*x - a*b
^3*e*x + 3*a^2*b*c*e*x + a^2*b^2*f*x - 2*a^3*c*f*x - a^3*b*g*x)/((a^3*b^2
- 4*a^4*c)*(c*x^4 + b*x^2 + a)) + 1/16*((10*b^5*c^2 - 78*a*b^3*c^3 + 152*a
^2*b*c^4 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5
 + 39*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c +
10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 76*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 38*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 5*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 19*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 10*(b^2 - 4*a*
c)*b^3*c^2 + 38*(b^2 - 4*a*c)*a*b*c^3)*(a^3*b^2 - 4*a^4*c)^2*d - (6*a*b^4*
c^2 - 44*a^2*b^2*c^3 + 80*a^3*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^3*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*c^3 - 6*(b^2 - 4*a*c)*a*b^2*c^2 + 20*(b^2 - 4*a*c)*a^2*c^3)*(...
```

3.130.9 Mupad [B] (verification not implemented)

Time = 12.57 (sec) , antiderivative size = 51386, normalized size of antiderivative = 94.81

$$\int \frac{d+ex^2+fx^4+gx^6}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4 + g*x^6)/(x^4*(a + b*x^2 + c*x^4)^2),x)
```

output
```
atan((((-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/
2) + a^4*b^11*f^2 + a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 80640
*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f
^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^10*
b*c^4*g^2 - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2
+ 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2
 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)
^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c
^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25
*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5
*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^2
 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d*
g - 6*a^3*b^12*e*f - 6*a^4*b^11*e*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f
 + 2*a^5*b^10*f*g + 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1
/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f - 168*
a^4*b^10*c*d*g + 152*a^4*b^10*c*e*f + 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*
g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)
^(1/2) - 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) -
 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*
a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161...
```

3.130. $\int \frac{d+ex^2+fx^4+gx^6}{x^4(a+bx^2+cx^4)^2}\,dx$

3.131 $\int x^2(a+bx^2+cx^4)^p\,(3a+b(5+2p)x^2+c(7+4p)x$

3.131.1 Optimal result

Integrand size = 42, antiderivative size = 20

$$\int x^2\left(a+bx^2+cx^4\right)^p\left(3a+b(5+2p)x^2+c(7+4p)x^4\right)\,dx=x^3\left(a+bx^2+cx^4\right)^{1+p}$$

output
```
x^3*(c*x^4+b*x^2+a)^(p+1)
```

3.131.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^2\left(a+bx^2+cx^4\right)^p\left(3a+b(5+2p)x^2+c(7+4p)x^4\right)\,dx=x^3\left(a+bx^2+cx^4\right)^{1+p}$$

input
```
Integrate[x^2*(a + b*x^2 + c*x^4)^p*(3*a + b*(5 + 2*p)*x^2 + c*(7 + 4*p)*x
^4),x]
```

output
```
x^3*(a + b*x^2 + c*x^4)^(1 + p)
```

3.131.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a + bx^2 + cx^4)^p \left(3a + b(2p+5)x^2 + c(4p+7)x^4\right) dx$$

$$\Big\downarrow \text{2021}$$

$$x^3 (a + bx^2 + cx^4)^{p+1}$$

input
```
Int[x^2*(a + b*x^2 + c*x^4)^p*(3*a + b*(5 + 2*p)*x^2 + c*(7 + 4*p)*x^4),x]
```

output
```
x^3*(a + b*x^2 + c*x^4)^(1 + p)
```

3.131.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.131.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
gosper	$x^3 (c\,x^4+b\,x^2+a)^{1+p}$	21
risch	$(c\,x^4+b\,x^2+a)^p\,x^3 (c\,x^4+b\,x^2+a)$	31
norman	$a\,x^3 e^{p\ln(c\,x^4+b\,x^2+a)}+b\,x^5 e^{p\ln(c\,x^4+b\,x^2+a)}+c\,x^7 e^{p\ln(c\,x^4+b\,x^2+a)}$	65
parallelrisch	$\frac{x^7 (c\,x^4+b\,x^2+a)^p ac+ab(c\,x^4+b\,x^2+a)^p x^5+a^2 (c\,x^4+b\,x^2+a)^p x^3}{a}$	67

input
```
int(x^2*(c*x^4+b*x^2+a)^p*(3*a+b*(5+2*p)*x^2+c*(7+4*p)*x^4),x,method=_RETU
RNVERBOSE)
```

output
```
x^3*(c*x^4+b*x^2+a)^(1+p)
```

3.131.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int x^2(a+bx^2+cx^4)^p\left(3a+b(5+2p)x^2+c(7+4p)x^4\right)\,dx$$
$$=\left(cx^7+bx^5+ax^3\right)\left(cx^4+bx^2+a\right)^p$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)^p*(3*a+b*(5+2*p)*x^2+c*(7+4*p)*x^4),x, algor
ithm="fricas")
```

output
```
(c*x^7 + b*x^5 + a*x^3)*(c*x^4 + b*x^2 + a)^p
```

3.131.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(17) = 34.

Time = 158.45 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.70

$$\int x^2(a+bx^2+cx^4)^p\left(3a+b(5+2p)x^2+c(7+4p)x^4\right)\,dx$$
$$=ax^3\left(a+bx^2+cx^4\right)^p+bx^5\left(a+bx^2+cx^4\right)^p+cx^7\left(a+bx^2+cx^4\right)^p$$

input
```
integrate(x**2*(c*x**4+b*x**2+a)**p*(3*a+b*(5+2*p)*x**2+c*(7+4*p)*x**4),x)
```

output
```
a*x**3*(a + b*x**2 + c*x**4)**p + b*x**5*(a + b*x**2 + c*x**4)**p + c*x**7
*(a + b*x**2 + c*x**4)**p
```

3.131.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int x^2(a+bx^2+cx^4)^p\left(3a+b(5+2p)x^2+c(7+4p)x^4\right)\,dx = (cx^7+bx^5+ax^3)(cx^4+bx^2+a)^p$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)^p*(3*a+b*(5+2*p)*x^2+c*(7+4*p)*x^4),x, algor
ithm="maxima")
```

output
```
(c*x^7 + b*x^5 + a*x^3)*(c*x^4 + b*x^2 + a)^p
```

3.131.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(20) = 40.

Time = 0.35 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.90

$$\int x^2(a+bx^2+cx^4)^p\left(3a+b(5+2p)x^2+c(7+4p)x^4\right)\,dx = (cx^4+bx^2+a)^pcx^7+(cx^4+bx^2+a)^pbx^5+(cx^4+bx^2+a)^pax^3$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)^p*(3*a+b*(5+2*p)*x^2+c*(7+4*p)*x^4),x, algor
ithm="giac")
```

output
```
(c*x^4 + b*x^2 + a)^p*c*x^7 + (c*x^4 + b*x^2 + a)^p*b*x^5 + (c*x^4 + b*x^2
 + a)^p*a*x^3
```

3.131.9 Mupad [B] (verification not implemented)

Time = 8.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int x^2(a+bx^2+cx^4)^p\left(3a+b(5+2p)x^2+c(7+4p)x^4\right)\,dx = (c\,x^7+b\,x^5+a\,x^3)\,(c\,x^4+b\,x^2+a)^p$$

input
```
int(x^2*(3*a + b*x^2*(2*p + 5) + c*x^4*(4*p + 7))*(a + b*x^2 + c*x^4)^p,x)
```

output
```
(a*x^3 + b*x^5 + c*x^7)*(a + b*x^2 + c*x^4)^p
```

3.132 $\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

3.132.1 Optimal result

Integrand size = 35, antiderivative size = 210

$$\begin{aligned}\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx &= -\frac{d^4(cd^4+bd^2e^2+ae^4)\sqrt{d-ex}\sqrt{d+ex}}{e^{10}} \\ &+ \frac{d^2(4cd^4+3bd^2e^2+2ae^4)(d-ex)^{3/2}(d+ex)^{3/2}}{3e^{10}} \\ &- \frac{(6cd^4+3bd^2e^2+ae^4)(d-ex)^{5/2}(d+ex)^{5/2}}{5e^{10}} \\ &+ \frac{(4cd^2+be^2)(d-ex)^{7/2}(d+ex)^{7/2}}{7e^{10}} - \frac{c(d-ex)^{9/2}(d+ex)^{9/2}}{9e^{10}}\end{aligned}$$

output
```
1/3*d^2*(2*a*e^4+3*b*d^2*e^2+4*c*d^4)*(-e*x+d)^(3/2)*(e*x+d)^(3/2)/e^10-1/
5*(a*e^4+3*b*d^2*e^2+6*c*d^4)*(-e*x+d)^(5/2)*(e*x+d)^(5/2)/e^10+1/7*(b*e^2
+4*c*d^2)*(-e*x+d)^(7/2)*(e*x+d)^(7/2)/e^10-1/9*c*(-e*x+d)^(9/2)*(e*x+d)^(
9/2)/e^10-d^4*(a*e^4+b*d^2*e^2+c*d^4)*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^10
```

3.132.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.71

$$\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-ex}\sqrt{d+ex}(21ae^4(8d^4+4d^2e^2x^2+3e^4x^4)+9b(16d^6e^2+8d^4e^4x^2+6d^2e^6x^4+5e^8x^6)+c(128d^8+\cdots}{315e^{10}}$$

input
```
Integrate[(x^5*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
-1/315*(Sqrt[d - e*x]*Sqrt[d + e*x]*(21*a*e^4*(8*d^4 + 4*d^2*e^2*x^2 + 3*e
^4*x^4) + 9*b*(16*d^6*e^2 + 8*d^4*e^4*x^2 + 6*d^2*e^6*x^4 + 5*e^8*x^6) + c
*(128*d^8 + 64*d^6*e^2*x^2 + 48*d^4*e^4*x^4 + 40*d^2*e^6*x^6 + 35*e^8*x^8)
))/e^10
```

3.132.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1905, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\left(a+bx^2+cx^4\right)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \; 1905$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{x^5\left(cx^4+bx^2+a\right)}{\sqrt{d^2-e^2x^2}}dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \; 1578$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{x^4\left(cx^4+bx^2+a\right)}{\sqrt{d^2-e^2x^2}}dx^2}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \; 1195$$

$$\frac{\sqrt{d^2-e^2x^2}\int\left(\frac{c(d^2-e^2x^2)^{7/2}}{e^8}+\frac{(-4cd^2-be^2)(d^2-e^2x^2)^{5/2}}{e^8}+\frac{(6cd^4+3be^2d^2+ae^4)(d^2-e^2x^2)^{3/2}}{e^8}+\frac{(-4cd^6-3be^2d^4-2ae^4d^2)\sqrt{d^2-e^2}}{e^8}\right.}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$\downarrow$ 2009

$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{2(d^2-e^2x^2)^{5/2}(ae^4+3bd^2e^2+6cd^4)}{5e^{10}}+\frac{2d^2(d^2-e^2x^2)^{3/2}(2ae^4+3bd^2e^2+4cd^4)}{3e^{10}}-\frac{2d^4\sqrt{d^2-e^2x^2}(ae^4+bd^2e^2+cd^4)}{e^{10}}+\frac{2(d^2}{}\right.}{2\sqrt{d-ex}\sqrt{d+ex}}$$

input

```
Int[(x^5*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output

```
(Sqrt[d^2 - e^2*x^2]*((-2*d^4*(c*d^4 + b*d^2*e^2 + a*e^4)*Sqrt[d^2 - e^2*x
^2])/e^10 + (2*d^2*(4*c*d^4 + 3*b*d^2*e^2 + 2*a*e^4)*(d^2 - e^2*x^2)^(3/2)
)/(3*e^10) - (2*(6*c*d^4 + 3*b*d^2*e^2 + a*e^4)*(d^2 - e^2*x^2)^(5/2))/(5*
e^10) + (2*(4*c*d^2 + b*e^2)*(d^2 - e^2*x^2)^(7/2))/(7*e^10) - (2*c*(d^2 -
 e^2*x^2)^(9/2))/(9*e^10)))/(2*Sqrt[d - e*x]*Sqrt[d + e*x])
```

3.132.3.1 Defintions of rubi rules used

rule 1195

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 1905

```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.132.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.69

method	result
gosper	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(35c\,x^8e^8+45b\,e^8x^6+40c\,d^2e^6x^6+63a\,e^8x^4+54b\,d^2e^6x^4+48c\,d^4e^4x^4+84a\,d^2e^6x^2+72b\,d^4e^4x^2+64c\,d^6e^2x^2+168}{315e^{10}}$
default	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(35c\,x^8e^8+45b\,e^8x^6+40c\,d^2e^6x^6+63a\,e^8x^4+54b\,d^2e^6x^4+48c\,d^4e^4x^4+84a\,d^2e^6x^2+72b\,d^4e^4x^2+64c\,d^6e^2x^2+168}{315e^{10}}$
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(35c\,x^8e^8+45b\,e^8x^6+40c\,d^2e^6x^6+63a\,e^8x^4+54b\,d^2e^6x^4+48c\,d^4e^4x^4+84a\,d^2e^6x^2+72b\,d^4e^4x^2+64c\,d^6e^2x^2+168}{315e^{10}}$

input
```
int(x^5*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/315*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(35*c*e^8*x^8+45*b*e^8*x^6+40*c*d^2*e^
6*x^6+63*a*e^8*x^4+54*b*d^2*e^6*x^4+48*c*d^4*e^4*x^4+84*a*d^2*e^6*x^2+72*b
*d^4*e^4*x^2+64*c*d^6*e^2*x^2+168*a*d^4*e^4+144*b*d^6*e^2+128*c*d^8)/e^10
```

3.132.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.66

$$\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(35\,ce^8x^8+128\,cd^8+144\,bd^6e^2+168\,ad^4e^4+5\,(8\,cd^2e^6+9\,be^8)x^6+3\,(16\,cd^4e^4+18\,bd^2e^6+21\,ae^8)x^4}{315\,e^{10}}$$

input
```
integrate(x^5*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/315*(35*c*e^8*x^8 + 128*c*d^8 + 144*b*d^6*e^2 + 168*a*d^4*e^4 + 5*(8*c*
d^2*e^6 + 9*b*e^8)*x^6 + 3*(16*c*d^4*e^4 + 18*b*d^2*e^6 + 21*a*e^8)*x^4 +
4*(16*c*d^6*e^2 + 18*b*d^4*e^4 + 21*a*d^2*e^6)*x^2)*sqrt(e*x + d)*sqrt(-e*
x + d)/e^10
```

3.132.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 19.88 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.75

$$\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{iad^5 G_{6,6}^{6,2}\left(\begin{matrix} -\frac{9}{4},-\frac{7}{4} & -2,-2,-\frac{3}{2},1 \\ -\frac{5}{2},-\frac{9}{4},-2,-\frac{7}{4},-\frac{3}{2},0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^6} - \frac{ad^5 G_{6,6}^{2,6}\left(\begin{matrix} -3,-\frac{11}{4},-\frac{5}{2},-\frac{9}{4},-2,1 & \\ -\frac{11}{4},-\frac{9}{4} & -3,-\frac{5}{2},-\frac{5}{2},0 \end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^6} - \frac{ibd^7 G_{6,6}^{6,2}\left(\begin{matrix} -\frac{13}{4},-\frac{11}{4} & -3,-3,-\frac{5}{2},1 \\ -\frac{7}{2},-\frac{13}{4},-3,-\frac{11}{4},-\frac{5}{2},0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^8} - \frac{bd^7 G_{6,6}^{2,6}\left(\begin{matrix} -4,-\frac{15}{4},-\frac{7}{2},-\frac{13}{4},-3,1 & \\ -\frac{15}{4},-\frac{13}{4} & -4,-\frac{7}{2},-\frac{7}{2},0 \end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^8} - \frac{icd^9 G_{6,6}^{6,2}\left(\begin{matrix} -\frac{17}{4},-\frac{15}{4} & -4,-4,-\frac{7}{2},1 \\ -\frac{9}{2},-\frac{17}{4},-4,-\frac{15}{4},-\frac{7}{2},0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^{10}} - \frac{cd^9 G_{6,6}^{2,6}\left(\begin{matrix} -5,-\frac{19}{4},-\frac{9}{2},-\frac{17}{4},-4,1 & \\ -\frac{19}{4},-\frac{17}{4} & -5,-\frac{9}{2},-\frac{9}{2},0 \end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^{10}}$$

input
```
integrate(x**5*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
-I*a*d**5*meijerg(((-9/4, -7/4), (-2, -2, -3/2, 1)), ((-5/2, -9/4, -2, -7/
4, -3/2, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**6) - a*d**5*meijerg(((
-3, -11/4, -5/2, -9/4, -2, 1), ()), ((-11/4, -9/4), (-3, -5/2, -5/2, 0)),
d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**6) - I*b*d**7*meijerg
(((-13/4, -11/4), (-3, -3, -5/2, 1)), ((-7/2, -13/4, -3, -11/4, -5/2, 0),
()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**8) - b*d**7*meijerg(((-4, -15/4, -7
/2, -13/4, -3, 1), ()), ((-15/4, -13/4), (-4, -7/2, -7/2, 0)), d**2*exp_po
lar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**8) - I*c*d**9*meijerg(((-17/4, -
15/4), (-4, -4, -7/2, 1)), ((-9/2, -17/4, -4, -15/4, -7/2, 0), ()), d**2/(
e**2*x**2))/(4*pi**(3/2)*e**10) - c*d**9*meijerg(((-5, -19/4, -9/2, -17/4,
 -4, 1), ()), ((-19/4, -17/4), (-5, -9/2, -9/2, 0)), d**2*exp_polar(-2*I*p
i)/(e**2*x**2))/(4*pi**(3/2)*e**10)
```

3.132.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.40

$$\begin{aligned}\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = &-\frac{\sqrt{-e^2x^2+d^2}cx^8}{9\,e^2}-\frac{8\sqrt{-e^2x^2+d^2}cd^2x^6}{63\,e^4}\\&-\frac{\sqrt{-e^2x^2+d^2}bx^6}{7\,e^2}-\frac{16\sqrt{-e^2x^2+d^2}cd^4x^4}{105\,e^6}\\&-\frac{6\sqrt{-e^2x^2+d^2}bd^2x^4}{35\,e^4}-\frac{\sqrt{-e^2x^2+d^2}ax^4}{5\,e^2}\\&-\frac{64\sqrt{-e^2x^2+d^2}cd^6x^2}{315\,e^8}-\frac{8\sqrt{-e^2x^2+d^2}bd^4x^2}{35\,e^6}\\&-\frac{4\sqrt{-e^2x^2+d^2}ad^2x^2}{15\,e^4}-\frac{128\sqrt{-e^2x^2+d^2}cd^8}{315\,e^{10}}\\&-\frac{16\sqrt{-e^2x^2+d^2}bd^6}{35\,e^8}-\frac{8\sqrt{-e^2x^2+d^2}ad^4}{15\,e^6}\end{aligned}$$

input
```
integrate(x^5*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="m
axima")
```

output
```
-1/9*sqrt(-e^2*x^2 + d^2)*c*x^8/e^2 - 8/63*sqrt(-e^2*x^2 + d^2)*c*d^2*x^6/
e^4 - 1/7*sqrt(-e^2*x^2 + d^2)*b*x^6/e^2 - 16/105*sqrt(-e^2*x^2 + d^2)*c*d
^4*x^4/e^6 - 6/35*sqrt(-e^2*x^2 + d^2)*b*d^2*x^4/e^4 - 1/5*sqrt(-e^2*x^2 +
 d^2)*a*x^4/e^2 - 64/315*sqrt(-e^2*x^2 + d^2)*c*d^6*x^2/e^8 - 8/35*sqrt(-e
^2*x^2 + d^2)*b*d^4*x^2/e^6 - 4/15*sqrt(-e^2*x^2 + d^2)*a*d^2*x^2/e^4 - 12
8/315*sqrt(-e^2*x^2 + d^2)*c*d^8/e^10 - 16/35*sqrt(-e^2*x^2 + d^2)*b*d^6/e
^8 - 8/15*sqrt(-e^2*x^2 + d^2)*a*d^4/e^6
```

3.132.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.09

$$\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx =$$
$$-\frac{(315\,cd^8+315\,bd^6e^2+315\,ad^4e^4-(840\,cd^7+630\,bd^5e^2+420\,ad^3e^4-(1932\,cd^6+1071\,bd^4e^2+462\,ad$$

input
```
integrate(x^5*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="g
iac")
```

output
```
-1/315*(315*c*d^8 + 315*b*d^6*e^2 + 315*a*d^4*e^4 - (840*c*d^7 + 630*b*d^5
*e^2 + 420*a*d^3*e^4 - (1932*c*d^6 + 1071*b*d^4*e^2 + 462*a*d^2*e^4 - (295
2*c*d^5 + 1116*b*d^3*e^2 + 252*a*d*e^4 - (3098*c*d^4 + 729*b*d^2*e^2 + 63*
a*e^4 - 5*(440*c*d^3 + 54*b*d*e^2 - (204*c*d^2 + 9*b*e^2 + 7*((e*x + d)*c
- 8*c*d)*(e*x + d))*(e*x + d))*(e*x + d))*(e*x + d))*(e*x + d))*(e*x + d))
*(e*x + d))*sqrt(e*x + d)*sqrt(-e*x + d)/e^10
```

3.132.9 Mupad [B] (verification not implemented)

Time = 8.42 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.37

$$\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx =$$
$$-\frac{\sqrt{d-e\,x}\left(\frac{128\,c\,d^9+144\,b\,d^7\,e^2+168\,a\,d^5\,e^4}{315\,e^{10}}+\frac{x^7\,(40\,c\,d^2\,e^7+45\,b\,e^9)}{315\,e^{10}}+\frac{x^2\,(64\,c\,d^7\,e^2+72\,b\,d^5\,e^4+84\,a\,d^3\,e^6)}{315\,e^{10}}+\frac{x^3\,(64\,c\,d^6\,e^3+72\,b\,d^4}{315\,e^{10}}$$

input
```
int((x^5*(a + b*x^2 + c*x^4))/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

output
```
-((d - e*x)^(1/2)*((128*c*d^9 + 168*a*d^5*e^4 + 144*b*d^7*e^2)/(315*e^10)
+ (x^7*(45*b*e^9 + 40*c*d^2*e^7))/(315*e^10) + (x^2*(84*a*d^3*e^6 + 72*b*d
^5*e^4 + 64*c*d^7*e^2))/(315*e^10) + (x^3*(84*a*d^2*e^7 + 72*b*d^4*e^5 + 6
4*c*d^6*e^3))/(315*e^10) + (c*x^9)/(9*e) + (x^5*(63*a*e^9 + 54*b*d^2*e^7 +
 48*c*d^4*e^5))/(315*e^10) + (x*(168*a*d^4*e^5 + 144*b*d^6*e^3 + 128*c*d^8
*e))/(315*e^10) + (x^6*(40*c*d^3*e^6 + 45*b*d*e^8))/(315*e^10) + (x^4*(54*
b*d^3*e^6 + 48*c*d^5*e^4 + 63*a*d*e^8))/(315*e^10) + (c*d*x^8)/(9*e^2)))/(
d + e*x)^(1/2)
```

3.133 $\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

3.133.1 Optimal result

Integrand size = 35, antiderivative size = 159

$$\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{d^2(cd^4+bd^2e^2+ae^4)\sqrt{d-ex}\sqrt{d+ex}}{e^8} + \frac{(3cd^4+2bd^2e^2+ae^4)(d-ex)^{3/2}(d+ex)^{3/2}}{3e^8} - \frac{(3cd^2+be^2)(d-ex)^{5/2}(d+ex)^{5/2}}{5e^8} + \frac{c(d-ex)^{7/2}(d+ex)^{7/2}}{7e^8}$$

output
```
1/3*(a*e^4+2*b*d^2*e^2+3*c*d^4)*(-e*x+d)^(3/2)*(e*x+d)^(3/2)/e^8-1/5*(b*e^
2+3*c*d^2)*(-e*x+d)^(5/2)*(e*x+d)^(5/2)/e^8+1/7*c*(-e*x+d)^(7/2)*(e*x+d)^(
7/2)/e^8-d^2*(a*e^4+b*d^2*e^2+c*d^4)*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^8
```

3.133.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.73

$$\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-ex}\sqrt{d+ex}(35ae^4(2d^2+e^2x^2)+7b(8d^4e^2+4d^2e^4x^2+3e^6x^4)+3c(16d^6+8d^4e^2x^2+6d^2e^4x^4+}{105e^8}$$

input
```
Integrate[(x^3*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
-1/105*(Sqrt[d - e*x]*Sqrt[d + e*x]*(35*a*e^4*(2*d^2 + e^2*x^2) + 7*b*(8*d
^4*e^2 + 4*d^2*e^4*x^2 + 3*e^6*x^4) + 3*c*(16*d^6 + 8*d^4*e^2*x^2 + 6*d^2*
e^4*x^4 + 5*e^6*x^6)))/e^8
```

3.133.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1905, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \text{1905}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{x^3(cx^4+bx^2+a)}{\sqrt{d^2-e^2x^2}}dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1578}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{x^2(cx^4+bx^2+a)}{\sqrt{d^2-e^2x^2}}dx^2}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1195}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \left(-\frac{c(d^2-e^2x^2)^{5/2}}{e^6}+\frac{(3cd^2+be^2)(d^2-e^2x^2)^{3/2}}{e^6}+\frac{(-3cd^4-2be^2d^2-ae^4)\sqrt{d^2-e^2x^2}}{e^6}+\frac{cd^6+be^2d^4+ae^4d^2}{e^6\sqrt{d^2-e^2x^2}}\right)dx^2}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{2(d^2-e^2x^2)^{3/2}(ae^4+2bd^2e^2+3cd^4)}{3e^8}-\frac{2d^2\sqrt{d^2-e^2x^2}(ae^4+bd^2e^2+cd^4)}{e^8}-\frac{2(d^2-e^2x^2)^{5/2}(be^2+3cd^2)}{5e^8}+\frac{2c(d^2-e^2x^2)^{7/2}}{7e^8}\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$

input
```
Int[(x^3*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*((-2*d^2*(c*d^4 + b*d^2*e^2 + a*e^4)*Sqrt[d^2 - e^2*x
^2])/e^8 + (2*(3*c*d^4 + 2*b*d^2*e^2 + a*e^4)*(d^2 - e^2*x^2)^(3/2))/(3*e^
8) - (2*(3*c*d^2 + b*e^2)*(d^2 - e^2*x^2)^(5/2))/(5*e^8) + (2*c*(d^2 - e^2
*x^2)^(7/2))/(7*e^8)))/(2*Sqrt[d - e*x]*Sqrt[d + e*x])
```

3.133.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.133.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.69

method	result	size
gosper	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\left(15c\,x^{6}e^{6}+21b\,e^{6}x^{4}+18c\,d^{2}e^{4}x^{4}+35a\,e^{6}x^{2}+28b\,d^{2}e^{4}x^{2}+24c\,d^{4}e^{2}x^{2}+70a\,d^{2}e^{4}+56b\,d^{4}e^{2}+48c\,d^{6}\right)}{105e^{8}}$	109
default	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\left(15c\,x^{6}e^{6}+21b\,e^{6}x^{4}+18c\,d^{2}e^{4}x^{4}+35a\,e^{6}x^{2}+28b\,d^{2}e^{4}x^{2}+24c\,d^{4}e^{2}x^{2}+70a\,d^{2}e^{4}+56b\,d^{4}e^{2}+48c\,d^{6}\right)}{105e^{8}}$	109
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\left(15c\,x^{6}e^{6}+21b\,e^{6}x^{4}+18c\,d^{2}e^{4}x^{4}+35a\,e^{6}x^{2}+28b\,d^{2}e^{4}x^{2}+24c\,d^{4}e^{2}x^{2}+70a\,d^{2}e^{4}+56b\,d^{4}e^{2}+48c\,d^{6}\right)}{105e^{8}}$	109

input
```
int(x^3*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/105*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(15*c*e^6*x^6+21*b*e^6*x^4+18*c*d^2*e^
4*x^4+35*a*e^6*x^2+28*b*d^2*e^4*x^2+24*c*d^4*e^2*x^2+70*a*d^2*e^4+56*b*d^4
*e^2+48*c*d^6)/e^8
```

3.133.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.65

$$\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx =$$
$$-\frac{(15\,ce^6x^6+48\,cd^6+56\,bd^4e^2+70\,ad^2e^4+3\,(6\,cd^2e^4+7\,be^6)x^4+(24\,cd^4e^2+28\,bd^2e^4+35\,ae^6)x^2)\sqrt{ex}}{105\,e^8}$$

input
```
integrate(x^3*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/105*(15*c*e^6*x^6 + 48*c*d^6 + 56*b*d^4*e^2 + 70*a*d^2*e^4 + 3*(6*c*d^2
*e^4 + 7*b*e^6)*x^4 + (24*c*d^4*e^2 + 28*b*d^2*e^4 + 35*a*e^6)*x^2)*sqrt(e
*x + d)*sqrt(-e*x + d)/e^8
```

3.133.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.90 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.31

$$\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{iad^3G_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1 \\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^4} - \frac{ad^3G_{6,6}^{2,6}\left(\begin{matrix} -2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0 \end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^4} - \frac{ibd^5G_{6,6}^{6,2}\left(\begin{matrix} -\frac{9}{4},-\frac{7}{4} & -2,-2,-\frac{3}{2},1 \\ -\frac{5}{2},-\frac{9}{4},-2,-\frac{7}{4},-\frac{3}{2},0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^6} - \frac{bd^5G_{6,6}^{2,6}\left(\begin{matrix} -3,-\frac{11}{4},-\frac{5}{2},-\frac{9}{4},-2,1 & \\ -\frac{11}{4},-\frac{9}{4} & -3,-\frac{5}{2},-\frac{5}{2},0 \end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^6} - \frac{icd^7G_{6,6}^{6,2}\left(\begin{matrix} -\frac{13}{4},-\frac{11}{4} & -3,-3,-\frac{5}{2},1 \\ -\frac{7}{2},-\frac{13}{4},-3,-\frac{11}{4},-\frac{5}{2},0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^8} - \frac{cd^7G_{6,6}^{2,6}\left(\begin{matrix} -4,-\frac{15}{4},-\frac{7}{2},-\frac{13}{4},-3,1 & \\ -\frac{15}{4},-\frac{13}{4} & -4,-\frac{7}{2},-\frac{7}{2},0 \end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^8}$$

input
```
integrate(x**3*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
-I*a*d**3*meijerg(((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/
4, -1/2, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**4) - a*d**3*meijerg(((
-2, -7/4, -3/2, -5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), d*
*2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**4) - I*b*d**5*meijerg((
(-9/4, -7/4), (-2, -2, -3/2, 1)), ((-5/2, -9/4, -2, -7/4, -3/2, 0), ()), d
**2/(e**2*x**2))/(4*pi**(3/2)*e**6) - b*d**5*meijerg(((-3, -11/4, -5/2, -9
/4, -2, 1), ()), ((-11/4, -9/4), (-3, -5/2, -5/2, 0)), d**2*exp_polar(-2*I
*pi)/(e**2*x**2))/(4*pi**(3/2)*e**6) - I*c*d**7*meijerg(((-13/4, -11/4), (
-3, -3, -5/2, 1)), ((-7/2, -13/4, -3, -11/4, -5/2, 0), ()), d**2/(e**2*x**
2))/(4*pi**(3/2)*e**8) - c*d**7*meijerg(((-4, -15/4, -7/2, -13/4, -3, 1),
()), ((-15/4, -13/4), (-4, -7/2, -7/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*
x**2))/(4*pi**(3/2)*e**8)
```

3.133.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.36

$$\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}cx^6}{7\,e^2} - \frac{6\,\sqrt{-e^2x^2+d^2}cd^2x^4}{35\,e^4} - \frac{\sqrt{-e^2x^2+d^2}bx^4}{5\,e^2} - \frac{8\,\sqrt{-e^2x^2+d^2}cd^4x^2}{35\,e^6} - \frac{4\,\sqrt{-e^2x^2+d^2}bd^2x^2}{15\,e^4} - \frac{\sqrt{-e^2x^2+d^2}ax^2}{3\,e^2} - \frac{16\,\sqrt{-e^2x^2+d^2}cd^6}{35\,e^8} - \frac{8\,\sqrt{-e^2x^2+d^2}bd^4}{15\,e^6} - \frac{2\,\sqrt{-e^2x^2+d^2}ad^2}{3\,e^4}$$

input
```
integrate(x^3*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="m
axima")
```

output
```
-1/7*sqrt(-e^2*x^2 + d^2)*c*x^6/e^2 - 6/35*sqrt(-e^2*x^2 + d^2)*c*d^2*x^4/
e^4 - 1/5*sqrt(-e^2*x^2 + d^2)*b*x^4/e^2 - 8/35*sqrt(-e^2*x^2 + d^2)*c*d^4
*x^2/e^6 - 4/15*sqrt(-e^2*x^2 + d^2)*b*d^2*x^2/e^4 - 1/3*sqrt(-e^2*x^2 + d
^2)*a*x^2/e^2 - 16/35*sqrt(-e^2*x^2 + d^2)*c*d^6/e^8 - 8/15*sqrt(-e^2*x^2
+ d^2)*b*d^4/e^6 - 2/3*sqrt(-e^2*x^2 + d^2)*a*d^2/e^4
```

3.133.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.03

$$\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(105\,cd^6+105\,bd^4e^2+105\,ad^2e^4-(210\,cd^5+140\,bd^3e^2+70\,ade^4-(357\,cd^4+154\,bd^2e^2+35\,ae^4-3}{}$$

input
```
integrate(x^3*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="g
iac")
```

output
```
-1/105*(105*c*d^6 + 105*b*d^4*e^2 + 105*a*d^2*e^4 - (210*c*d^5 + 140*b*d^3
*e^2 + 70*a*d*e^4 - (357*c*d^4 + 154*b*d^2*e^2 + 35*a*e^4 - 3*(124*c*d^3 +
 28*b*d*e^2 - (81*c*d^2 + 7*b*e^2 + 5*((e*x + d)*c - 6*c*d)*(e*x + d))*(e*
x + d))*(e*x + d))*(e*x + d))*(e*x + d))*sqrt(e*x + d)*sqrt(-e*x + d)/e^8
```

3.133.9 Mupad [B] (verification not implemented)

Time = 8.29 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.35

$$\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-e\,x}\left(\frac{48\,c\,d^7+56\,b\,d^5\,e^2+70\,a\,d^3\,e^4}{105\,e^8}+\frac{x^5\,(18\,c\,d^2\,e^5+21\,b\,e^7)}{105\,e^8}+\frac{c\,x^7}{7\,e}+\frac{x^3\,(24\,c\,d^4\,e^3+28\,b\,d^2\,e^5+35\,a\,e^7)}{105\,e^8}+\frac{x\,(48\,c\,d^6\,e+56\,b\,d^4}{105\,e}\right.}{\sqrt{d+e\,x}}$$

input
```
int((x^3*(a + b*x^2 + c*x^4))/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

output
```
-((d - e*x)^(1/2)*((48*c*d^7 + 70*a*d^3*e^4 + 56*b*d^5*e^2)/(105*e^8) + (x
^5*(21*b*e^7 + 18*c*d^2*e^5))/(105*e^8) + (c*x^7)/(7*e) + (x^3*(35*a*e^7 +
 28*b*d^2*e^5 + 24*c*d^4*e^3))/(105*e^8) + (x*(70*a*d^2*e^5 + 56*b*d^4*e^3
 + 48*c*d^6*e))/(105*e^8) + (x^4*(18*c*d^3*e^4 + 21*b*d*e^6))/(105*e^8) +
(x^2*(28*b*d^3*e^4 + 24*c*d^5*e^2 + 35*a*d*e^6))/(105*e^8) + (c*d*x^6)/(7*
e^2)))/(d + e*x)^(1/2)
```

3.134 $\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

3.134.1 Optimal result

Integrand size = 33, antiderivative size = 109

$$\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(cd^4+bd^2e^2+ae^4)\sqrt{d-ex}\sqrt{d+ex}}{e^6} + \frac{(2cd^2+be^2)(d-ex)^{3/2}(d+ex)^{3/2}}{3e^6} - \frac{c(d-ex)^{5/2}(d+ex)^{5/2}}{5e^6}$$

output
```
1/3*(b*e^2+2*c*d^2)*(-e*x+d)^(3/2)*(e*x+d)^(3/2)/e^6-1/5*c*(-e*x+d)^(5/2)*
(e*x+d)^(5/2)/e^6-(a*e^4+b*d^2*e^2+c*d^4)*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^6
```

3.134.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.73

$$\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-ex}\sqrt{d+ex}(5(2bd^2e^2+3ae^4+be^4x^2)+c(8d^4+4d^2e^2x^2+3e^4x^4))}{15e^6}$$

input
```
Integrate[(x*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
-1/15*(Sqrt[d - e*x]*Sqrt[d + e*x]*(5*(2*b*d^2*e^2 + 3*a*e^4 + b*e^4*x^2)
+ c*(8*d^4 + 4*d^2*e^2*x^2 + 3*e^4*x^4)))/e^6
```

3.134.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.28, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1905, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \text{1905}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{x(cx^4+bx^2+a)}{\sqrt{d^2-e^2x^2}}dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1576}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^4+bx^2+a}{\sqrt{d^2-e^2x^2}}dx^2}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1140}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \left(\frac{c(d^2-e^2x^2)^{3/2}}{e^4}+\frac{(-2cd^2-be^2)\sqrt{d^2-e^2x^2}}{e^4}+\frac{cd^4+be^2d^2+ae^4}{e^4\sqrt{d^2-e^2x^2}}\right)dx^2}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{2\sqrt{d^2-e^2x^2}(ae^4+bd^2e^2+cd^4)}{e^6}+\frac{2(d^2-e^2x^2)^{3/2}(be^2+2cd^2)}{3e^6}-\frac{2c(d^2-e^2x^2)^{5/2}}{5e^6}\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$

input
```
Int[(x*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*((-2*(c*d^4 + b*d^2*e^2 + a*e^4)*Sqrt[d^2 - e^2*x^2])
/e^6 + (2*(2*c*d^2 + b*e^2)*(d^2 - e^2*x^2)^(3/2))/(3*e^6) - (2*c*(d^2 - e
^2*x^2)^(5/2))/(5*e^6)))/(2*Sqrt[d - e*x]*Sqrt[d + e*x])
```

3.134.3.1 Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.134.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.67

method	result	size
gosper	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(3c\,x^4e^4+5b\,e^4x^2+4c\,d^2e^2x^2+15e^4a+10e^2d^2b+8d^4c)}{15e^6}$	73
default	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(3c\,x^4e^4+5b\,e^4x^2+4c\,d^2e^2x^2+15e^4a+10e^2d^2b+8d^4c)}{15e^6}$	73
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(3c\,x^4e^4+5b\,e^4x^2+4c\,d^2e^2x^2+15e^4a+10e^2d^2b+8d^4c)}{15e^6}$	73

input
```
int(x*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/15*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(3*c*e^4*x^4+5*b*e^4*x^2+4*c*d^2*e^2*x^
2+15*a*e^4+10*b*d^2*e^2+8*c*d^4)/e^6
```

3.134.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.65

$$\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$$
$$= -\frac{(3\,ce^4x^4+8\,cd^4+10\,bd^2e^2+15\,ae^4+(4\,cd^2e^2+5\,be^4)x^2)\sqrt{ex+d}\sqrt{-ex+d}}{15\,e^6}$$

input
```
integrate(x*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fri
cas")
```

output
```
-1/15*(3*c*e^4*x^4 + 8*c*d^4 + 10*b*d^2*e^2 + 15*a*e^4 + (4*c*d^2*e^2 + 5*
b*e^4)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d)/e^6
```

3.134.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.52 (sec) , antiderivative size = 350, normalized size of antiderivative = 3.21

$$\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{iadG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^2} - \frac{adG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^2} - \frac{ibd^3G_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1 \\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^4} - \frac{bd^3G_{6,6}^{2,6}\left(\begin{matrix} -2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0 \end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^4} - \frac{icd^5G_{6,6}^{6,2}\left(\begin{matrix} -\frac{9}{4},-\frac{7}{4} & -2,-2,-\frac{3}{2},1 \\ -\frac{5}{2},-\frac{9}{4},-2,-\frac{7}{4},-\frac{3}{2},0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^6} - \frac{cd^5G_{6,6}^{2,6}\left(\begin{matrix} -3,-\frac{11}{4},-\frac{5}{2},-\frac{9}{4},-2,1 & \\ -\frac{11}{4},-\frac{9}{4} & -3,-\frac{5}{2},-\frac{5}{2},0 \end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^6}$$

input
```
integrate(x*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
-I*a*d*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0
), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**2) - a*d*meijerg(((-1, -3/4, -1/
2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), d**2*exp_polar(-
2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**2) - I*b*d**3*meijerg(((-5/4, -3/4),
(-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), d**2/(e**2*x**2
))/(4*pi**(3/2)*e**4) - b*d**3*meijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ())
, ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2)
)/(4*pi**(3/2)*e**4) - I*c*d**5*meijerg(((-9/4, -7/4), (-2, -2, -3/2, 1)),
 ((-5/2, -9/4, -2, -7/4, -3/2, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**
6) - c*d**5*meijerg(((-3, -11/4, -5/2, -9/4, -2, 1), ()), ((-11/4, -9/4),
(-3, -5/2, -5/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e*
*6)
```

3.134. $\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

3.134.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.28

$$\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}cx^4}{5\,e^2} - \frac{4\,\sqrt{-e^2x^2+d^2}cd^2x^2}{15\,e^4} - \frac{\sqrt{-e^2x^2+d^2}bx^2}{3\,e^2} - \frac{8\,\sqrt{-e^2x^2+d^2}cd^4}{15\,e^6} - \frac{2\,\sqrt{-e^2x^2+d^2}bd^2}{3\,e^4} - \frac{\sqrt{-e^2x^2+d^2}a}{e^2}$$

input
```
integrate(x*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="max
ima")
```

output
```
-1/5*sqrt(-e^2*x^2 + d^2)*c*x^4/e^2 - 4/15*sqrt(-e^2*x^2 + d^2)*c*d^2*x^2/
e^4 - 1/3*sqrt(-e^2*x^2 + d^2)*b*x^2/e^2 - 8/15*sqrt(-e^2*x^2 + d^2)*c*d^4
/e^6 - 2/3*sqrt(-e^2*x^2 + d^2)*b*d^2/e^4 - sqrt(-e^2*x^2 + d^2)*a/e^2
```

3.134.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(15\,cd^4+15\,bd^2e^2+15\,ae^4-(20\,cd^3+10\,bde^2-(22\,cd^2+5\,be^2+3\,((ex+d)c-4\,cd)(ex+d))(ex+d}{15\,e^6}$$

input
```
integrate(x*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gia
c")
```

output
```
-1/15*(15*c*d^4 + 15*b*d^2*e^2 + 15*a*e^4 - (20*c*d^3 + 10*b*d*e^2 - (22*c
*d^2 + 5*b*e^2 + 3*((e*x + d)*c - 4*c*d)*(e*x + d))*(e*x + d))*(e*x + d))*
sqrt(e*x + d)*sqrt(-e*x + d)/e^6
```

3.134.9 Mupad [B] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.31

$$\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-e\,x}\left(\frac{8\,c\,d^5+10\,b\,d^3\,e^2+15\,a\,d\,e^4}{15\,e^6}+\frac{x^3\,\left(4\,c\,d^2\,e^3+5\,b\,e^5\right)}{15\,e^6}+\frac{c\,x^5}{5\,e}+\frac{x^2\,\left(4\,c\,d^3\,e^2+5\,b\,d\,e^4\right)}{15\,e^6}+\frac{x\,\left(8\,c\,d^4\,e+10\,b\,d^2\,e^3+15\,a\,e^5\right)}{15\,e^6}+\right.}{\sqrt{d+e\,x}}$$

input
```
int((x*(a + b*x^2 + c*x^4))/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

output
```
-((d - e*x)^(1/2)*((8*c*d^5 + 10*b*d^3*e^2 + 15*a*d*e^4)/(15*e^6) + (x^3*(
5*b*e^5 + 4*c*d^2*e^3))/(15*e^6) + (c*x^5)/(5*e) + (x^2*(4*c*d^3*e^2 + 5*b
*d*e^4))/(15*e^6) + (x*(15*a*e^5 + 10*b*d^2*e^3 + 8*c*d^4*e))/(15*e^6) + (
c*d*x^4)/(5*e^2)))/(d + e*x)^(1/2)
```

3.135 $\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx$

3.135.1 Optimal result

Integrand size = 35, antiderivative size = 93

$$\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(cd^2+be^2)\sqrt{d-ex}\sqrt{d+ex}}{e^4} + \frac{c(d-ex)^{3/2}(d+ex)^{3/2}}{3e^4} - \frac{a\operatorname{arctanh}\left(\frac{\sqrt{d-ex}\sqrt{d+ex}}{d}\right)}{d}$$

output
```
1/3*c*(-e*x+d)^(3/2)*(e*x+d)^(3/2)/e^4-a*arctanh((-e*x+d)^(1/2)*(e*x+d)^(1
/2)/d)/d-(b*e^2+c*d^2)*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^4
```

3.135.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.14

$$\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-ex}\sqrt{d+ex}(2cd^2+3be^2+ce^2x^2)}{3e^4} + \frac{a\log\left(-1+\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{d} - \frac{a\log\left(d+\frac{d\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{d}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(x*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
-1/3*(Sqrt[d - e*x]*Sqrt[d + e*x]*(2*c*d^2 + 3*b*e^2 + c*e^2*x^2))/e^4 + (
a*Log[-1 + Sqrt[d + e*x]/Sqrt[d - e*x]])/d - (a*Log[d + (d*Sqrt[d + e*x])/
Sqrt[d - e*x]])/d
```

3.135.3 Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {1905, 1578, 1192, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx$$
$$\Big\downarrow \text{ 1905}$$
$$\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^4+bx^2+a}{x\sqrt{d^2-e^2x^2}}dx}{\sqrt{d-ex}\sqrt{d+ex}}$$
$$\Big\downarrow \text{ 1578}$$
$$\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^4+bx^2+a}{x^2\sqrt{d^2-e^2x^2}}dx^2}{2\sqrt{d-ex}\sqrt{d+ex}}$$
$$\Big\downarrow \text{ 1192}$$
$$\frac{\sqrt{d^2-e^2x^2}\int -\frac{cx^8-(2cd^2+be^2)x^4+cd^4+ae^4+bd^2e^2}{d^2-x^4}d\sqrt{d^2-e^2x^2}}{e^4\sqrt{d-ex}\sqrt{d+ex}}$$
$$\Big\downarrow \text{ 25}$$
$$-\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^8-(2cd^2+be^2)x^4+cd^4+ae^4+bd^2e^2}{d^2-x^4}d\sqrt{d^2-e^2x^2}}{e^4\sqrt{d-ex}\sqrt{d+ex}}$$
$$\Big\downarrow \text{ 1467}$$
$$-\frac{\sqrt{d^2-e^2x^2}\int \left(\frac{ae^4}{d^2-x^4}+be^2-cx^4+cd^2\right)d\sqrt{d^2-e^2x^2}}{e^4\sqrt{d-ex}\sqrt{d+ex}}$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{ae^4\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\sqrt{d^2-e^2x^2}\left(be^2+cd^2\right)+\frac{cx^6}{3}\right)}{e^4\sqrt{d-ex}\sqrt{d+ex}}$$

input
```
Int[(a + b*x^2 + c*x^4)/(x*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*((c*x^6)/3 - (c*d^2 + b*e^2)*Sqrt[d^2 - e^2*x^2] - (a
*e^4*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d))/(e^4*Sqrt[d - e*x]*Sqrt[d + e*x])
```

3.135.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.135.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.44 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.54

method	result
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\left(\operatorname{csgn}(d)cd\,e^{2}x^{2}\sqrt{-e^{2}x^{2}+d^{2}}+3\sqrt{-e^{2}x^{2}+d^{2}}\,\operatorname{csgn}(d)bd\,e^{2}+2\sqrt{-e^{2}x^{2}+d^{2}}\,\operatorname{csgn}(d)c\,d^{3}+3\ln\left(\frac{2d\left(\sqrt{-e^{2}x^{2}+d^{2}}\,\operatorname{csg}\right.}{x}\right.\right.}{3d\sqrt{-e^{2}x^{2}+d^{2}}\,e^{4}}$

input
```
int((c*x^4+b*x^2+a)/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/3*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d*(csgn(d)*c*d*e^2*x^2*(-e^2*x^2+d^2)^(1
/2)+3*(-e^2*x^2+d^2)^(1/2)*csgn(d)*b*d*e^2+2*(-e^2*x^2+d^2)^(1/2)*csgn(d)*
c*d^3+3*ln(2*d*((-e^2*x^2+d^2)^(1/2)*csgn(d)+d)/x)*a*e^4)*csgn(d)/(-e^2*x^
2+d^2)^(1/2)/e^4
```

3.135.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{3\,ae^4\log\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{x}\right)-\left(cde^2x^2+2\,cd^3+3\,bde^2\right)\sqrt{ex+d}\sqrt{-ex+d}}{3\,de^4}$$

input
```
integrate((c*x^4+b*x^2+a)/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fri
cas")
```

output
```
1/3*(3*a*e^4*log((sqrt(e*x + d)*sqrt(-e*x + d) - d)/x) - (c*d*e^2*x^2 + 2*
c*d^3 + 3*b*d*e^2)*sqrt(e*x + d)*sqrt(-e*x + d))/(d*e^4)
```

3.135.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 18.14 (sec) , antiderivative size = 304, normalized size of antiderivative = 3.27

$$\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{iaG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle|\,\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d} - \frac{aG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix}\middle|\,\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d} - \frac{ibdG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\,\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^2} - \frac{bdG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle|\,\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^2} - \frac{icd^3G_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1 \\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle|\,\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^4} - \frac{cd^3G_{6,6}^{2,6}\left(\begin{matrix} -2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0 \end{matrix}\middle|\,\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^4}$$

input
```
integrate((c*x**4+b*x**2+a)/x/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
I*a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)),
 d**2/(e**2*x**2))/(4*pi**(3/2)*d) - a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1),
()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/
(4*pi**(3/2)*d) - I*b*d*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/
4, 0, 1/4, 1/2, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**2) - b*d*meijer
g(((-1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)),
 d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**2) - I*c*d**3*meijer
g(((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ())
, d**2/(e**2*x**2))/(4*pi**(3/2)*e**4) - c*d**3*meijerg(((-2, -7/4, -3/2,
-5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), d**2*exp_polar(-2*
I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**4)
```

3.135.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.13

$$\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}cx^2}{3\,e^2} - \frac{a\log\left(\frac{2\,d^2}{|x|} + \frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{d} - \frac{2\sqrt{-e^2x^2+d^2}cd^2}{3\,e^4} - \frac{\sqrt{-e^2x^2+d^2}b}{e^2}$$

input
```
integrate((c*x^4+b*x^2+a)/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="max
ima")
```

output
```
-1/3*sqrt(-e^2*x^2 + d^2)*c*x^2/e^2 - a*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2
 + d^2)*d/abs(x))/d - 2/3*sqrt(-e^2*x^2 + d^2)*c*d^2/e^4 - sqrt(-e^2*x^2 +
 d^2)*b/e^2
```

3.135.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(79) = 158$.

Time = 0.44 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.03

$$\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\frac{3\,ae^4\log\left(\left|-\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}+\frac{\sqrt{ex+d}}{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}+2\right|\right)}{d} - \frac{3\,ae^4\log\left(\left|-\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}+\frac{\sqrt{ex+d}}{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}-2\right|\right)}{d} + (3\,cd^2+3\,be^2+((ex-}{3\,e^4}$$

input
```
integrate((c*x^4+b*x^2+a)/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gia
c")
```

output
```
-1/3*(3*a*e^4*log(abs(-(sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) +
sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)) + 2))/d - 3*a*e^4*log(abs
(-(sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) + sqrt(e*x + d)/(sqrt(2
)*sqrt(d) - sqrt(-e*x + d)) - 2))/d + (3*c*d^2 + 3*b*e^2 + ((e*x + d)*c -
2*c*d)*(e*x + d))*sqrt(e*x + d)*sqrt(-e*x + d))/e^4
```

3.135.9 Mupad [B] (verification not implemented)

Time = 9.75 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.73

$$\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{a\left(\ln\left(\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-1\right)-\ln\left(\frac{\sqrt{d+e\,x}-\sqrt{d}}{\sqrt{d-e\,x}-\sqrt{d}}\right)\right)}{d} - \frac{\sqrt{d-e\,x}\left(\frac{2\,c\,d^3}{3\,e^4}+\frac{c\,x^3}{3\,e}+\frac{c\,d\,x^2}{3\,e^2}+\frac{2\,c\,d^2\,x}{3\,e^3}\right)}{\sqrt{d+e\,x}} - \frac{\left(\frac{b\,d}{e^2}+\frac{b\,x}{e}\right)\sqrt{d-e\,x}}{\sqrt{d+e\,x}}$$

input
```
int((a + b*x^2 + c*x^4)/(x*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

output
```
(a*(log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 - 1) -
 log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2)))))/d - ((d -
e*x)^(1/2)*((2*c*d^3)/(3*e^4) + (c*x^3)/(3*e) + (c*d*x^2)/(3*e^2) + (2*c*d
^2*x)/(3*e^3)))/(d + e*x)^(1/2) - (((b*d)/e^2 + (b*x)/e)*(d - e*x)^(1/2))/
(d + e*x)^(1/2)
```

3.136 $\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx$

3.136.1 Optimal result

Integrand size = 35, antiderivative size = 99

$$\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx=-\frac{c\sqrt{d-ex}\sqrt{d+ex}}{e^2}-\frac{a\sqrt{d-ex}\sqrt{d+ex}}{2d^2x^2}-\frac{(2bd^2+ae^2)\,\text{arctanh}\left(\frac{\sqrt{d-ex}\sqrt{d+ex}}{d}\right)}{2d^3}$$

output
```
-1/2*(a*e^2+2*b*d^2)*arctanh((-e*x+d)^(1/2)*(e*x+d)^(1/2)/d)/d^3-c*(-e*x+d
)^(1/2)*(e*x+d)^(1/2)/e^2-1/2*a*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2/x^2
```

3.136.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.86

$$\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx=-\frac{\frac{\sqrt{d-ex}\sqrt{d+ex}(ade^2+2cd^3x^2)}{e^2x^2}+2(2bd^2+ae^2)\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{2d^3}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(x^3*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
-1/2*((Sqrt[d - e*x]*Sqrt[d + e*x]*(a*d*e^2 + 2*c*d^3*x^2))/(e^2*x^2) + 2*
(2*b*d^2 + a*e^2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d - e*x]])/d^3
```

3.136.3 Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.51, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1905, 1578, 1192, 1471, 25, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \text{1905}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^4+bx^2+a}{x^3\sqrt{d^2-e^2x^2}}dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1578}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^4+bx^2+a}{x^4\sqrt{d^2-e^2x^2}}dx^2}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1192}$$

$$-\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^8-(2cd^2+be^2)x^4+cd^4+ae^4+bd^2e^2}{(d^2-x^4)^2}d\sqrt{d^2-e^2x^2}}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1471}$$

$$-\frac{\sqrt{d^2-e^2x^2}\left(\frac{ae^4\sqrt{d^2-e^2x^2}}{2d^2(d^2-x^4)}-\frac{\int -\frac{2cd^4-2cx^4d^2+2be^2d^2+ae^4}{d^2-x^4}d\sqrt{d^2-e^2x^2}}{2d^2}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{d^2-e^2x^2}\left(\frac{\int \frac{2cd^4-2cx^4d^2+2be^2d^2+ae^4}{d^2-x^4}d\sqrt{d^2-e^2x^2}}{2d^2}+\frac{ae^4\sqrt{d^2-e^2x^2}}{2d^2(d^2-x^4)}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{299}$$

$$-\frac{\sqrt{d^2-e^2x^2}\left(\frac{e^2(ae^2+2bd^2)\int \frac{1}{d^2-x^4}d\sqrt{d^2-e^2x^2}+2cd^2\sqrt{d^2-e^2x^2}}{2d^2}+\frac{ae^4\sqrt{d^2-e^2x^2}}{2d^2(d^2-x^4)}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{e^2(ae^2+2bd^2)\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}+2cd^2\sqrt{d^2-e^2x^2}}{2d^2}+\frac{ae^4\sqrt{d^2-e^2x^2}}{2d^2(d^2-x^4)}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$

input
```
Int[(a + b*x^2 + c*x^4)/(x^3*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
-((Sqrt[d^2 - e^2*x^2]*((a*e^4*Sqrt[d^2 - e^2*x^2])/(2*d^2*(d^2 - x^4)) +
(2*c*d^2*Sqrt[d^2 - e^2*x^2] + (e^2*(2*b*d^2 + a*e^2)*ArcTanh[Sqrt[d^2 - e
^2*x^2]/d])/d)/(2*d^2)))/(e^2*Sqrt[d - e*x]*Sqrt[d + e*x]))
```

3.136.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

3.136.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.38

method	result
risch	$-\frac{a\sqrt{-ex+d}\,\sqrt{ex+d}}{2d^2x^2}+\frac{\left(-\frac{2c\,d^2\sqrt{-(ex-d)(ex+d)}}{e^2}-\frac{\left(a\,e^2+2b\,d^2\right)\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)\sqrt{(ex+d)(-ex+d)}}{2d^2\sqrt{ex+d}\,\sqrt{-ex+d}}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\left(2\,\mathrm{csgn}(d)c\,d^3x^2\sqrt{-e^2x^2+d^2}+\ln\left(\frac{2d\left(\sqrt{-e^2x^2+d^2}\,\mathrm{csgn}(d)+d\right)}{x}\right)a\,e^4x^2+2\ln\left(\frac{2d\left(\sqrt{-e^2x^2+d^2}\,\mathrm{csgn}(d)+d\right)}{x}\right)b\,d^2e^2\cdots\right.}{2d^3\sqrt{-e^2x^2+d^2}\,x^2e^2}$

input
```
int((c*x^4+b*x^2+a)/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/2*a*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2/x^2+1/2/d^2*(-2*c*d^2/e^2*(-(e*x-d
)*(e*x+d))^(1/2)-(a*e^2+2*b*d^2)/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2
*x^2+d^2)^(1/2))/x))*((e*x+d)*(-e*x+d))^(1/2)/(e*x+d)^(1/2)/(-e*x+d)^(1/2)
```

3.136.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99

$$\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{2\,cd^4x^2-(2\,bd^2e^2+ae^4)x^2\log\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{x}\right)+(2\,cd^3x^2+ade^2)\sqrt{ex+d}\sqrt{-ex+d}}{2\,d^3e^2x^2}$$

input
```
integrate((c*x^4+b*x^2+a)/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/2*(2*c*d^4*x^2 - (2*b*d^2*e^2 + a*e^4)*x^2*log((sqrt(e*x + d)*sqrt(-e*x
 + d) - d)/x) + (2*c*d^3*x^2 + a*d*e^2)*sqrt(e*x + d)*sqrt(-e*x + d))/(d^3
*e^2*x^2)
```

3.136.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**4+b*x**2+a)/x**3/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
Timed out
```

3.136.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.24

$$\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{b\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{d} - \frac{ae^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{2\,d^3} - \frac{\sqrt{-e^2x^2+d^2}c}{e^2} - \frac{\sqrt{-e^2x^2+d^2}a}{2\,d^2x^2}$$

input
```
integrate((c*x^4+b*x^2+a)/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="m
axima")
```

output
```
-b*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d - 1/2*a*e^2*log(2
*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 - sqrt(-e^2*x^2 + d^2)*
c/e^2 - 1/2*sqrt(-e^2*x^2 + d^2)*a/(d^2*x^2)
```

3.136.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(83) = 166$.

Time = 0.46 (sec) , antiderivative size = 374, normalized size of antiderivative = 3.78

$$\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{2\sqrt{ex+d}\sqrt{-ex+d}c + \frac{(2\,bd^2e^2+ae^4)\log\left(\left|-\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}+\frac{\sqrt{ex+d}}{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}+2\right|\right)}{d^3} - \frac{(2\,bd^2e^2+ae^4)\log\left(\left|-\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}+\cdots\right|\right)}{d^3}\cdots}{2\,e^2}$$

input
```
integrate((c*x^4+b*x^2+a)/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="g
iac")
```

output
```
-1/2*(2*sqrt(e*x + d)*sqrt(-e*x + d)*c + (2*b*d^2*e^2 + a*e^4)*log(abs(-(s
qrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) + sqrt(e*x + d)/(sqrt(2)*sq
rt(d) - sqrt(-e*x + d)) + 2))/d^3 - (2*b*d^2*e^2 + a*e^4)*log(abs(-(sqrt(2
)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) + sqrt(e*x + d)/(sqrt(2)*sqrt(d)
 - sqrt(-e*x + d)) - 2))/d^3 - 4*(a*e^4*((sqrt(2)*sqrt(d) - sqrt(-e*x + d)
)/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^3 + 4*
a*e^4*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(s
qrt(2)*sqrt(d) - sqrt(-e*x + d))))/((((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/s
qrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^2 - 4)^2*
d^3))/e^2
```

3.136.9 Mupad [B] (verification not implemented)

Time = 10.86 (sec) , antiderivative size = 422, normalized size of antiderivative = 4.26

$$\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{b\left(\ln\left(\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-1\right)-\ln\left(\frac{\sqrt{d+e\,x}-\sqrt{d}}{\sqrt{d-e\,x}-\sqrt{d}}\right)\right)}{d} - \frac{\left(\frac{c\,d}{e^2}+\frac{c\,x}{e}\right)\sqrt{d-e\,x}}{\sqrt{d+e\,x}} - \frac{\frac{a\,e^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-\frac{a\,e^2}{2}+\frac{15\,a\,e^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)^4}{2\left(\sqrt{d-e\,x}-\sqrt{d}\right)^4}}{\frac{16\,d^3\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-\frac{32\,d^3\left(\sqrt{d+e\,x}-\sqrt{d}\right)^4}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^4}+\frac{16\,d^3\left(\sqrt{d+e\,x}-\sqrt{d}\right)^6}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^6}} - \frac{a\,e^2\ln\left(\frac{\sqrt{d+e\,x}-\sqrt{d}}{\sqrt{d-e\,x}-\sqrt{d}}\right)}{2\,d^3} + \frac{a\,e^2\ln\left(\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-1\right)}{2\,d^3} + \frac{a\,e^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{32\,d^3\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}$$

input
```
int((a + b*x^2 + c*x^4)/(x^3*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

output
```
(b*(log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 - 1) -
 log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2)))))/d - (((c*d
)/e^2 + (c*x)/e)*(d - e*x)^(1/2))/(d + e*x)^(1/2) - ((a*e^2*((d + e*x)^(1/
2) - d^(1/2))^2)/((d - e*x)^(1/2) - d^(1/2))^2 - (a*e^2)/2 + (15*a*e^2*((d
 + e*x)^(1/2) - d^(1/2))^4)/(2*((d - e*x)^(1/2) - d^(1/2))^4))/((16*d^3*((
d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) - d^(1/2))^2 - (32*d^3*((d +
 e*x)^(1/2) - d^(1/2))^4)/((d - e*x)^(1/2) - d^(1/2))^4 + (16*d^3*((d + e*
x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2) - d^(1/2))^6) - (a*e^2*log(((d + e
*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/(2*d^3) + (a*e^2*log(((
d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 - 1))/(2*d^3) +
(a*e^2*((d + e*x)^(1/2) - d^(1/2))^2)/(32*d^3*((d - e*x)^(1/2) - d^(1/2))^
2)
```

3.137 $\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx$

3.137.1 Optimal result

Integrand size = 35, antiderivative size = 126

$$\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{a\sqrt{d-ex}\sqrt{d+ex}}{4d^2x^4} - \frac{(4bd^2+3ae^2)\sqrt{d-ex}\sqrt{d+ex}}{8d^4x^2} - \frac{(8cd^4+4bd^2e^2+3ae^4)\operatorname{arctanh}\left(\frac{\sqrt{d-ex}\sqrt{d+ex}}{d}\right)}{8d^5}$$

output
```
-1/8*(3*a*e^4+4*b*d^2*e^2+8*c*d^4)*arctanh((-e*x+d)^(1/2)*(e*x+d)^(1/2)/d)
/d^5-1/4*a*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2/x^4-1/8*(3*a*e^2+4*b*d^2)*(-e*
x+d)^(1/2)*(e*x+d)^(1/2)/d^4/x^2
```

3.137.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.81

$$\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\frac{d\sqrt{d-ex}\sqrt{d+ex}(2ad^2+4bd^2x^2+3ae^2x^2)}{x^4} + 2(8cd^4+4bd^2e^2+3ae^4)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{8d^5}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(x^5*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
-1/8*((d*Sqrt[d - e*x]*Sqrt[d + e*x]*(2*a*d^2 + 4*b*d^2*x^2 + 3*a*e^2*x^2)
)/x^4 + 2*(8*c*d^4 + 4*b*d^2*e^2 + 3*a*e^4)*ArcTanh[Sqrt[d + e*x]/Sqrt[d -
 e*x]])/d^5
```

3.137.3 Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.43, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {1905, 1578, 1192, 25, 1471, 25, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \text{1905}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^4+bx^2+a}{x^5\sqrt{d^2-e^2x^2}}dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1578}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^4+bx^2+a}{x^6\sqrt{d^2-e^2x^2}}dx^2}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1192}$$

$$\frac{\sqrt{d^2-e^2x^2}\int -\frac{cx^8-(2cd^2+be^2)x^4+cd^4+ae^4+bd^2e^2}{(d^2-x^4)^3}d\sqrt{d^2-e^2x^2}}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^8-(2cd^2+be^2)x^4+cd^4+ae^4+bd^2e^2}{(d^2-x^4)^3}d\sqrt{d^2-e^2x^2}}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1471}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\int -\frac{4cd^4-4cx^4d^2+4be^2d^2+3ae^4}{\left(d^2-x^4\right)^2}d\sqrt{d^2-e^2x^2}}{4d^2}-\frac{ae^4\sqrt{d^2-e^2x^2}}{4d^2(d^2-x^4)^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{\int \frac{4cd^4-4cx^4d^2+4be^2d^2+3ae^4}{\left(d^2-x^4\right)^2}d\sqrt{d^2-e^2x^2}}{4d^2}-\frac{ae^4\sqrt{d^2-e^2x^2}}{4d^2(d^2-x^4)^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{298}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{\frac{\left(3ae^4+4bd^2e^2+8cd^4\right)\int \frac{1}{d^2-x^4}d\sqrt{d^2-e^2x^2}}{2d^2}+\frac{e^2\sqrt{d^2-e^2x^2}\left(\frac{3ae^2}{d^2}+4b\right)}{2\left(d^2-x^4\right)}}{4d^2}-\frac{ae^4\sqrt{d^2-e^2x^2}}{4d^2(d^2-x^4)^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\left(3ae^4+4bd^2e^2+8cd^4\right)}{2d^3}+\frac{e^2\sqrt{d^2-e^2x^2}\left(\frac{3ae^2}{d^2}+4b\right)}{2\left(d^2-x^4\right)}}{4d^2}-\frac{ae^4\sqrt{d^2-e^2x^2}}{4d^2(d^2-x^4)^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

input
```
Int[(a + b*x^2 + c*x^4)/(x^5*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-1/4*(a*e^4*Sqrt[d^2 - e^2*x^2])/(d^2*(d^2 - x^4)^2)
 - ((e^2*(4*b + (3*a*e^2)/d^2)*Sqrt[d^2 - e^2*x^2])/(2*(d^2 - x^4)) + ((8*
c*d^4 + 4*b*d^2*e^2 + 3*a*e^4)*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^3))/(4
*d^2)))/(Sqrt[d - e*x]*Sqrt[d + e*x])
```

3.137.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

3.137.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\left(3a\,e^{2}x^{2}+4b\,d^{2}x^{2}+2a\,d^{2}\right)}{8d^{4}x^{4}}-\frac{\left(3e^{4}a+4e^{2}d^{2}b+8d^{4}c\right)\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)\sqrt{(ex+d)(-ex+d)}}{8d^{4}\sqrt{d^{2}}\,\sqrt{ex+d}\,\sqrt{-ex+d}}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\left(3\ln\left(\frac{2d\left(\sqrt{-e^{2}x^{2}+d^{2}}\,\operatorname{csgn}(d)+d\right)}{x}\right)a\,e^{4}x^{4}+4\ln\left(\frac{2d\left(\sqrt{-e^{2}x^{2}+d^{2}}\,\operatorname{csgn}(d)+d\right)}{x}\right)b\,d^{2}e^{2}x^{4}+8\ln\left(\frac{2d\left(\sqrt{-e^{2}x^{2}+d^{2}}\,\operatorname{csg}\right.}{x}\right.}{8d^{5}\sqrt{-e^{2}x^{2}+d^{2}}}$

input
```
int((c*x^4+b*x^2+a)/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/8*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(3*a*e^2*x^2+4*b*d^2*x^2+2*a*d^2)/d^4/x^
4-1/8/d^4*(3*a*e^4+4*b*d^2*e^2+8*c*d^4)/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2
)*(-e^2*x^2+d^2)^(1/2))/x)*((e*x+d)*(-e*x+d))^(1/2)/(e*x+d)^(1/2)/(-e*x+d)
^(1/2)
```

3.137.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.81

$$\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx$$
$$= \frac{(8\,cd^4+4\,bd^2e^2+3\,ae^4)x^4\log\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{x}\right)-(2\,ad^3+(4\,bd^3+3\,ade^2)x^2)\sqrt{ex+d}\sqrt{-ex+d}}{8\,d^5x^4}$$

input
```
integrate((c*x^4+b*x^2+a)/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="f
ricas")
```

output
```
1/8*((8*c*d^4 + 4*b*d^2*e^2 + 3*a*e^4)*x^4*log((sqrt(e*x + d)*sqrt(-e*x +
d) - d)/x) - (2*a*d^3 + (4*b*d^3 + 3*a*d*e^2)*x^2)*sqrt(e*x + d)*sqrt(-e*x
 + d))/(d^5*x^4)
```

3.137.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**4+b*x**2+a)/x**5/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
Timed out
```

3.137.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.53

$$\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{c\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{d} - \frac{be^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{2\,d^3} - \frac{3\,ae^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{8\,d^5} - \frac{\sqrt{-e^2x^2+d^2}b}{2\,d^2x^2} - \frac{3\,\sqrt{-e^2x^2+d^2}ae^2}{8\,d^4x^2} - \frac{\sqrt{-e^2x^2+d^2}a}{4\,d^2x^4}$$

input
```
integrate((c*x^4+b*x^2+a)/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="m
axima")
```

output
```
-c*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d - 1/2*b*e^2*log(2
*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 - 3/8*a*e^4*log(2*d^2/a
bs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^5 - 1/2*sqrt(-e^2*x^2 + d^2)*b/
(d^2*x^2) - 3/8*sqrt(-e^2*x^2 + d^2)*a*e^2/(d^4*x^2) - 1/4*sqrt(-e^2*x^2 +
 d^2)*a/(d^2*x^4)
```

3.137.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 767 vs. $2(108) = 216$.

Time = 0.61 (sec) , antiderivative size = 767, normalized size of antiderivative = 6.09

$$\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\left(8\,cd^4e+4\,bd^2e^3+3\,ae^5\right)\log\left(\left|-\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}+\frac{\sqrt{ex+d}}{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}+2\right|\right)}{d^5} - \frac{\left(8\,cd^4e+4\,bd^2e^3+3\,ae^5\right)\log\left(\left|-\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}+\frac{\sqrt{ex+d}}{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}-\right.\right.}{d^5} - \frac{}{}$$

input
```
integrate((c*x^4+b*x^2+a)/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="g
iac")
```

output
```
-1/8*((8*c*d^4*e + 4*b*d^2*e^3 + 3*a*e^5)*log(abs(-(sqrt(2)*sqrt(d) - sqrt
(-e*x + d))/sqrt(e*x + d) + sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d
)) + 2))/d^5 - (8*c*d^4*e + 4*b*d^2*e^3 + 3*a*e^5)*log(abs(-(sqrt(2)*sqrt(
d) - sqrt(-e*x + d))/sqrt(e*x + d) + sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt
(-e*x + d)) - 2))/d^5 - 4*(4*b*d^2*e^3*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))
/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^7 + 5*a
*e^5*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sq
rt(2)*sqrt(d) - sqrt(-e*x + d)))^7 - 16*b*d^2*e^3*((sqrt(2)*sqrt(d) - sqrt
(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d
)))^5 + 12*a*e^5*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(
e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^5 - 64*b*d^2*e^3*((sqrt(2)*sq
rt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - s
qrt(-e*x + d)))^3 + 48*a*e^5*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x
+ d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^3 + 256*b*d^2*e^3
*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2
)*sqrt(d) - sqrt(-e*x + d))) + 320*a*e^5*((sqrt(2)*sqrt(d) - sqrt(-e*x + d
))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d))))/((((
sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*s
qrt(d) - sqrt(-e*x + d)))^2 - 4)^4*d^5))/e
```

3.137. $\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx$

3.137.9 Mupad [B] (verification not implemented)

Time = 15.50 (sec) , antiderivative size = 932, normalized size of antiderivative = 7.40

$$\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx$$
$$= \frac{\frac{a\,e^4}{4}+\frac{6\,a\,e^4\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-\frac{53\,a\,e^4\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^4}{2\,\left(\sqrt{d-e\,x}-\sqrt{d}\right)^4}-\frac{87\,a\,e^4\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^6}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^6}+\frac{657\,a\,e^4\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^8}{4\,\left(\sqrt{d-e\,x}-\sqrt{d}\right)^8}-\frac{121\,a\,e^4\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^{10}}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^{10}}}{\frac{256\,d^5\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^4}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^4}-\frac{1024\,d^5\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^6}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^6}+\frac{1536\,d^5\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^8}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^8}-\frac{1024\,d^5\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^{10}}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^{10}}+\frac{256\,d^5\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^{12}}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^{12}}}$$
$$-\frac{\frac{b\,e^2\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-\frac{b\,e^2}{2}+\frac{15\,b\,e^2\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^4}{2\,\left(\sqrt{d-e\,x}-\sqrt{d}\right)^4}}{\frac{16\,d^3\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-\frac{32\,d^3\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^4}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^4}+\frac{16\,d^3\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^6}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^6}}$$
$$+\frac{c\left(\ln\left(\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-1\right)-\ln\left(\frac{\sqrt{d+e\,x}-\sqrt{d}}{\sqrt{d-e\,x}-\sqrt{d}}\right)\right)}{d}-\frac{3\,a\,e^4\,\ln\left(\frac{\sqrt{d+e\,x}-\sqrt{d}}{\sqrt{d-e\,x}-\sqrt{d}}\right)}{8\,d^5}$$
$$-\frac{b\,e^2\,\ln\left(\frac{\sqrt{d+e\,x}-\sqrt{d}}{\sqrt{d-e\,x}-\sqrt{d}}\right)}{2\,d^3}+\frac{3\,a\,e^4\,\ln\left(\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-1\right)}{8\,d^5}+\frac{b\,e^2\,\ln\left(\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-1\right)}{2\,d^3}$$
$$+\frac{7\,a\,e^4\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{256\,d^5\,\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}+\frac{a\,e^4\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^4}{1024\,d^5\,\left(\sqrt{d-e\,x}-\sqrt{d}\right)^4}+\frac{b\,e^2\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{32\,d^3\,\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}$$

input
```
int((a + b*x^2 + c*x^4)/(x^5*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

output
```
((a*e^4)/4 + (6*a*e^4*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) - d^
(1/2))^2 - (53*a*e^4*((d + e*x)^(1/2) - d^(1/2))^4)/(2*((d - e*x)^(1/2) -
d^(1/2))^4) - (87*a*e^4*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2) -
d^(1/2))^6 + (657*a*e^4*((d + e*x)^(1/2) - d^(1/2))^8)/(4*((d - e*x)^(1/2)
 - d^(1/2))^8) - (121*a*e^4*((d + e*x)^(1/2) - d^(1/2))^10)/((d - e*x)^(1/
2) - d^(1/2))^10)/((256*d^5*((d + e*x)^(1/2) - d^(1/2))^4)/((d - e*x)^(1/2
) - d^(1/2))^4 - (1024*d^5*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2)
 - d^(1/2))^6 + (1536*d^5*((d + e*x)^(1/2) - d^(1/2))^8)/((d - e*x)^(1/2)
- d^(1/2))^8 - (1024*d^5*((d + e*x)^(1/2) - d^(1/2))^10)/((d - e*x)^(1/2)
- d^(1/2))^10 + (256*d^5*((d + e*x)^(1/2) - d^(1/2))^12)/((d - e*x)^(1/2)
- d^(1/2))^12) - ((b*e^2*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) -
 d^(1/2))^2 - (b*e^2)/2 + (15*b*e^2*((d + e*x)^(1/2) - d^(1/2))^4)/(2*((d
- e*x)^(1/2) - d^(1/2))^4))/((16*d^3*((d + e*x)^(1/2) - d^(1/2))^2)/((d -
e*x)^(1/2) - d^(1/2))^2 - (32*d^3*((d + e*x)^(1/2) - d^(1/2))^4)/((d - e*x
)^(1/2) - d^(1/2))^4 + (16*d^3*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(
1/2) - d^(1/2))^6) + (c*(log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2
) - d^(1/2))^2 - 1) - log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d
^(1/2)))))/d - (3*a*e^4*log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) -
 d^(1/2))))/(8*d^5) - (b*e^2*log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1
/2) - d^(1/2))))/(2*d^3) + (3*a*e^4*log(((d + e*x)^(1/2) - d^(1/2))^2/(...
```

3.137. $\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx$

3.138 $\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx$

3.138.1 Optimal result

Integrand size = 35, antiderivative size = 212

$$\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{a\sqrt{d-ex}\sqrt{d+ex}}{6d^2x^6} - \frac{(6bd^2+5ae^2)\sqrt{d-ex}\sqrt{d+ex}}{24d^4x^4} - \frac{(8cd^4+6bd^2e^2+5ae^4)\sqrt{d-ex}\sqrt{d+ex}}{16d^6x^2} - \frac{e^2(8cd^4+6bd^2e^2+5ae^4)\sqrt{d^2-e^2x^2}\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^7\sqrt{d-ex}\sqrt{d+ex}}$$

output
```
-1/6*a*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2/x^6-1/24*(5*a*e^2+6*b*d^2)*(-e*x+d
)^(1/2)*(e*x+d)^(1/2)/d^4/x^4-1/16*(5*a*e^4+6*b*d^2*e^2+8*c*d^4)*(-e*x+d)^
(1/2)*(e*x+d)^(1/2)/d^6/x^2-1/16*e^2*(5*a*e^4+6*b*d^2*e^2+8*c*d^4)*arctanh
((-e^2*x^2+d^2)^(1/2)/d)*(-e^2*x^2+d^2)^(1/2)/d^7/(-e*x+d)^(1/2)/(e*x+d)^(
1/2)
```

3.138.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.67

$$\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\frac{d\sqrt{d-ex}\sqrt{d+ex}\left(6\left(2bd^4x^2+4cd^4x^4+3bd^2e^2x^4\right)+a\left(8d^4+10d^2e^2x^2+15e^4x^4\right)\right)}{x^6} + 6e^2(8cd^4+6bd^2e^2+5ae^4)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{48d^7}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(x^7*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
-1/48*((d*Sqrt[d - e*x]*Sqrt[d + e*x]*(6*(2*b*d^4*x^2 + 4*c*d^4*x^4 + 3*b*
d^2*e^2*x^4) + a*(8*d^4 + 10*d^2*e^2*x^2 + 15*e^4*x^4)))/x^6 + 6*e^2*(8*c*
d^4 + 6*b*d^2*e^2 + 5*a*e^4)*ArcTanh[Sqrt[d + e*x]/Sqrt[d - e*x]])/d^7
```

3.138.3 Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {1905, 1578, 1192, 1471, 25, 298, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{1905}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^4+bx^2+a}{x^7\sqrt{d^2-e^2x^2}}dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1578}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^4+bx^2+a}{x^8\sqrt{d^2-e^2x^2}}dx^2}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1192}$$

$$-\frac{e^2\sqrt{d^2-e^2x^2}\int \frac{cx^8-(2cd^2+be^2)x^4+cd^4+ae^4+bd^2e^2}{(d^2-x^4)^4}d\sqrt{d^2-e^2x^2}}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1471}$$

$$-\frac{e^2\sqrt{d^2-e^2x^2}\left(\frac{ae^4\sqrt{d^2-e^2x^2}}{6d^2(d^2-x^4)^3}-\frac{\int -\frac{6cd^4-6cx^4d^2+6be^2d^2+5ae^4}{\left(d^2-x^4\right)^3}d\sqrt{d^2-e^2x^2}}{6d^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{e^2\sqrt{d^2-e^2x^2}\left(\frac{\int \frac{6cd^4-6cx^4d^2+6be^2d^2+5ae^4}{(d^2-x^4)^3}d\sqrt{d^2-e^2x^2}}{6d^2}+\frac{ae^4\sqrt{d^2-e^2x^2}}{6d^2(d^2-x^4)^3}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{298}$$

$$-\frac{e^2\sqrt{d^2-e^2x^2}\left(\frac{\frac{3(5ae^4+6bd^2e^2+8cd^4)\int \frac{1}{(d^2-x^4)^2}d\sqrt{d^2-e^2x^2}}{4d^2}+\frac{e^2\sqrt{d^2-e^2x^2}\left(\frac{5ae^2}{d^2}+6b\right)}{4(d^2-x^4)^2}}{6d^2}+\frac{ae^4\sqrt{d^2-e^2x^2}}{6d^2(d^2-x^4)^3}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{215}$$

$$-\frac{e^2\sqrt{d^2-e^2x^2}\left(\frac{\frac{3(5ae^4+6bd^2e^2+8cd^4)\left(\frac{\int \frac{1}{d^2-x^4}d\sqrt{d^2-e^2x^2}}{2d^2}+\frac{\sqrt{d^2-e^2x^2}}{2d^2(d^2-x^4)}\right)}{4d^2}+\frac{e^2\sqrt{d^2-e^2x^2}\left(\frac{5ae^2}{d^2}+6b\right)}{4(d^2-x^4)^2}}{6d^2}+\frac{ae^4\sqrt{d^2-e^2x^2}}{6d^2(d^2-x^4)^3}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{e^2\sqrt{d^2-e^2x^2}\left(\frac{\frac{3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^3}+\frac{\sqrt{d^2-e^2x^2}}{2d^2(d^2-x^4)}\right)(5ae^4+6bd^2e^2+8cd^4)}{4d^2}+\frac{e^2\sqrt{d^2-e^2x^2}\left(\frac{5ae^2}{d^2}+6b\right)}{4(d^2-x^4)^2}}{6d^2}+\frac{ae^4\sqrt{d^2-e^2x^2}}{6d^2(d^2-x^4)^3}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

input

```
Int[(a + b*x^2 + c*x^4)/(x^7*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output

```
-((e^2*Sqrt[d^2 - e^2*x^2]*((a*e^4*Sqrt[d^2 - e^2*x^2])/(6*d^2*(d^2 - x^4)
^3) + ((e^2*(6*b + (5*a*e^2)/d^2)*Sqrt[d^2 - e^2*x^2])/(4*(d^2 - x^4)^2) +
 (3*(8*c*d^4 + 6*b*d^2*e^2 + 5*a*e^4)*(Sqrt[d^2 - e^2*x^2]/(2*d^2*(d^2 - x
^4)) + ArcTanh[Sqrt[d^2 - e^2*x^2]/d]/(2*d^3)))/(4*d^2))/(6*d^2)))/(Sqrt[d
 - e*x]*Sqrt[d + e*x]))
```

3.138. $\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx$

3.138.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.138. $\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx$

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

3.138.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(15a\,e^4x^4+18b\,d^2e^2x^4+24c\,d^4x^4+10a\,d^2e^2x^2+12b\,d^4x^2+8a\,d^4)}{48d^6x^6}-\frac{e^2(5e^4a+6e^2d^2b+8d^4c)\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e}}{x}\right.}{16d^6\sqrt{d^2}\,\sqrt{ex+d}\,\sqrt{-}}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\left(15\ln\left(\frac{2d\left(\sqrt{-e^2x^2+d^2}\,\mathrm{csgn}(d)+d\right)}{x}\right)a\,e^6x^6+18\ln\left(\frac{2d\left(\sqrt{-e^2x^2+d^2}\,\mathrm{csgn}(d)+d\right)}{x}\right)b\,d^2e^4x^6+24\ln\left(\frac{2d\left(\sqrt{-e^2x^2+d^2}\right.}{x}\right.\right.}{}$

input
```
int((c*x^4+b*x^2+a)/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/48*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(15*a*e^4*x^4+18*b*d^2*e^2*x^4+24*c*d^4
*x^4+10*a*d^2*e^2*x^2+12*b*d^4*x^2+8*a*d^4)/d^6/x^6-1/16*e^2*(5*a*e^4+6*b*
d^2*e^2+8*c*d^4)/d^6/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1
/2))/x)*((e*x+d)*(-e*x+d))^(1/2)/(e*x+d)^(1/2)/(-e*x+d)^(1/2)
```

3.138.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.65

$$\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx$$
$$=\frac{3\,(8\,cd^4e^2+6\,bd^2e^4+5\,ae^6)x^6\log\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{x}\right)-(8\,ad^5+3\,(8\,cd^5+6\,bd^3e^2+5\,ade^4)x^4+2\,(6\,bd^5+}{48\,d^7x^6}$$

input
```
integrate((c*x^4+b*x^2+a)/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="f
ricas")
```

output
```
1/48*(3*(8*c*d^4*e^2 + 6*b*d^2*e^4 + 5*a*e^6)*x^6*log((sqrt(e*x + d)*sqrt(
-e*x + d) - d)/x) - (8*a*d^5 + 3*(8*c*d^5 + 6*b*d^3*e^2 + 5*a*d*e^4)*x^4 +
 2*(6*b*d^5 + 5*a*d^3*e^2)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d))/(d^7*x^6)
```

3.138.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**4+b*x**2+a)/x**7/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
Timed out
```

3.138.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.28

$$\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{ce^2\log\left(\frac{2d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{2d^3} - \frac{3be^4\log\left(\frac{2d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{8d^5} - \frac{5ae^6\log\left(\frac{2d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{16d^7} - \frac{\sqrt{-e^2x^2+d^2}c}{2d^2x^2} - \frac{3\sqrt{-e^2x^2+d^2}be^2}{8d^4x^2} - \frac{5\sqrt{-e^2x^2+d^2}ae^4}{16d^6x^2} - \frac{\sqrt{-e^2x^2+d^2}b}{4d^2x^4} - \frac{5\sqrt{-e^2x^2+d^2}ae^2}{24d^4x^4} - \frac{\sqrt{-e^2x^2+d^2}a}{6d^2x^6}$$

input
```
integrate((c*x^4+b*x^2+a)/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="m
axima")
```

output
```
-1/2*c*e^2*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 - 3/8*b
*e^4*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^5 - 5/16*a*e^6*
log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^7 - 1/2*sqrt(-e^2*x^
2 + d^2)*c/(d^2*x^2) - 3/8*sqrt(-e^2*x^2 + d^2)*b*e^2/(d^4*x^2) - 5/16*sqr
t(-e^2*x^2 + d^2)*a*e^4/(d^6*x^2) - 1/4*sqrt(-e^2*x^2 + d^2)*b/(d^2*x^4) -
 5/24*sqrt(-e^2*x^2 + d^2)*a*e^2/(d^4*x^4) - 1/6*sqrt(-e^2*x^2 + d^2)*a/(d
^2*x^6)
```

3.138.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1434 vs. 2(184) = 368.

Time = 0.77 (sec) , antiderivative size = 1434, normalized size of antiderivative = 6.76

$$\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^4+b*x^2+a)/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="g
iac")
```

output
```
-1/48*(3*(8*c*d^4*e^3 + 6*b*d^2*e^5 + 5*a*e^7)*log(abs(-(sqrt(2)*sqrt(d) -
 sqrt(-e*x + d))/sqrt(e*x + d) + sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*
x + d)) + 2))/d^7 - 3*(8*c*d^4*e^3 + 6*b*d^2*e^5 + 5*a*e^7)*log(abs(-(sqrt
(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) + sqrt(e*x + d)/(sqrt(2)*sqrt(
d) - sqrt(-e*x + d)) - 2))/d^7 - 4*(24*c*d^4*e^3*((sqrt(2)*sqrt(d) - sqrt(
-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)
))^11 + 30*b*d^2*e^5*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - s
qrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^11 + 33*a*e^7*((sqrt(2)*s
qrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) -
sqrt(-e*x + d)))^11 - 288*c*d^4*e^3*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sq
rt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^9 - 168*b*
d^2*e^5*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/
(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^9 + 20*a*e^7*((sqrt(2)*sqrt(d) - sqrt(
-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)
))^9 + 768*c*d^4*e^3*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - s
qrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^7 + 192*b*d^2*e^5*((sqrt(
2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d
) - sqrt(-e*x + d)))^7 + 1440*a*e^7*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sq
rt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^7 + 3072*c
*d^4*e^3*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x +...
```

3.138.9 Mupad [B] (verification not implemented)

Time = 24.81 (sec) , antiderivative size = 1621, normalized size of antiderivative = 7.65

$$\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^2 + c*x^4)/(x^7*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

output
```
((b*e^4)/4 + (6*b*e^4*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) - d^
(1/2))^2 - (53*b*e^4*((d + e*x)^(1/2) - d^(1/2))^4)/(2*((d - e*x)^(1/2) -
d^(1/2))^4) - (87*b*e^4*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2) -
d^(1/2))^6 + (657*b*e^4*((d + e*x)^(1/2) - d^(1/2))^8)/(4*((d - e*x)^(1/2)
 - d^(1/2))^8) - (121*b*e^4*((d + e*x)^(1/2) - d^(1/2))^10)/((d - e*x)^(1/
2) - d^(1/2))^10)/((256*d^5*((d + e*x)^(1/2) - d^(1/2))^4)/((d - e*x)^(1/2
) - d^(1/2))^4 - (1024*d^5*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2)
 - d^(1/2))^6 + (1536*d^5*((d + e*x)^(1/2) - d^(1/2))^8)/((d - e*x)^(1/2)
- d^(1/2))^8 - (1024*d^5*((d + e*x)^(1/2) - d^(1/2))^10)/((d - e*x)^(1/2)
- d^(1/2))^10 + (256*d^5*((d + e*x)^(1/2) - d^(1/2))^12)/((d - e*x)^(1/2)
- d^(1/2))^12) - ((c*e^2*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) -
 d^(1/2))^2 - (c*e^2)/2 + (15*c*e^2*((d + e*x)^(1/2) - d^(1/2))^4)/(2*((d
- e*x)^(1/2) - d^(1/2))^4))/((16*d^3*((d + e*x)^(1/2) - d^(1/2))^2)/((d -
e*x)^(1/2) - d^(1/2))^2 - (32*d^3*((d + e*x)^(1/2) - d^(1/2))^4)/((d - e*x
)^(1/2) - d^(1/2))^4 + (16*d^3*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(
1/2) - d^(1/2))^6) + ((a*e^6)/6 + (4*a*e^6*((d + e*x)^(1/2) - d^(1/2))^2)/
((d - e*x)^(1/2) - d^(1/2))^2 + (71*a*e^6*((d + e*x)^(1/2) - d^(1/2))^4)/(
(d - e*x)^(1/2) - d^(1/2))^4 - (1558*a*e^6*((d + e*x)^(1/2) - d^(1/2))^6)/
(3*((d - e*x)^(1/2) - d^(1/2))^6) - (540*a*e^6*((d + e*x)^(1/2) - d^(1/2))
^8)/((d - e*x)^(1/2) - d^(1/2))^8 + (4248*a*e^6*((d + e*x)^(1/2) - d^(1...
```

3.138. $\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx$

3.139 $\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

3.139.1 Optimal result

Integrand size = 35, antiderivative size = 216

$$\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(5cd^4+6bd^2e^2+8ae^4)\,x\sqrt{d-ex}\sqrt{d+ex}}{16e^6} - \frac{(5cd^2+6be^2)\,x^3\sqrt{d-ex}\sqrt{d+ex}}{24e^4} + \frac{cx^5(-d+ex)\sqrt{d+ex}}{6e^2\sqrt{d-ex}} + \frac{d^2(5cd^4+6bd^2e^2+8ae^4)\sqrt{d^2-e^2x^2}\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^7\sqrt{d-ex}\sqrt{d+ex}}$$

output
```
1/6*c*x^5*(e*x-d)*(e*x+d)^(1/2)/e^2/(-e*x+d)^(1/2)-1/16*(8*a*e^4+6*b*d^2*e
^2+5*c*d^4)*x*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^6-1/24*(6*b*e^2+5*c*d^2)*x^3*
(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^4+1/16*d^2*(8*a*e^4+6*b*d^2*e^2+5*c*d^4)*ar
ctan(e*x/(-e^2*x^2+d^2)^(1/2))*(-e^2*x^2+d^2)^(1/2)/e^7/(-e*x+d)^(1/2)/(e*
x+d)^(1/2)
```

3.139.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.62

$$\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$$
$$= \frac{-ex\sqrt{d-ex}\sqrt{d+ex}(6(3bd^2e^2+4ae^4+2be^4x^2)+c(15d^4+10d^2e^2x^2+8e^4x^4))+6d^2(5cd^4+6bd^2e^2+8}{48e^7}$$

input
```
Integrate[(x^2*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
(-(e*x*Sqrt[d - e*x]*Sqrt[d + e*x]*(6*(3*b*d^2*e^2 + 4*a*e^4 + 2*b*e^4*x^2
) + c*(15*d^4 + 10*d^2*e^2*x^2 + 8*e^4*x^4))) + 6*d^2*(5*c*d^4 + 6*b*d^2*e
^2 + 8*a*e^4)*ArcTan[Sqrt[d + e*x]/Sqrt[d - e*x]])/(48*e^7)
```

3.139.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1905, 1590, 25, 363, 262, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+bx^2+cx^4\right)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$$
$$\Big\downarrow \; 1905$$
$$\frac{\sqrt{d^2-e^2x^2}\int \frac{x^2\left(cx^4+bx^2+a\right)}{\sqrt{d^2-e^2x^2}}dx}{\sqrt{d-ex}\sqrt{d+ex}}$$
$$\Big\downarrow \; 1590$$
$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{\int -\frac{x^2\left(6ae^2+\left(5cd^2+6be^2\right)x^2\right)}{\sqrt{d^2-e^2x^2}}dx}{6e^2}-\frac{cx^5\sqrt{d^2-e^2x^2}}{6e^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$
$$\Big\downarrow \; 25$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\int \frac{x^2\left(6ae^2+\left(5cd^2+6be^2\right)x^2\right)}{\sqrt{d^2-e^2x^2}}dx}{6e^2}-\frac{cx^5\sqrt{d^2-e^2x^2}}{6e^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$\big\downarrow$ 363

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{3\left(8ae^4+6bd^2e^2+5cd^4\right)\int \frac{x^2}{\sqrt{d^2-e^2x^2}}dx}{4e^2}-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}\left(6b+\frac{5cd^2}{e^2}\right)}{6e^2}-\frac{cx^5\sqrt{d^2-e^2x^2}}{6e^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$\big\downarrow$ 262

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{3\left(8ae^4+6bd^2e^2+5cd^4\right)\left(\frac{d^2\int \frac{1}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{x\sqrt{d^2-e^2x^2}}{2e^2}\right)}{4e^2}-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}\left(6b+\frac{5cd^2}{e^2}\right)}{6e^2}-\frac{cx^5\sqrt{d^2-e^2x^2}}{6e^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$\big\downarrow$ 224

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{3\left(8ae^4+6bd^2e^2+5cd^4\right)\left(\frac{d^2\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}}{2e^2}-\frac{x\sqrt{d^2-e^2x^2}}{2e^2}\right)}{4e^2}-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}\left(6b+\frac{5cd^2}{e^2}\right)}{6e^2}-\frac{cx^5\sqrt{d^2-e^2x^2}}{6e^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$\big\downarrow$ 216

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{3\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}-\frac{x\sqrt{d^2-e^2x^2}}{2e^2}\right)\left(8ae^4+6bd^2e^2+5cd^4\right)}{4e^2}-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}\left(6b+\frac{5cd^2}{e^2}\right)}{6e^2}-\frac{cx^5\sqrt{d^2-e^2x^2}}{6e^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

input
```
Int[(x^2*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

3.139. $\int \frac{x^2\left(a+bx^2+cx^4\right)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

output
```
(Sqrt[d^2 - e^2*x^2]*(-1/6*(c*x^5*Sqrt[d^2 - e^2*x^2])/e^2 + (-1/4*((6*b +
 (5*c*d^2)/e^2)*x^3*Sqrt[d^2 - e^2*x^2]) + (3*(5*c*d^4 + 6*b*d^2*e^2 + 8*a
*e^4)*(-1/2*(x*Sqrt[d^2 - e^2*x^2])/e^2 + (d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2
*x^2]])/(2*e^3)))/(4*e^2))/(6*e^2)))/(Sqrt[d - e*x]*Sqrt[d + e*x])
```

3.139.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

3.139. $\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

3.139.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{x(8c x^4e^4+12b e^4x^2+10c d^2e^2x^2+24e^4a+18e^2d^2b+15d^4c)\sqrt{-ex+d}\,\sqrt{ex+d}}{48e^6}+\frac{d^2(8e^4a+6e^2d^2b+5d^4c)\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)\sqrt{}}{16e^6\sqrt{e^2}\,\sqrt{ex+d}\,\sqrt{-ex+d}}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\left(8\ \mathrm{csgn}(e)c e^5x^5\sqrt{-e^2x^2+d^2}+12\ \mathrm{csgn}(e)b e^5x^3\sqrt{-e^2x^2+d^2}+10\ \mathrm{csgn}(e)c d^2e^3x^3\sqrt{-e^2x^2+d^2}+24\sqrt{-e^2x^2+d^2}\right.}{}$

input
```
int(x^2*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/48*x*(8*c*e^4*x^4+12*b*e^4*x^2+10*c*d^2*e^2*x^2+24*a*e^4+18*b*d^2*e^2+1
5*c*d^4)/e^6*(-e*x+d)^(1/2)*(e*x+d)^(1/2)+1/16*d^2*(8*a*e^4+6*b*d^2*e^2+5*
c*d^4)/e^6/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))*((e*x+d)
*(-e*x+d))^(1/2)/(e*x+d)^(1/2)/(-e*x+d)^(1/2)
```

3.139.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.62

$$\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(8\,ce^5x^5+2\,(5\,cd^2e^3+6\,be^5)x^3+3\,(5\,cd^4e+6\,bd^2e^3+8\,ae^5)x)\sqrt{ex+d}\sqrt{-ex+d}+6\,(5\,cd^6+6\,bd^4e^2}{48\,e^7}$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/48*((8*c*e^5*x^5 + 2*(5*c*d^2*e^3 + 6*b*e^5)*x^3 + 3*(5*c*d^4*e + 6*b*d
^2*e^3 + 8*a*e^5)*x)*sqrt(e*x + d)*sqrt(-e*x + d) + 6*(5*c*d^6 + 6*b*d^4*e
^2 + 8*a*d^2*e^4)*arctan((sqrt(e*x + d)*sqrt(-e*x + d) - d)/(e*x)))/e^7
```

3.139.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
Timed out
```

3.139.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.05

$$\begin{aligned}\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = &-\frac{\sqrt{-e^2x^2+d^2}cx^5}{6\,e^2} - \frac{5\,\sqrt{-e^2x^2+d^2}cd^2x^3}{24\,e^4} - \frac{\sqrt{-e^2x^2+d^2}bx^3}{4\,e^2} \\ &+ \frac{5\,cd^6\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{16\,\sqrt{e^2}e^6} + \frac{3\,bd^4\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{8\,\sqrt{e^2}e^4} + \frac{ad^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\,\sqrt{e^2}e^2} \\ &- \frac{5\,\sqrt{-e^2x^2+d^2}cd^4x}{16\,e^6} - \frac{3\,\sqrt{-e^2x^2+d^2}bd^2x}{8\,e^4} - \frac{\sqrt{-e^2x^2+d^2}ax}{2\,e^2}\end{aligned}$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="m
axima")
```

output
```
-1/6*sqrt(-e^2*x^2 + d^2)*c*x^5/e^2 - 5/24*sqrt(-e^2*x^2 + d^2)*c*d^2*x^3/
e^4 - 1/4*sqrt(-e^2*x^2 + d^2)*b*x^3/e^2 + 5/16*c*d^6*arcsin(e^2*x/(d*sqrt
(e^2)))/(sqrt(e^2)*e^6) + 3/8*b*d^4*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)
*e^4) + 1/2*a*d^2*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^2) - 5/16*sqrt(
-e^2*x^2 + d^2)*c*d^4*x/e^6 - 3/8*sqrt(-e^2*x^2 + d^2)*b*d^2*x/e^4 - 1/2*s
qrt(-e^2*x^2 + d^2)*a*x/e^2
```

3.139.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.82

$$\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$$
$$= \frac{(33\,cd^5+30\,bd^3e^2+24\,ade^4-(85\,cd^4+54\,bd^2e^2+24\,ae^4-2\,(55\,cd^3+18\,bde^2-(45\,cd^2+6\,be^2+4\,((ex$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="g
iac")
```

output
```
1/48*((33*c*d^5 + 30*b*d^3*e^2 + 24*a*d*e^4 - (85*c*d^4 + 54*b*d^2*e^2 + 2
4*a*e^4 - 2*(55*c*d^3 + 18*b*d*e^2 - (45*c*d^2 + 6*b*e^2 + 4*((e*x + d)*c
- 5*c*d)*(e*x + d))*(e*x + d))*(e*x + d))*(e*x + d))*sqrt(e*x + d)*sqrt(-e
*x + d) + 6*(5*c*d^6 + 6*b*d^4*e^2 + 8*a*d^2*e^4)*arcsin(1/2*sqrt(2)*sqrt(
e*x + d)/sqrt(d)))/e^7
```

3.139.9 Mupad [B] (verification not implemented)

Time = 27.91 (sec) , antiderivative size = 1132, normalized size of antiderivative = 5.24

$$\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Too large to display}$$

input
```
int((x^2*(a + b*x^2 + c*x^4))/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

output
```
((14*a*d^2*((d + e*x)^(1/2) - d^(1/2))^3)/((d - e*x)^(1/2) - d^(1/2))^3 -
(14*a*d^2*((d + e*x)^(1/2) - d^(1/2))^5)/((d - e*x)^(1/2) - d^(1/2))^5 + (
2*a*d^2*((d + e*x)^(1/2) - d^(1/2))^7)/((d - e*x)^(1/2) - d^(1/2))^7 - (2*
a*d^2*((d + e*x)^(1/2) - d^(1/2)))/((d - e*x)^(1/2) - d^(1/2)))/(e^3*(((d
+ e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 + 1)^4) - ((175*c*
d^6*((d + e*x)^(1/2) - d^(1/2))^3)/(12*((d - e*x)^(1/2) - d^(1/2))^3) + (3
11*c*d^6*((d + e*x)^(1/2) - d^(1/2))^5)/(4*((d - e*x)^(1/2) - d^(1/2))^5)
- (8361*c*d^6*((d + e*x)^(1/2) - d^(1/2))^7)/(4*((d - e*x)^(1/2) - d^(1/2)
)^7) + (42259*c*d^6*((d + e*x)^(1/2) - d^(1/2))^9)/(6*((d - e*x)^(1/2) - d
^(1/2))^9) - (25295*c*d^6*((d + e*x)^(1/2) - d^(1/2))^11)/(2*((d - e*x)^(1
/2) - d^(1/2))^11) + (25295*c*d^6*((d + e*x)^(1/2) - d^(1/2))^13)/(2*((d -
 e*x)^(1/2) - d^(1/2))^13) - (42259*c*d^6*((d + e*x)^(1/2) - d^(1/2))^15)/
(6*((d - e*x)^(1/2) - d^(1/2))^15) + (8361*c*d^6*((d + e*x)^(1/2) - d^(1/2
))^17)/(4*((d - e*x)^(1/2) - d^(1/2))^17) - (311*c*d^6*((d + e*x)^(1/2) -
d^(1/2))^19)/(4*((d - e*x)^(1/2) - d^(1/2))^19) - (175*c*d^6*((d + e*x)^(1
/2) - d^(1/2))^21)/(12*((d - e*x)^(1/2) - d^(1/2))^21) - (5*c*d^6*((d + e*
x)^(1/2) - d^(1/2))^23)/(4*((d - e*x)^(1/2) - d^(1/2))^23) + (5*c*d^6*((d
+ e*x)^(1/2) - d^(1/2)))/(4*((d - e*x)^(1/2) - d^(1/2))))/(e^7*(((d + e*x)
^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 + 1)^12) - ((23*b*d^4*((
d + e*x)^(1/2) - d^(1/2))^3)/(2*((d - e*x)^(1/2) - d^(1/2))^3) - (333*b...
```

3.139. $\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

3.140 $\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

3.140.1 Optimal result

Integrand size = 32, antiderivative size = 128

$$\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx=-\frac{(3cd^2+4be^2)\,x\sqrt{d-ex}\sqrt{d+ex}}{8e^4}+\frac{cx^3(-d+ex)\sqrt{d+ex}}{4e^2\sqrt{d-ex}}-\frac{(3cd^4+4bd^2e^2+8ae^4)\arctan\left(\frac{\sqrt{d-ex}}{\sqrt{d+ex}}\right)}{4e^5}$$

output
```
-1/4*(8*a*e^4+4*b*d^2*e^2+3*c*d^4)*arctan((-e*x+d)^(1/2)/(e*x+d)^(1/2))/e^
5+1/4*c*x^3*(e*x-d)*(e*x+d)^(1/2)/e^2/(-e*x+d)^(1/2)-1/8*(4*b*e^2+3*c*d^2)
*x*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^4
```

3.140.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.77

$$\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx=\frac{-ex\sqrt{d-ex}\sqrt{d+ex}(3cd^2+4be^2+2ce^2x^2)+2(3cd^4+4bd^2e^2+8ae^4)\arctan\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{8e^5}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
(-(e*x*Sqrt[d - e*x]*Sqrt[d + e*x]*(3*c*d^2 + 4*b*e^2 + 2*c*e^2*x^2)) + 2*
(3*c*d^4 + 4*b*d^2*e^2 + 8*a*e^4)*ArcTan[Sqrt[d + e*x]/Sqrt[d - e*x]])/(8*
e^5)
```

3.140.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1789, 1473, 25, 299, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{1789}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^4+bx^2+a}{\sqrt{d^2-e^2x^2}}dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1473}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{\int -\frac{4ae^2+\left(3cd^2+4be^2\right)x^2}{\sqrt{d^2-e^2x^2}}dx}{4e^2}-\frac{cx^3\sqrt{d^2-e^2x^2}}{4e^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\int \frac{4ae^2+\left(3cd^2+4be^2\right)x^2}{\sqrt{d^2-e^2x^2}}dx}{4e^2}-\frac{cx^3\sqrt{d^2-e^2x^2}}{4e^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{299}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{\left(8ae^4+4bd^2e^2+3cd^4\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{1}{2}x\sqrt{d^2-e^2x^2}\left(4b+\frac{3cd^2}{e^2}\right)}{4e^2}-\frac{cx^3\sqrt{d^2-e^2x^2}}{4e^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{\left(8ae^4+4bd^2e^2+3cd^4\right)\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}}{2e^2}-\frac{1}{2}x\sqrt{d^2-e^2x^2}\left(4b+\frac{3cd^2}{e^2}\right)}{4e^2}-\frac{cx^3\sqrt{d^2-e^2x^2}}{4e^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(8ae^4+4bd^2e^2+3cd^4\right)}{2e^3}-\frac{1}{2}x\sqrt{d^2-e^2x^2}\left(4b+\frac{3cd^2}{e^2}\right)}{4e^2}-\frac{cx^3\sqrt{d^2-e^2x^2}}{4e^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

input
```
Int[(a + b*x^2 + c*x^4)/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-1/4*(c*x^3*Sqrt[d^2 - e^2*x^2])/e^2 + (-1/2*((4*b +
 (3*c*d^2)/e^2)*x*Sqrt[d^2 - e^2*x^2]) + ((3*c*d^4 + 4*b*d^2*e^2 + 8*a*e^4
)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3))/(4*e^2)))/(Sqrt[d - e*x]*Sqr
t[d + e*x])
```

3.140.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 1789
```
Int[((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)*(x_)^(non2_.))^(q_
.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Simp[(d
1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[q]/(d1*d2 + e1*e2*x
^n)^FracPart[q])   Int[(d1*d2 + e1*e2*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x],
 x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[
non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

3.140.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{x\left(2c\,x^{2}e^{2}+4b\,e^{2}+3c\,d^{2}\right)\sqrt{-ex+d}\,\sqrt{ex+d}}{8e^{4}}+\frac{\left(8e^{4}a+4e^{2}d^{2}b+3d^{4}c\right)\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)\sqrt{\left(ex+d\right)\left(-ex+d\right)}}{8e^{4}\sqrt{e^{2}}\,\sqrt{ex+d}\,\sqrt{-ex+d}}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\left(2\,\mathrm{csgn}(e)c\,e^{3}x^{3}\sqrt{-e^{2}x^{2}+d^{2}}+4\,\mathrm{csgn}(e)e^{3}\sqrt{-e^{2}x^{2}+d^{2}}\,bx+3\,\mathrm{csgn}(e)e\sqrt{-e^{2}x^{2}+d^{2}}\,c\,d^{2}x-8\arctan\left(\frac{\mathrm{csgn}(e)ex}{\sqrt{-e^{2}x^{2}+d}}\right.\right.}{8e^{5}\sqrt{-e^{2}x^{2}+d^{2}}}$

input
```
int((c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*x*(2*c*e^2*x^2+4*b*e^2+3*c*d^2)/e^4*(-e*x+d)^(1/2)*(e*x+d)^(1/2)+1/8*
(8*a*e^4+4*b*d^2*e^2+3*c*d^4)/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x
^2+d^2)^(1/2))*((e*x+d)*(-e*x+d))^(1/2)/(e*x+d)^(1/2)/(-e*x+d)^(1/2)
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.78

$$\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\left(2\,ce^3x^3+\left(3\,cd^2e+4\,be^3\right)x\right)\sqrt{ex+d}\sqrt{-ex+d}+2\left(3\,cd^4+4\,bd^2e^2+8\,ae^4\right)\arctan\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{ex}\right)}{8\,e^5}$$

input
```
integrate((c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="frica
s")
```

output
```
-1/8*((2*c*e^3*x^3 + (3*c*d^2*e + 4*b*e^3)*x)*sqrt(e*x + d)*sqrt(-e*x + d)
 + 2*(3*c*d^4 + 4*b*d^2*e^2 + 8*a*e^4)*arctan((sqrt(e*x + d)*sqrt(-e*x + d
) - d)/(e*x)))/e^5
```

3.140.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
Timed out
```

3.140.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.14

$$\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}cx^3}{4\,e^2}+\frac{a\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}}+\frac{3\,cd^4\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{8\,\sqrt{e^2}e^4}+\frac{bd^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\,\sqrt{e^2}e^2}-\frac{3\,\sqrt{-e^2x^2+d^2}cd^2x}{8\,e^4}-\frac{\sqrt{-e^2x^2+d^2}bx}{2\,e^2}$$

input
```
integrate((c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxim
a")
```

output
```
-1/4*sqrt(-e^2*x^2 + d^2)*c*x^3/e^2 + a*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e
^2) + 3/8*c*d^4*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^4) + 1/2*b*d^2*ar
csin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^2) - 3/8*sqrt(-e^2*x^2 + d^2)*c*d^2
*x/e^4 - 1/2*sqrt(-e^2*x^2 + d^2)*b*x/e^2
```

3.140.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.90

$$\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$$
$$= \frac{(5\,cd^3 + 4\,bde^2 - (9\,cd^2 + 4\,be^2 + 2\,((ex+d)c - 3\,cd)(ex+d))(ex+d))\sqrt{ex+d}\sqrt{-ex+d} + 2\,(3\,cd^4 + 4}{8\,e^5}$$

input
```
integrate((c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac"
)
```

output
```
1/8*((5*c*d^3 + 4*b*d*e^2 - (9*c*d^2 + 4*b*e^2 + 2*((e*x + d)*c - 3*c*d)*(
e*x + d))*(e*x + d))*sqrt(e*x + d)*sqrt(-e*x + d) + 2*(3*c*d^4 + 4*b*d^2*e
^2 + 8*a*e^4)*arcsin(1/2*sqrt(2)*sqrt(e*x + d)/sqrt(d)))/e^5
```

3.140.9 Mupad [B] (verification not implemented)

Time = 17.26 (sec) , antiderivative size = 651, normalized size of antiderivative = 5.09

$$\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$$
$$= \frac{\frac{14\,b\,d^2\,{\left(\sqrt{d+e\,x}-\sqrt{d}\right)}^3}{{\left(\sqrt{d-e\,x}-\sqrt{d}\right)}^3}-\frac{14\,b\,d^2\,{\left(\sqrt{d+e\,x}-\sqrt{d}\right)}^5}{{\left(\sqrt{d-e\,x}-\sqrt{d}\right)}^5}+\frac{2\,b\,d^2\,{\left(\sqrt{d+e\,x}-\sqrt{d}\right)}^7}{{\left(\sqrt{d-e\,x}-\sqrt{d}\right)}^7}-\frac{2\,b\,d^2\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)}{\sqrt{d-e\,x}-\sqrt{d}}}{e^3\,{\left(\frac{{\left(\sqrt{d+e\,x}-\sqrt{d}\right)}^2}{{\left(\sqrt{d-e\,x}-\sqrt{d}\right)}^2}+1\right)}^4}$$
$$-\frac{4\,a\,\mathrm{atan}\left(\frac{e\,\left(\sqrt{d-e\,x}-\sqrt{d}\right)}{\sqrt{e^2}\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)}\right)}{\sqrt{e^2}}$$
$$-\frac{\frac{23\,c\,d^4\,{\left(\sqrt{d+e\,x}-\sqrt{d}\right)}^3}{2\,{\left(\sqrt{d-e\,x}-\sqrt{d}\right)}^3}-\frac{333\,c\,d^4\,{\left(\sqrt{d+e\,x}-\sqrt{d}\right)}^5}{2\,{\left(\sqrt{d-e\,x}-\sqrt{d}\right)}^5}+\frac{671\,c\,d^4\,{\left(\sqrt{d+e\,x}-\sqrt{d}\right)}^7}{2\,{\left(\sqrt{d-e\,x}-\sqrt{d}\right)}^7}-\frac{671\,c\,d^4\,{\left(\sqrt{d+e\,x}-\sqrt{d}\right)}^9}{2\,{\left(\sqrt{d-e\,x}-\sqrt{d}\right)}^9}+\frac{333\,c\,d^4\,{\left(\sqrt{d+e\,x}-\sqrt{d}\right)}^{11}}{2\,{\left(\sqrt{d-e\,x}-\sqrt{d}\right)}^{11}}}{e^5\,{\left(\frac{{\left(\sqrt{d+e\,x}-\sqrt{d}\right)}^2}{{\left(\sqrt{d-e\,x}-\sqrt{d}\right)}^2}+1\right)}^8}$$
$$+\frac{2\,b\,d^2\,\mathrm{atan}\left(\frac{\sqrt{d+e\,x}-\sqrt{d}}{\sqrt{d-e\,x}-\sqrt{d}}\right)}{e^3}+\frac{3\,c\,d^4\,\mathrm{atan}\left(\frac{\sqrt{d+e\,x}-\sqrt{d}}{\sqrt{d-e\,x}-\sqrt{d}}\right)}{2\,e^5}$$

input
```
int((a + b*x^2 + c*x^4)/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

output
```
((14*b*d^2*((d + e*x)^(1/2) - d^(1/2))^3)/((d - e*x)^(1/2) - d^(1/2))^3 -
(14*b*d^2*((d + e*x)^(1/2) - d^(1/2))^5)/((d - e*x)^(1/2) - d^(1/2))^5 + (
2*b*d^2*((d + e*x)^(1/2) - d^(1/2))^7)/((d - e*x)^(1/2) - d^(1/2))^7 - (2*
b*d^2*((d + e*x)^(1/2) - d^(1/2)))/((d - e*x)^(1/2) - d^(1/2)))/(e^3*(((d
+ e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 + 1)^4) - (4*a*ata
n((e*((d - e*x)^(1/2) - d^(1/2)))/((e^2)^(1/2)*((d + e*x)^(1/2) - d^(1/2))
)))/(e^2)^(1/2) - ((23*c*d^4*((d + e*x)^(1/2) - d^(1/2))^3)/(2*((d - e*x)^
(1/2) - d^(1/2))^3) - (333*c*d^4*((d + e*x)^(1/2) - d^(1/2))^5)/(2*((d - e
*x)^(1/2) - d^(1/2))^5) + (671*c*d^4*((d + e*x)^(1/2) - d^(1/2))^7)/(2*((d
 - e*x)^(1/2) - d^(1/2))^7) - (671*c*d^4*((d + e*x)^(1/2) - d^(1/2))^9)/(2
*((d - e*x)^(1/2) - d^(1/2))^9) + (333*c*d^4*((d + e*x)^(1/2) - d^(1/2))^1
1)/(2*((d - e*x)^(1/2) - d^(1/2))^11) - (23*c*d^4*((d + e*x)^(1/2) - d^(1/
2))^13)/(2*((d - e*x)^(1/2) - d^(1/2))^13) - (3*c*d^4*((d + e*x)^(1/2) - d
^(1/2))^15)/(2*((d - e*x)^(1/2) - d^(1/2))^15) + (3*c*d^4*((d + e*x)^(1/2)
 - d^(1/2)))/(2*((d - e*x)^(1/2) - d^(1/2))))/(e^5*(((d + e*x)^(1/2) - d^(
1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 + 1)^8) + (2*b*d^2*atan(((d + e*x)^(
1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/e^3 + (3*c*d^4*atan(((d + e*
x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/(2*e^5)
```

3.140. $\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

3.141 $\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx$

3.141.1 Optimal result

Integrand size = 35, antiderivative size = 102

$$\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{a\sqrt{d-ex}\sqrt{d+ex}}{d^2x} + \frac{cx(-d+ex)\sqrt{d+ex}}{2e^2\sqrt{d-ex}} - \frac{(cd^2+2be^2)\arctan\left(\frac{\sqrt{d-ex}}{\sqrt{d+ex}}\right)}{e^3}$$

output
```
-(2*b*e^2+c*d^2)*arctan((-e*x+d)^(1/2)/(e*x+d)^(1/2))/e^3+1/2*c*x*(e*x-d)*
(e*x+d)^(1/2)/e^2/(-e*x+d)^(1/2)-a*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2/x
```

3.141.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.84

$$\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{-\frac{e\sqrt{d-ex}\sqrt{d+ex}(2ae^2+cd^2x^2)}{d^2x} + 2(cd^2+2be^2)\arctan\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{2e^3}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(x^2*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
(-((e*Sqrt[d - e*x]*Sqrt[d + e*x]*(2*a*e^2 + c*d^2*x^2))/(d^2*x)) + 2*(c*d
^2 + 2*b*e^2)*ArcTan[Sqrt[d + e*x]/Sqrt[d - e*x]])/(2*e^3)
```

3.141.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.24, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1905, 1588, 25, 27, 299, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{ 1905}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^4+bx^2+a}{x^2\sqrt{d^2-e^2x^2}}dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 1588}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{\int -\frac{d^2(cx^2+b)}{\sqrt{d^2-e^2x^2}}dx}{d^2}-\frac{a\sqrt{d^2-e^2x^2}}{d^2x}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\int \frac{d^2(cx^2+b)}{\sqrt{d^2-e^2x^2}}dx}{d^2}-\frac{a\sqrt{d^2-e^2x^2}}{d^2x}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\int \frac{cx^2+b}{\sqrt{d^2-e^2x^2}}dx-\frac{a\sqrt{d^2-e^2x^2}}{d^2x}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{1}{2}\left(2b+\frac{cd^2}{e^2}\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{a\sqrt{d^2-e^2x^2}}{d^2x}-\frac{cx\sqrt{d^2-e^2x^2}}{2e^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{1}{2}\left(2b+\frac{cd^2}{e^2}\right)\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{a\sqrt{d^2-e^2x^2}}{d^2x}-\frac{cx\sqrt{d^2-e^2x^2}}{2e^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{a\sqrt{d^2-e^2x^2}}{d^2x}+\frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(2b+\frac{cd^2}{e^2}\right)}{2e}-\frac{cx\sqrt{d^2-e^2x^2}}{2e^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

input
```
Int[(a + b*x^2 + c*x^4)/(x^2*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-((a*Sqrt[d^2 - e^2*x^2])/(d^2*x)) - (c*x*Sqrt[d^2 -
 e^2*x^2])/(2*e^2) + ((2*b + (c*d^2)/e^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]
])/(2*e)))/(Sqrt[d - e*x]*Sqrt[d + e*x])
```

3.141.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

3.141.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.14

method	result
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\left(c\,d^{2} x^{2}+2 a\,e^{2}\right)}{2 e^{2} d^{2} x}+\frac{\left(2 b\,e^{2}+c\,d^{2}\right) \arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right) \sqrt{\left(ex+d\right)\left(-ex+d\right)}}{2 e^{2} \sqrt{e^{2}}\,\sqrt{ex+d}\,\sqrt{-ex+d}}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\left(\operatorname{csgn}\left(e\right) c\,d^{2} e\,x^{2} \sqrt{-e^{2} x^{2}+d^{2}}-2 \arctan\left(\frac{\operatorname{csgn}\left(e\right) e x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right) b\,d^{2} e^{2} x-\arctan\left(\frac{\operatorname{csgn}\left(e\right) e x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right) c\,d^{4} x+2\ \operatorname{csgn}\left(e\right) e^{3} \sqrt{-e^{2} \cdots}\right)}{2 d^{2} e^{3} \sqrt{-e^{2} x^{2}+d^{2}}\, x}$

input
```
int((c*x^4+b*x^2+a)/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/2*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(c*d^2*x^2+2*a*e^2)/e^2/d^2/x+1/2*(2*b*e
^2+c*d^2)/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))*((e*x
+d)*(-e*x+d))^(1/2)/(e*x+d)^(1/2)/(-e*x+d)^(1/2)
```

3.141.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{2\,(cd^4+2\,bd^2e^2)x\arctan\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{ex}\right)+(cd^2ex^2+2\,ae^3)\sqrt{ex+d}\sqrt{-ex+d}}{2\,d^2e^3x}$$

input
```
integrate((c*x^4+b*x^2+a)/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/2*(2*(c*d^4 + 2*b*d^2*e^2)*x*arctan((sqrt(e*x + d)*sqrt(-e*x + d) - d)/
(e*x)) + (c*d^2*e*x^2 + 2*a*e^3)*sqrt(e*x + d)*sqrt(-e*x + d))/(d^2*e^3*x)
```

3.141.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**4+b*x**2+a)/x**2/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
Timed out
```

3.141.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.92

$$\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{b\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}}+\frac{cd^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\sqrt{e^2}e^2}-\frac{\sqrt{-e^2x^2+d^2}cx}{2\,e^2}-\frac{\sqrt{-e^2x^2+d^2}a}{d^2x}$$

input
```
integrate((c*x^4+b*x^2+a)/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="m
axima")
```

output
```
b*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 1/2*c*d^2*arcsin(e^2*x/(d*sqrt(e
^2)))/(sqrt(e^2)*e^2) - 1/2*sqrt(-e^2*x^2 + d^2)*c*x/e^2 - sqrt(-e^2*x^2 +
 d^2)*a/(d^2*x)
```

3.141.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(88) = 176$.

Time = 0.37 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.34

$$\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\frac{8\,ae^4\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}-\frac{\sqrt{ex+d}}{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}\right)}{\left(\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}-\frac{\sqrt{ex+d}}{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}\right)^2-4\right)d^2}-\left(\pi+2\,\arctan\left(\frac{\sqrt{ex+d}\left(\frac{\left(\sqrt{2}\sqrt{d}-\sqrt{-ex+d}\right)^2}{ex+d}-1\right)}{2\left(\sqrt{2}\sqrt{d}-\sqrt{-ex+d}\right)}\right)\right)(cd^2+2\,be^2)+((ex}{2\,e^3}$$

input
```
integrate((c*x^4+b*x^2+a)/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="g
iac")
```

output
```
-1/2*(8*a*e^4*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x
 + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))/((((sqrt(2)*sqrt(d) - sqrt(-e*x
+ d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^2
- 4)*d^2) - (pi + 2*arctan(1/2*sqrt(e*x + d)*((sqrt(2)*sqrt(d) - sqrt(-e*x
 + d))^2/(e*x + d) - 1)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d))))*(c*d^2 + 2*b*
e^2) + ((e*x + d)*c - c*d)*sqrt(e*x + d)*sqrt(-e*x + d))/e^3
```

3.141.9 Mupad [B] (verification not implemented)

Time = 11.99 (sec) , antiderivative size = 306, normalized size of antiderivative = 3.00

$$\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\frac{14\,c\,d^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)^3}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^3}-\frac{14\,c\,d^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)^5}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^5}+\frac{2\,c\,d^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)^7}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^7}-\frac{2\,c\,d^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)}{\sqrt{d-e\,x}-\sqrt{d}}}{e^3\left(\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}+1\right)^4} - \frac{4\,b\,\mathrm{atan}\left(\frac{e\left(\sqrt{d-e\,x}-\sqrt{d}\right)}{\sqrt{e^2}\left(\sqrt{d+e\,x}-\sqrt{d}\right)}\right)}{\sqrt{e^2}}+\frac{2\,c\,d^2\,\mathrm{atan}\left(\frac{\sqrt{d+e\,x}-\sqrt{d}}{\sqrt{d-e\,x}-\sqrt{d}}\right)}{e^3}-\frac{\left(\frac{a}{d}+\frac{a\,e\,x}{d^2}\right)\sqrt{d-e\,x}}{x\,\sqrt{d+e\,x}}$$

input
```
int((a + b*x^2 + c*x^4)/(x^2*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

output
```
((14*c*d^2*((d + e*x)^(1/2) - d^(1/2))^3)/((d - e*x)^(1/2) - d^(1/2))^3 -
(14*c*d^2*((d + e*x)^(1/2) - d^(1/2))^5)/((d - e*x)^(1/2) - d^(1/2))^5 + (
2*c*d^2*((d + e*x)^(1/2) - d^(1/2))^7)/((d - e*x)^(1/2) - d^(1/2))^7 - (2*
c*d^2*((d + e*x)^(1/2) - d^(1/2)))/((d - e*x)^(1/2) - d^(1/2)))/(e^3*(((d
+ e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 + 1)^4) - (4*b*ata
n((e*((d - e*x)^(1/2) - d^(1/2)))/((e^2)^(1/2)*((d + e*x)^(1/2) - d^(1/2))
)))/(e^2)^(1/2) + (2*c*d^2*atan(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/
2) - d^(1/2))))/e^3 - ((a/d + (a*e*x)/d^2)*(d - e*x)^(1/2))/(x*(d + e*x)^(
1/2))
```

3.141. $\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx$

3.142 $\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx$

3.142.1 Optimal result

Integrand size = 35, antiderivative size = 157

$$\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{a(d^2-e^2x^2)}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{(3bd^2+2ae^2)(d^2-e^2x^2)}{3d^4x\sqrt{d-ex}\sqrt{d+ex}} + \frac{c\sqrt{d^2-e^2x^2}\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e\sqrt{d-ex}\sqrt{d+ex}}$$

output
```
-1/3*a*(-e^2*x^2+d^2)/d^2/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/3*(2*a*e^2+3*
b*d^2)*(-e^2*x^2+d^2)/d^4/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2)+c*arctan(e*x/(-e^
2*x^2+d^2)^(1/2))*(-e^2*x^2+d^2)^(1/2)/e/(-e*x+d)^(1/2)/(e*x+d)^(1/2)
```

3.142.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.52

$$\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-ex}\sqrt{d+ex}(3bd^2x^2+a(d^2+2e^2x^2))}{3d^4x^3} + \frac{2c\arctan\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{e}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(x^4*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
-1/3*(Sqrt[d - e*x]*Sqrt[d + e*x]*(3*b*d^2*x^2 + a*(d^2 + 2*e^2*x^2)))/(d^
4*x^3) + (2*c*ArcTan[Sqrt[d + e*x]/Sqrt[d - e*x]])/e
```

3.142.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.86, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {1905, 1588, 25, 358, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{1905}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^4+bx^2+a}{x^4\sqrt{d^2-e^2x^2}}dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1588}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{\int -\frac{3cx^2d^2+3bd^2+2ae^2}{x^2\sqrt{d^2-e^2x^2}}dx}{3d^2}-\frac{a\sqrt{d^2-e^2x^2}}{3d^2x^3}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\int \frac{3cx^2d^2+3bd^2+2ae^2}{x^2\sqrt{d^2-e^2x^2}}dx}{3d^2}-\frac{a\sqrt{d^2-e^2x^2}}{3d^2x^3}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{358}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{3cd^2\int \frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{\sqrt{d^2-e^2x^2}\left(\frac{2ae^2}{d^2}+3b\right)}{x}}{3d^2}-\frac{a\sqrt{d^2-e^2x^2}}{3d^2x^3}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{3cd^2\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}\left(\frac{2ae^2}{d^2}+3b\right)}{x}}{3d^2}-\frac{a\sqrt{d^2-e^2x^2}}{3d^2x^3}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{3cd^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}-\frac{\sqrt{d^2-e^2x^2}\left(\frac{2ae^2}{d^2}+3b\right)}{x}}{3d^2}-\frac{a\sqrt{d^2-e^2x^2}}{3d^2x^3}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

input
```
Int[(a + b*x^2 + c*x^4)/(x^4*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-1/3*(a*Sqrt[d^2 - e^2*x^2])/(d^2*x^3) + (-(((3*b +
(2*a*e^2)/d^2)*Sqrt[d^2 - e^2*x^2])/x) + (3*c*d^2*ArcTan[(e*x)/Sqrt[d^2 -
e^2*x^2]])/e)/(3*d^2)))/(Sqrt[d - e*x]*Sqrt[d + e*x])
```

3.142.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

3.142.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(2a\,e^2x^2+3b\,d^2x^2+a\,d^2)}{3d^4x^3}+\frac{c\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)\sqrt{(ex+d)(-ex+d)}}{\sqrt{e^2}\,\sqrt{ex+d}\,\sqrt{-ex+d}}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\left(-3\arctan\left(\frac{\mathrm{csgn}(e)ex}{\sqrt{-e^2x^2+d^2}}\right)c\,d^4x^3+2\,\mathrm{csgn}(e)e^3\sqrt{-e^2x^2+d^2}\,a\,x^2+3\,\mathrm{csgn}(e)e\sqrt{-e^2x^2+d^2}\,b\,d^2x^2+a\sqrt{-e^2x^2+d^2}}{3d^4\sqrt{-e^2x^2+d^2}\,x^3e}$

input
```
int((c*x^4+b*x^2+a)/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/3*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(2*a*e^2*x^2+3*b*d^2*x^2+a*d^2)/d^4/x^3+
c/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))*((e*x+d)*(-e*x+d)
)^(1/2)/(e*x+d)^(1/2)/(-e*x+d)^(1/2)
```

3.142.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.57

$$\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx$$
$$=-\frac{6\,cd^4x^3\arctan\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{ex}\right)+(ad^2e+(3\,bd^2e+2\,ae^3)x^2)\sqrt{ex+d}\sqrt{-ex+d}}{3\,d^4ex^3}$$

input
```
integrate((c*x^4+b*x^2+a)/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/3*(6*c*d^4*x^3*arctan((sqrt(e*x + d)*sqrt(-e*x + d) - d)/(e*x)) + (a*d^
2*e + (3*b*d^2*e + 2*a*e^3)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d))/(d^4*e*x^3)
```

3.142.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 18.20 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.64

$$\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{iae^3G_{6,6}^{5,3}\left(\begin{matrix} \frac{9}{4},\frac{11}{4},1 & \frac{5}{2},\frac{5}{2},3 \\ 2,\frac{9}{4},\frac{5}{2},\frac{11}{4},3 & 0 \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4} + \frac{ae^3G_{6,6}^{2,6}\left(\begin{matrix} \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2},1 & \\ \frac{7}{4},\frac{9}{4} & \frac{3}{2},2,2,0 \end{matrix}\middle| \frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4} + \frac{ibeG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2 \\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0 \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2} + \frac{beG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0 \end{matrix}\middle| \frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2} - \frac{icG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e} + \frac{cG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix}\middle| \frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e}$$

input
```
integrate((c*x**4+b*x**2+a)/x**4/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
I*a*e**3*meijerg(((9/4, 11/4, 1), (5/2, 5/2, 3)), ((2, 9/4, 5/2, 11/4, 3),
 (0,)), d**2/(e**2*x**2))/(4*pi**(3/2)*d**4) + a*e**3*meijerg(((3/2, 7/4,
2, 9/4, 5/2, 1), ()), ((7/4, 9/4), (3/2, 2, 2, 0)), d**2*exp_polar(-2*I*pi
)/(e**2*x**2))/(4*pi**(3/2)*d**4) + I*b*e*meijerg(((5/4, 7/4, 1), (3/2, 3/
2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)), d**2/(e**2*x**2))/(4*pi**(3/2)*d**2
) + b*e*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1,
 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*d**2) - I*c*meijer
g(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), d**2/(e*
*2*x**2))/(4*pi**(3/2)*e) + c*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1), ()),
((-1/4, 1/4), (-1/2, 0, 0, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi
**(3/2)*e)
```

3.142.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.60

$$\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{c\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} - \frac{\sqrt{-e^2x^2+d^2}b}{d^2x} - \frac{2\sqrt{-e^2x^2+d^2}ae^2}{3\,d^4x} - \frac{\sqrt{-e^2x^2+d^2}a}{3\,d^2x^3}$$

input
```
integrate((c*x^4+b*x^2+a)/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="m
axima")
```

output
```
c*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - sqrt(-e^2*x^2 + d^2)*b/(d^2*x) -
 2/3*sqrt(-e^2*x^2 + d^2)*a*e^2/(d^4*x) - 1/3*sqrt(-e^2*x^2 + d^2)*a/(d^2*
x^3)
```

3.142.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(139) = 278$.

Time = 0.41 (sec) , antiderivative size = 530, normalized size of antiderivative = 3.38

$$\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{3\left(\pi + 2\arctan\left(\frac{\sqrt{ex+d}\left(\frac{\left(\sqrt{2}\sqrt{d}-\sqrt{-ex+d}\right)^2}{ex+d}-1\right)}{2\left(\sqrt{2}\sqrt{d}-\sqrt{-ex+d}\right)}\right)\right)c - \frac{4\left(3\,bd^2e^2\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}-\frac{\sqrt{ex+d}}{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}\right)^5+3\,ae^4\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}-\right.\right.}{}}{}$$

input
```
integrate((c*x^4+b*x^2+a)/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="g
iac")
```

output
```
1/3*(3*(pi + 2*arctan(1/2*sqrt(e*x + d)*((sqrt(2)*sqrt(d) - sqrt(-e*x + d)
)^2/(e*x + d) - 1)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d))))*c - 4*(3*b*d^2*e^2
*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2
)*sqrt(d) - sqrt(-e*x + d)))^5 + 3*a*e^4*((sqrt(2)*sqrt(d) - sqrt(-e*x + d
))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^5 - 2
4*b*d^2*e^2*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x +
 d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^3 - 8*a*e^4*((sqrt(2)*sqrt(d) - sq
rt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x +
 d)))^3 + 48*b*d^2*e^2*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) -
 sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d))) + 48*a*e^4*((sqrt(2)*sq
rt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - s
qrt(-e*x + d))))/((((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqr
t(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^2 - 4)^3*d^4))/e
```

3.142.9 Mupad [B] (verification not implemented)

Time = 8.75 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{4\,c\,\mathrm{atan}\left(\frac{e\left(\sqrt{d-e\,x}-\sqrt{d}\right)}{\sqrt{e^2}\left(\sqrt{d+e\,x}-\sqrt{d}\right)}\right)}{\sqrt{e^2}} - \frac{\left(\frac{b}{d}+\frac{b\,e\,x}{d^2}\right)\sqrt{d-e\,x}}{x\,\sqrt{d+e\,x}} - \frac{\sqrt{d-e\,x}\left(\frac{a}{3\,d}+\frac{2\,a\,e^2\,x^2}{3\,d^3}+\frac{2\,a\,e^3\,x^3}{3\,d^4}+\frac{a\,e\,x}{3\,d^2}\right)}{x^3\,\sqrt{d+e\,x}}$$

input
```
int((a + b*x^2 + c*x^4)/(x^4*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

output
```
- (4*c*atan((e*((d - e*x)^(1/2) - d^(1/2)))/((e^2)^(1/2)*((d + e*x)^(1/2)
- d^(1/2)))))/(e^2)^(1/2) - ((b/d + (b*e*x)/d^2)*(d - e*x)^(1/2))/(x*(d +
e*x)^(1/2)) - ((d - e*x)^(1/2)*(a/(3*d) + (2*a*e^2*x^2)/(3*d^3) + (2*a*e^3
*x^3)/(3*d^4) + (a*e*x)/(3*d^2)))/(x^3*(d + e*x)^(1/2))
```

3.143 $\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx$

3.143.1 Optimal result

Integrand size = 35, antiderivative size = 160

$$\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{a(d^2-e^2x^2)}{5d^2x^5\sqrt{d-ex}\sqrt{d+ex}} - \frac{(5bd^2+4ae^2)(d^2-e^2x^2)}{15d^4x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{(15cd^4+10bd^2e^2+8ae^4)(d^2-e^2x^2)}{15d^6x\sqrt{d-ex}\sqrt{d+ex}}$$

output
```
-1/5*a*(-e^2*x^2+d^2)/d^2/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/15*(4*a*e^2+5
*b*d^2)*(-e^2*x^2+d^2)/d^4/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/15*(8*a*e^4+
10*b*d^2*e^2+15*c*d^4)*(-e^2*x^2+d^2)/d^6/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2)
```

3.143.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.54

$$\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-ex}\sqrt{d+ex}(15cd^4x^4+5bd^2x^2(d^2+2e^2x^2)+a(3d^4+4d^2e^2x^2+8e^4x^4))}{15d^6x^5}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(x^6*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
-1/15*(Sqrt[d - e*x]*Sqrt[d + e*x]*(15*c*d^4*x^4 + 5*b*d^2*x^2*(d^2 + 2*e^
2*x^2) + a*(3*d^4 + 4*d^2*e^2*x^2 + 8*e^4*x^4)))/(d^6*x^5)
```

3.143.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1905, 1588, 25, 359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{1905}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^4+bx^2+a}{x^6\sqrt{d^2-e^2x^2}}dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1588}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{\int -\frac{5cx^2d^2+5bd^2+4ae^2}{x^4\sqrt{d^2-e^2x^2}}dx}{5d^2}-\frac{a\sqrt{d^2-e^2x^2}}{5d^2x^5}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\int \frac{5cx^2d^2+5bd^2+4ae^2}{x^4\sqrt{d^2-e^2x^2}}dx}{5d^2}-\frac{a\sqrt{d^2-e^2x^2}}{5d^2x^5}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{359}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{\left(8ae^4+10bd^2e^2+15cd^4\right)\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx}{3d^2}-\frac{\sqrt{d^2-e^2x^2}\left(\frac{4ae^2}{d^2}+5b\right)}{3x^3}}{5d^2}-\frac{a\sqrt{d^2-e^2x^2}}{5d^2x^5}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{242}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{-\frac{\sqrt{d^2-e^2x^2}\left(8ae^4+10bd^2e^2+15cd^4\right)}{3d^4x}-\frac{\sqrt{d^2-e^2x^2}\left(\frac{4ae^2}{d^2}+5b\right)}{3x^3}}{5d^2}-\frac{a\sqrt{d^2-e^2x^2}}{5d^2x^5}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

input
```
Int[(a + b*x^2 + c*x^4)/(x^6*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-1/5*(a*Sqrt[d^2 - e^2*x^2])/(d^2*x^5) + (-1/3*((5*b
 + (4*a*e^2)/d^2)*Sqrt[d^2 - e^2*x^2])/x^3 - ((15*c*d^4 + 10*b*d^2*e^2 + 8
*a*e^4)*Sqrt[d^2 - e^2*x^2])/(3*d^4*x))/(5*d^2)))/(Sqrt[d - e*x]*Sqrt[d +
e*x])
```

3.143.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

3.143. $\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx$

3.143.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(8a\,e^4x^4+10b\,d^2e^2x^4+15c\,d^4x^4+4a\,d^2e^2x^2+5b\,d^4x^2+3a\,d^4)}{15x^5d^6}$	82
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(8a\,e^4x^4+10b\,d^2e^2x^4+15c\,d^4x^4+4a\,d^2e^2x^2+5b\,d^4x^2+3a\,d^4)}{15x^5d^6}$	82
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\mathrm{csgn}(e)^2\,(8a\,e^4x^4+10b\,d^2e^2x^4+15c\,d^4x^4+4a\,d^2e^2x^2+5b\,d^4x^2+3a\,d^4)}{15d^6x^5}$	86

input
```
int((c*x^4+b*x^2+a)/x^6/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/15*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(8*a*e^4*x^4+10*b*d^2*e^2*x^4+15*c*d^4*
x^4+4*a*d^2*e^2*x^2+5*b*d^4*x^2+3*a*d^4)/x^5/d^6
```

3.143.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.48

$$\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(3\,ad^4+(15\,cd^4+10\,bd^2e^2+8\,ae^4)x^4+(5\,bd^4+4\,ad^2e^2)x^2)\sqrt{ex+d}\sqrt{-ex+d}}{15\,d^6x^5}$$

input
```
integrate((c*x^4+b*x^2+a)/x^6/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/15*(3*a*d^4 + (15*c*d^4 + 10*b*d^2*e^2 + 8*a*e^4)*x^4 + (5*b*d^4 + 4*a*
d^2*e^2)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d)/(d^6*x^5)
```

3.143.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx=\text{Timed out}$$

input
```
integrate((c*x**4+b*x**2+a)/x**6/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
Timed out
```

3.143.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.92

$$\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx=-\frac{\sqrt{-e^2x^2+d^2}c}{d^2x}-\frac{2\sqrt{-e^2x^2+d^2}be^2}{3\,d^4x}-\frac{8\sqrt{-e^2x^2+d^2}ae^4}{15\,d^6x}-\frac{\sqrt{-e^2x^2+d^2}b}{3\,d^2x^3}-\frac{4\sqrt{-e^2x^2+d^2}ae^2}{15\,d^4x^3}-\frac{\sqrt{-e^2x^2+d^2}a}{5\,d^2x^5}$$

input
```
integrate((c*x^4+b*x^2+a)/x^6/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="m
axima")
```

output
```
-sqrt(-e^2*x^2 + d^2)*c/(d^2*x) - 2/3*sqrt(-e^2*x^2 + d^2)*b*e^2/(d^4*x) -
 8/15*sqrt(-e^2*x^2 + d^2)*a*e^4/(d^6*x) - 1/3*sqrt(-e^2*x^2 + d^2)*b/(d^2
*x^3) - 4/15*sqrt(-e^2*x^2 + d^2)*a*e^2/(d^4*x^3) - 1/5*sqrt(-e^2*x^2 + d^
2)*a/(d^2*x^5)
```

3.143.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1055 vs. 2(145) = 290.

Time = 0.53 (sec) , antiderivative size = 1055, normalized size of antiderivative = 6.59

$$\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx=$$

$$-\frac{4\left(15\,cd^4e^2\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}-\frac{\sqrt{ex+d}}{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}\right)^9+15\,bd^2e^4\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}-\frac{\sqrt{ex+d}}{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}\right)^9+15\,ae^6\left(\frac{\sqrt{2}\sqrt{}\right.}{}$$

input
```
integrate((c*x^4+b*x^2+a)/x^6/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="g
iac")
```

output
```
-4/15*(15*c*d^4*e^2*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sq
rt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^9 + 15*b*d^2*e^4*((sqrt(2)
*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d)
- sqrt(-e*x + d)))^9 + 15*a*e^6*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e
*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^9 - 240*c*d^4*
e^2*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqr
t(2)*sqrt(d) - sqrt(-e*x + d)))^7 - 160*b*d^2*e^4*((sqrt(2)*sqrt(d) - sqrt
(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d
)))^7 - 80*a*e^6*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(
e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^7 + 1440*c*d^4*e^2*((sqrt(2)*
sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) -
 sqrt(-e*x + d)))^5 + 800*b*d^2*e^4*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sq
rt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^5 + 928*a*
e^6*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqr
t(2)*sqrt(d) - sqrt(-e*x + d)))^5 - 3840*c*d^4*e^2*((sqrt(2)*sqrt(d) - sqr
t(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x +
d)))^3 - 2560*b*d^2*e^4*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d)
- sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^3 - 1280*a*e^6*((sqrt(
2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d
) - sqrt(-e*x + d)))^3 + 3840*c*d^4*e^2*((sqrt(2)*sqrt(d) - sqrt(-e*x +...
```

3.143.9 Mupad [B] (verification not implemented)

Time = 8.49 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.91

$$\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-e\,x}\left(\frac{a}{5\,d}+\frac{x^4\,\left(15\,c\,d^5+10\,b\,d^3\,e^2+8\,a\,d\,e^4\right)}{15\,d^6}+\frac{x^5\,\left(15\,c\,d^4\,e+10\,b\,d^2\,e^3+8\,a\,e^5\right)}{15\,d^6}+\frac{x^2\,\left(5\,b\,d^5+4\,a\,d^3\,e^2\right)}{15\,d^6}+\frac{x^3\,\left(5\,b\,d^4\,e+4\,a\,d^2\,e^3\right)}{15\,d^6}\right.}{x^5\,\sqrt{d+e\,x}}$$

input
```
int((a + b*x^2 + c*x^4)/(x^6*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

output
```
-((d - e*x)^(1/2)*(a/(5*d) + (x^4*(15*c*d^5 + 10*b*d^3*e^2 + 8*a*d*e^4))/(
15*d^6) + (x^5*(8*a*e^5 + 10*b*d^2*e^3 + 15*c*d^4*e))/(15*d^6) + (x^2*(5*b
*d^5 + 4*a*d^3*e^2))/(15*d^6) + (x^3*(4*a*d^2*e^3 + 5*b*d^4*e))/(15*d^6) +
 (a*e*x)/(5*d^2)))/(x^5*(d + e*x)^(1/2))
```

3.144 $\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx$

3.144.1 Optimal result

Integrand size = 35, antiderivative size = 226

$$\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{a(d^2-e^2x^2)}{7d^2x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{(7bd^2+6ae^2)\,(d^2-e^2x^2)}{35d^4x^5\sqrt{d-ex}\sqrt{d+ex}} - \frac{(35cd^4+28bd^2e^2+24ae^4)\,(d^2-e^2x^2)}{105d^6x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{2e^2(35cd^4+28bd^2e^2+24ae^4)\,(d^2-e^2x^2)}{105d^8x\sqrt{d-ex}\sqrt{d+ex}}$$

output
```
-1/7*a*(-e^2*x^2+d^2)/d^2/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/35*(6*a*e^2+7
*b*d^2)*(-e^2*x^2+d^2)/d^4/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/105*(24*a*e^
4+28*b*d^2*e^2+35*c*d^4)*(-e^2*x^2+d^2)/d^6/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/
2)-2/105*e^2*(24*a*e^4+28*b*d^2*e^2+35*c*d^4)*(-e^2*x^2+d^2)/d^8/x/(-e*x+d
)^(1/2)/(e*x+d)^(1/2)
```

3.144.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.55

$$\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-ex}\sqrt{d+ex}(35cd^4x^4(d^2+2e^2x^2)+7b(3d^6x^2+4d^4e^2x^4+8d^2e^4x^6)+3a(5d^6+6d^4e^2x^2+8d^2e^4x^4}{105d^8x^7}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(x^8*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
-1/105*(Sqrt[d - e*x]*Sqrt[d + e*x]*(35*c*d^4*x^4*(d^2 + 2*e^2*x^2) + 7*b*
(3*d^6*x^2 + 4*d^4*e^2*x^4 + 8*d^2*e^4*x^6) + 3*a*(5*d^6 + 6*d^4*e^2*x^2 +
 8*d^2*e^4*x^4 + 16*e^6*x^6)))/(d^8*x^7)
```

3.144.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {1905, 1588, 25, 359, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{1905}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^4+bx^2+a}{x^8\sqrt{d^2-e^2x^2}}dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1588}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{\int -\frac{7cx^2d^2+7bd^2+6ae^2}{x^6\sqrt{d^2-e^2x^2}}dx}{7d^2}-\frac{a\sqrt{d^2-e^2x^2}}{7d^2x^7}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\int \frac{7cx^2d^2+7bd^2+6ae^2}{x^6\sqrt{d^2-e^2x^2}}dx}{7d^2}-\frac{a\sqrt{d^2-e^2x^2}}{7d^2x^7}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{359}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{\left(24ae^4+28bd^2e^2+35cd^4\right)\int \frac{1}{x^4\sqrt{d^2-e^2x^2}}dx}{5d^2}-\frac{\sqrt{d^2-e^2x^2}\left(\frac{6ae^2}{d^2}+7b\right)}{5x^5}}{7d^2}-\frac{a\sqrt{d^2-e^2x^2}}{7d^2x^7}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{245}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{\left(24ae^4+28bd^2e^2+35cd^4\right)\left(\frac{2e^2\int\frac{1}{x^2\sqrt{d^2-e^2x^2}}dx}{3d^2}-\frac{\sqrt{d^2-e^2x^2}}{3d^2x^3}\right)}{5d^2}-\frac{\sqrt{d^2-e^2x^2}\left(\frac{6ae^2}{d^2}+7b\right)}{5x^5}}{7d^2}-\frac{a\sqrt{d^2-e^2x^2}}{7d^2x^7}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \ 242$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{\left(-\frac{\sqrt{d^2-e^2x^2}}{3d^2x^3}-\frac{2e^2\sqrt{d^2-e^2x^2}}{3d^4x}\right)\left(24ae^4+28bd^2e^2+35cd^4\right)}{5d^2}-\frac{\sqrt{d^2-e^2x^2}\left(\frac{6ae^2}{d^2}+7b\right)}{5x^5}}{7d^2}-\frac{a\sqrt{d^2-e^2x^2}}{7d^2x^7}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

input
```
Int[(a + b*x^2 + c*x^4)/(x^8*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-1/7*(a*Sqrt[d^2 - e^2*x^2])/(d^2*x^7) + (-1/5*((7*b
 + (6*a*e^2)/d^2)*Sqrt[d^2 - e^2*x^2])/x^5 + ((35*c*d^4 + 28*b*d^2*e^2 + 2
4*a*e^4)*(-1/3*Sqrt[d^2 - e^2*x^2]/(d^2*x^3) - (2*e^2*Sqrt[d^2 - e^2*x^2])
/(3*d^4*x)))/(5*d^2))/(7*d^2)))/(Sqrt[d - e*x]*Sqrt[d + e*x])
```

3.144.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

3.144.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.52

method	result
gosper	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(48a\,e^6x^6+56b\,d^2e^4x^6+70c\,d^4e^2x^6+24a\,d^2e^4x^4+28b\,d^4e^2x^4+35c\,d^6x^4+18a\,d^4e^2x^2+21b\,d^6x^2+15a\,d^6)}{105x^7d^8}$
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(48a\,e^6x^6+56b\,d^2e^4x^6+70c\,d^4e^2x^6+24a\,d^2e^4x^4+28b\,d^4e^2x^4+35c\,d^6x^4+18a\,d^4e^2x^2+21b\,d^6x^2+15a\,d^6)}{105x^7d^8}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\operatorname{csgn}(e)^2(48a\,e^6x^6+56b\,d^2e^4x^6+70c\,d^4e^2x^6+24a\,d^2e^4x^4+28b\,d^4e^2x^4+35c\,d^6x^4+18a\,d^4e^2x^2+21b\,d^6x^2+15a\,d^6}{105d^8x^7}$

input
```
int((c*x^4+b*x^2+a)/x^8/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/105*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(48*a*e^6*x^6+56*b*d^2*e^4*x^6+70*c*d^
4*e^2*x^6+24*a*d^2*e^4*x^4+28*b*d^4*e^2*x^4+35*c*d^6*x^4+18*a*d^4*e^2*x^2+
21*b*d^6*x^2+15*a*d^6)/x^7/d^8
```

3.144.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.49

$$\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(15\,ad^6+2\,(35\,cd^4e^2+28\,bd^2e^4+24\,ae^6)x^6+(35\,cd^6+28\,bd^4e^2+24\,ad^2e^4)x^4+3\,(7\,bd^6+6\,ad^4e^2)x^2)}{105\,d^8x^7}$$

input
```
integrate((c*x^4+b*x^2+a)/x^8/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/105*(15*a*d^6 + 2*(35*c*d^4*e^2 + 28*b*d^2*e^4 + 24*a*e^6)*x^6 + (35*c*
d^6 + 28*b*d^4*e^2 + 24*a*d^2*e^4)*x^4 + 3*(7*b*d^6 + 6*a*d^4*e^2)*x^2)*sq
rt(e*x + d)*sqrt(-e*x + d)/(d^8*x^7)
```

3.144.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**4+b*x**2+a)/x**8/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
Timed out
```

3.144.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx = &-\frac{2\sqrt{-e^2x^2+d^2}ce^2}{3\,d^4x}-\frac{8\sqrt{-e^2x^2+d^2}be^4}{15\,d^6x}-\frac{16\sqrt{-e^2x^2+d^2}ae^6}{35\,d^8x}\\&-\frac{\sqrt{-e^2x^2+d^2}c}{3\,d^2x^3}-\frac{4\sqrt{-e^2x^2+d^2}be^2}{15\,d^4x^3}-\frac{8\sqrt{-e^2x^2+d^2}ae^4}{35\,d^6x^3}\\&-\frac{\sqrt{-e^2x^2+d^2}b}{5\,d^2x^5}-\frac{6\sqrt{-e^2x^2+d^2}ae^2}{35\,d^4x^5}-\frac{\sqrt{-e^2x^2+d^2}a}{7\,d^2x^7}\end{aligned}$$

input
```
integrate((c*x^4+b*x^2+a)/x^8/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="m
axima")
```

output
```
-2/3*sqrt(-e^2*x^2 + d^2)*c*e^2/(d^4*x) - 8/15*sqrt(-e^2*x^2 + d^2)*b*e^4/
(d^6*x) - 16/35*sqrt(-e^2*x^2 + d^2)*a*e^6/(d^8*x) - 1/3*sqrt(-e^2*x^2 + d
^2)*c/(d^2*x^3) - 4/15*sqrt(-e^2*x^2 + d^2)*b*e^2/(d^4*x^3) - 8/35*sqrt(-e
^2*x^2 + d^2)*a*e^4/(d^6*x^3) - 1/5*sqrt(-e^2*x^2 + d^2)*b/(d^2*x^5) - 6/3
5*sqrt(-e^2*x^2 + d^2)*a*e^2/(d^4*x^5) - 1/7*sqrt(-e^2*x^2 + d^2)*a/(d^2*x
^7)
```

3.144.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1451 vs. $2(206) = 412$.

Time = 0.69 (sec) , antiderivative size = 1451, normalized size of antiderivative = 6.42

$$\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^4+b*x^2+a)/x^8/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="g
iac")
```

output
```
-4/105*(105*c*d^4*e^4*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) -
sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^13 + 105*b*d^2*e^6*((sqr
t(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt
(d) - sqrt(-e*x + d)))^13 + 105*a*e^8*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/
sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^13 - 196
0*c*d^4*e^4*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x +
 d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^11 - 1400*b*d^2*e^6*((sqrt(2)*sqrt
(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqr
t(-e*x + d)))^11 - 840*a*e^8*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x
+ d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^11 + 16240*c*d^4*
e^4*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqr
t(2)*sqrt(d) - sqrt(-e*x + d)))^9 + 12656*b*d^2*e^6*((sqrt(2)*sqrt(d) - sq
rt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x +
 d)))^9 + 14448*a*e^8*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) -
sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^9 - 80640*c*d^4*e^4*((sq
rt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqr
t(d) - sqrt(-e*x + d)))^7 - 69888*b*d^2*e^6*((sqrt(2)*sqrt(d) - sqrt(-e*x
+ d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^7
- 40704*a*e^8*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x
 + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^7 + 259840*c*d^4*e^4*((sqrt(2...
```

3.144.9 Mupad [B] (verification not implemented)

Time = 8.53 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.96

$$\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-e\,x}\left(\frac{a}{7\,d}+\frac{x^2\,\left(21\,b\,d^7+18\,a\,d^5\,e^2\right)}{105\,d^8}+\frac{x^4\,\left(35\,c\,d^7+28\,b\,d^5\,e^2+24\,a\,d^3\,e^4\right)}{105\,d^8}+\frac{x^7\,\left(70\,c\,d^4\,e^3+56\,b\,d^2\,e^5+48\,a\,e^7\right)}{105\,d^8}+\frac{x^3\,\left(21\,b\,d^6\,e+1\cdots\right.}{105\,d^8}\cdots\right.}{x^7\,\sqrt{d+e\,x}}$$

input
```
int((a + b*x^2 + c*x^4)/(x^8*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

output
```
-((d - e*x)^(1/2)*(a/(7*d) + (x^2*(21*b*d^7 + 18*a*d^5*e^2))/(105*d^8) + (
x^4*(35*c*d^7 + 24*a*d^3*e^4 + 28*b*d^5*e^2))/(105*d^8) + (x^7*(48*a*e^7 +
 56*b*d^2*e^5 + 70*c*d^4*e^3))/(105*d^8) + (x^3*(18*a*d^4*e^3 + 21*b*d^6*e
))/(105*d^8) + (x^5*(24*a*d^2*e^5 + 28*b*d^4*e^3 + 35*c*d^6*e))/(105*d^8)
+ (x^6*(56*b*d^3*e^4 + 70*c*d^5*e^2 + 48*a*d*e^6))/(105*d^8) + (a*e*x)/(7*
d^2)))/(x^7*(d + e*x)^(1/2))
```

3.145 $\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx$

3.145.1 Optimal result

Integrand size = 35, antiderivative size = 292

$$\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{a(d^2-e^2x^2)}{9d^2x^9\sqrt{d-ex}\sqrt{d+ex}} - \frac{(9bd^2+8ae^2)(d^2-e^2x^2)}{63d^4x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{(21cd^4+18bd^2e^2+16ae^4)(d^2-e^2x^2)}{105d^6x^5\sqrt{d-ex}\sqrt{d+ex}} - \frac{4e^2(21cd^4+18bd^2e^2+16ae^4)(d^2-e^2x^2)}{315d^8x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{8e^4(21cd^4+18bd^2e^2+16ae^4)(d^2-e^2x^2)}{315d^{10}x\sqrt{d-ex}\sqrt{d+ex}}$$

output
```
-1/9*a*(-e^2*x^2+d^2)/d^2/x^9/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/63*(8*a*e^2+9
*b*d^2)*(-e^2*x^2+d^2)/d^4/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/105*(16*a*e^
4+18*b*d^2*e^2+21*c*d^4)*(-e^2*x^2+d^2)/d^6/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/
2)-4/315*e^2*(16*a*e^4+18*b*d^2*e^2+21*c*d^4)*(-e^2*x^2+d^2)/d^8/x^3/(-e*x
+d)^(1/2)/(e*x+d)^(1/2)-8/315*e^4*(16*a*e^4+18*b*d^2*e^2+21*c*d^4)*(-e^2*x
^2+d^2)/d^10/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2)
```

3.145.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.54

$$\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx =$$
$$-\frac{\sqrt{d-ex}\sqrt{d+ex}(21cd^4x^4(3d^4+4d^2e^2x^2+8e^4x^4)+9b(5d^8x^2+6d^6e^2x^4+8d^4e^4x^6+16d^2e^6x^8)+a(35}{315d^{10}x^9}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(x^10*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
-1/315*(Sqrt[d - e*x]*Sqrt[d + e*x]*(21*c*d^4*x^4*(3*d^4 + 4*d^2*e^2*x^2 +
 8*e^4*x^4) + 9*b*(5*d^8*x^2 + 6*d^6*e^2*x^4 + 8*d^4*e^4*x^6 + 16*d^2*e^6*
x^8) + a*(35*d^8 + 40*d^6*e^2*x^2 + 48*d^4*e^4*x^4 + 64*d^2*e^6*x^6 + 128*
e^8*x^8)))/(d^10*x^9)
```

3.145.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.79, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1905, 1588, 25, 359, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \ 1905$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^4+bx^2+a}{x^{10}\sqrt{d^2-e^2x^2}}dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \ 1588$$

$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{\int -\frac{9cx^2d^2+9bd^2+8ae^2}{x^8\sqrt{d^2-e^2x^2}}dx}{9d^2}-\frac{a\sqrt{d^2-e^2x^2}}{9d^2x^9}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\int \frac{9cx^2d^2+9bd^2+8ae^2}{x^8\sqrt{d^2-e^2x^2}}dx}{9d^2}-\frac{a\sqrt{d^2-e^2x^2}}{9d^2x^9}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \ 359$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{3\left(16ae^4+18bd^2e^2+21cd^4\right)\int \frac{1}{x^6\sqrt{d^2-e^2x^2}}dx}{7d^2}-\frac{\sqrt{d^2-e^2x^2}\left(\frac{8ae^2}{d^2}+9b\right)}{7x^7}}{9d^2}-\frac{a\sqrt{d^2-e^2x^2}}{9d^2x^9}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \ 245$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{3\left(16ae^4+18bd^2e^2+21cd^4\right)\left(\frac{4e^2\int \frac{1}{x^4\sqrt{d^2-e^2x^2}}dx}{5d^2}-\frac{\sqrt{d^2-e^2x^2}}{5d^2x^5}\right)}{7d^2}-\frac{\sqrt{d^2-e^2x^2}\left(\frac{8ae^2}{d^2}+9b\right)}{7x^7}}{9d^2}-\frac{a\sqrt{d^2-e^2x^2}}{9d^2x^9}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \ 245$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{3\left(16ae^4+18bd^2e^2+21cd^4\right)\left(\frac{4e^2\left(\frac{2e^2\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx}{3d^2}-\frac{\sqrt{d^2-e^2x^2}}{3d^2x^3}\right)}{5d^2}-\frac{\sqrt{d^2-e^2x^2}}{5d^2x^5}\right)}{7d^2}-\frac{\sqrt{d^2-e^2x^2}\left(\frac{8ae^2}{d^2}+9b\right)}{7x^7}}{9d^2}-\frac{a\sqrt{d^2-e^2x^2}}{9d^2x^9}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \ 242$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{3\left(\frac{4e^2\left(-\frac{\sqrt{d^2-e^2x^2}}{3d^2x^3}-\frac{2e^2\sqrt{d^2-e^2x^2}}{3d^4x}\right)}{5d^2}-\frac{\sqrt{d^2-e^2x^2}}{5d^2x^5}\right)\left(16ae^4+18bd^2e^2+21cd^4\right)}{7d^2}-\frac{\sqrt{d^2-e^2x^2}\left(\frac{8ae^2}{d^2}+9b\right)}{7x^7}}{9d^2}-\frac{a\sqrt{d^2-e^2x^2}}{9d^2x^9}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

input
```
Int[(a + b*x^2 + c*x^4)/(x^10*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

3.145. $\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx$

output
```
(Sqrt[d^2 - e^2*x^2]*(-1/9*(a*Sqrt[d^2 - e^2*x^2])/(d^2*x^9) + (-1/7*((9*b
 + (8*a*e^2)/d^2)*Sqrt[d^2 - e^2*x^2])/x^7 + (3*(21*c*d^4 + 18*b*d^2*e^2 +
 16*a*e^4)*(-1/5*Sqrt[d^2 - e^2*x^2]/(d^2*x^5) + (4*e^2*(-1/3*Sqrt[d^2 - e
^2*x^2]/(d^2*x^3) - (2*e^2*Sqrt[d^2 - e^2*x^2])/(3*d^4*x)))/(5*d^2)))/(7*d
^2))/(9*d^2)))/(Sqrt[d - e*x]*Sqrt[d + e*x])
```

3.145.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

3.145.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.53

method	result
gosper	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(128a\,e^{8}x^{8}+144b\,d^{2}e^{6}x^{8}+168c\,d^{4}e^{4}x^{8}+64a\,d^{2}e^{6}x^{6}+72b\,d^{4}e^{4}x^{6}+84c\,d^{6}e^{2}x^{6}+48a\,d^{4}e^{4}x^{4}+54b\,d^{6}e^{2}x^{4}+63c\,d^{8}x}{315x^{9}d^{10}}$
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(128a\,e^{8}x^{8}+144b\,d^{2}e^{6}x^{8}+168c\,d^{4}e^{4}x^{8}+64a\,d^{2}e^{6}x^{6}+72b\,d^{4}e^{4}x^{6}+84c\,d^{6}e^{2}x^{6}+48a\,d^{4}e^{4}x^{4}+54b\,d^{6}e^{2}x^{4}+63c\,d^{8}x}{315x^{9}d^{10}}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\operatorname{csgn}(e)^{2}(128a\,e^{8}x^{8}+144b\,d^{2}e^{6}x^{8}+168c\,d^{4}e^{4}x^{8}+64a\,d^{2}e^{6}x^{6}+72b\,d^{4}e^{4}x^{6}+84c\,d^{6}e^{2}x^{6}+48a\,d^{4}e^{4}x^{4}+54b\,d^{6}e^{2}x^{4}}{315d^{10}x^{9}}$

input
```
int((c*x^4+b*x^2+a)/x^10/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
-1/315*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(128*a*e^8*x^8+144*b*d^2*e^6*x^8+168*c
*d^4*e^4*x^8+64*a*d^2*e^6*x^6+72*b*d^4*e^4*x^6+84*c*d^6*e^2*x^6+48*a*d^4*e
^4*x^4+54*b*d^6*e^2*x^4+63*c*d^8*x^4+40*a*d^6*e^2*x^2+45*b*d^8*x^2+35*a*d^
8)/x^9/d^10
```

3.145.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.49

$$\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx =$$
$$-\frac{(35\,ad^8+8\,(21\,cd^4e^4+18\,bd^2e^6+16\,ae^8)x^8+4\,(21\,cd^6e^2+18\,bd^4e^4+16\,ad^2e^6)x^6+3\,(21\,cd^8+18\,bd^6}{315\,d^{10}x^9}$$

input
```
integrate((c*x^4+b*x^2+a)/x^10/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="
fricas")
```

output
```
-1/315*(35*a*d^8 + 8*(21*c*d^4*e^4 + 18*b*d^2*e^6 + 16*a*e^8)*x^8 + 4*(21*
c*d^6*e^2 + 18*b*d^4*e^4 + 16*a*d^2*e^6)*x^6 + 3*(21*c*d^8 + 18*b*d^6*e^2
+ 16*a*d^4*e^4)*x^4 + 5*(9*b*d^8 + 8*a*d^6*e^2)*x^2)*sqrt(e*x + d)*sqrt(-e
*x + d)/(d^10*x^9)
```

3.145.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**4+b*x**2+a)/x**10/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
Timed out
```

3.145.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.04

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx =& -\frac{8\sqrt{-e^2x^2+d^2}ce^4}{15\,d^6x} - \frac{16\sqrt{-e^2x^2+d^2}be^6}{35\,d^8x} \\ & - \frac{128\sqrt{-e^2x^2+d^2}ae^8}{315\,d^{10}x} - \frac{4\sqrt{-e^2x^2+d^2}ce^2}{15\,d^4x^3} \\ & - \frac{8\sqrt{-e^2x^2+d^2}be^4}{35\,d^6x^3} - \frac{64\sqrt{-e^2x^2+d^2}ae^6}{315\,d^8x^3} \\ & - \frac{\sqrt{-e^2x^2+d^2}c}{5\,d^2x^5} - \frac{6\sqrt{-e^2x^2+d^2}be^2}{35\,d^4x^5} - \frac{16\sqrt{-e^2x^2+d^2}ae^4}{105\,d^6x^5} \\ & - \frac{\sqrt{-e^2x^2+d^2}b}{7\,d^2x^7} - \frac{8\sqrt{-e^2x^2+d^2}ae^2}{63\,d^4x^7} - \frac{\sqrt{-e^2x^2+d^2}a}{9\,d^2x^9}\end{aligned}$$

input
```
integrate((c*x^4+b*x^2+a)/x^10/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="
maxima")
```

output
```
-8/15*sqrt(-e^2*x^2 + d^2)*c*e^4/(d^6*x) - 16/35*sqrt(-e^2*x^2 + d^2)*b*e^
6/(d^8*x) - 128/315*sqrt(-e^2*x^2 + d^2)*a*e^8/(d^10*x) - 4/15*sqrt(-e^2*x
^2 + d^2)*c*e^2/(d^4*x^3) - 8/35*sqrt(-e^2*x^2 + d^2)*b*e^4/(d^6*x^3) - 64
/315*sqrt(-e^2*x^2 + d^2)*a*e^6/(d^8*x^3) - 1/5*sqrt(-e^2*x^2 + d^2)*c/(d^
2*x^5) - 6/35*sqrt(-e^2*x^2 + d^2)*b*e^2/(d^4*x^5) - 16/105*sqrt(-e^2*x^2
+ d^2)*a*e^4/(d^6*x^5) - 1/7*sqrt(-e^2*x^2 + d^2)*b/(d^2*x^7) - 8/63*sqrt(
-e^2*x^2 + d^2)*a*e^2/(d^4*x^7) - 1/9*sqrt(-e^2*x^2 + d^2)*a/(d^2*x^9)
```

3.145.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1847 vs. $2(267) = 534$.

Time = 0.94 (sec) , antiderivative size = 1847, normalized size of antiderivative = 6.33

$$\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^4+b*x^2+a)/x^10/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="
giac")
```

output
```
-4/315*(315*c*d^4*e^6*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) -
sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^17 + 315*b*d^2*e^8*((sqr
t(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt
(d) - sqrt(-e*x + d)))^17 + 315*a*e^10*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))
/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^17 - 67
20*c*d^4*e^6*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x
+ d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^15 - 5040*b*d^2*e^8*((sqrt(2)*sqr
t(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sq
rt(-e*x + d)))^15 - 3360*a*e^10*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e
*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^15 + 76608*c*d
^4*e^6*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(
sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^13 + 68544*b*d^2*e^8*((sqrt(2)*sqrt(d)
- sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e
*x + d)))^13 + 76608*a*e^10*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x +
 d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^13 - 580608*c*d^4*
e^6*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqr
t(2)*sqrt(d) - sqrt(-e*x + d)))^11 - 509184*b*d^2*e^8*((sqrt(2)*sqrt(d) -
sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x
 + d)))^11 - 327168*a*e^10*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x +
d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^11 + 2892288*c*d...
```

3.145.9 Mupad [B] (verification not implemented)

Time = 8.56 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.99

$$\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-e\,x}\,\left(\frac{a}{9\,d}+\frac{x^2\,\left(45\,b\,d^9+40\,a\,d^7\,e^2\right)}{315\,d^{10}}+\frac{x^6\,\left(84\,c\,d^7\,e^2+72\,b\,d^5\,e^4+64\,a\,d^3\,e^6\right)}{315\,d^{10}}+\frac{x^7\,\left(84\,c\,d^6\,e^3+72\,b\,d^4\,e^5+64\,a\,d^2\,e^7\right)}{315\,d^{10}}+\frac{x^4\,\left(63\,c\,d\right.}{}\right.}{}$$

input
```
int((a + b*x^2 + c*x^4)/(x^10*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

output
```
-((d - e*x)^(1/2)*(a/(9*d) + (x^2*(45*b*d^9 + 40*a*d^7*e^2))/(315*d^10) +
(x^6*(64*a*d^3*e^6 + 72*b*d^5*e^4 + 84*c*d^7*e^2))/(315*d^10) + (x^7*(64*a
*d^2*e^7 + 72*b*d^4*e^5 + 84*c*d^6*e^3))/(315*d^10) + (x^4*(63*c*d^9 + 48*
a*d^5*e^4 + 54*b*d^7*e^2))/(315*d^10) + (x^9*(128*a*e^9 + 144*b*d^2*e^7 +
168*c*d^4*e^5))/(315*d^10) + (x^3*(40*a*d^6*e^3 + 45*b*d^8*e))/(315*d^10)
+ (x^5*(48*a*d^4*e^5 + 54*b*d^6*e^3 + 63*c*d^8*e))/(315*d^10) + (x^8*(144*
b*d^3*e^6 + 168*c*d^5*e^4 + 128*a*d*e^8))/(315*d^10) + (a*e*x)/(9*d^2)))/(
x^9*(d + e*x)^(1/2))
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:  #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])  #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =" "
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```